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MONOPHONIC DISTANCE LAPLACIAN ENERGY OF SOME PRODUCT GRAPHS

Diana R

Research Scholar, Reg.Number:20113162092015, Department of Mathematics,Scott Christian College(Autonomous),Nagercoil-629 003,TamilNadu,India, (Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, TamilNadu,India)

Email: dianajino@gmail.com

Binu Selin T

Assistant Professor, Department of Mathematics,Scott Christian College(Autonomous),Nagercoil-629 003,TamilNadu,India,(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, TamilNadu,India)

Email: binuselin@gmail.com

Abstract

Let G be a simple connected graph of order n, v_i its vertex. Let $\delta_1^L, \delta_2^L \dots.., \delta_n^L$ be the eigenvalues of the distance Laplacian matrix D^L of G. We studied the Monophonic Distance Laplacian energy in [3] , $LE_M(G) = \sum_{i=1}^{n} \left|\mu_i^L - \frac{1}{n}\sum_{j=1}^{n} MT_G(v_j)\right|$, where $MT_G(v_j)$ is the j th row sum of Monophonic Distance matrix $M(G)$, and $\mu_1^L \leq \mu_2^L \dots.. \leq \mu_n^L$ be the eigen values of Monophonic Distance Laplacian matrix $M^L(G)$. In this paper we find the Monophonic Distance Laplacian energy of $K_n \oplus K_n$, $P_2 \otimes K_{n,n}$, $C_3 \bullet K_n$ graphs.

Keywords: Monophonic Distance Laplacian spectrum, Monophonic Distance Laplacian energy, product of graphs, lexicographic product, cartesian product, tensor product graphs.

AMS Subject Classification: 05C12, 05C50

1 Introduction

I.Gutman introduced the concept of graph energy in 1978 [5]. Consider the graph G, which has n vertices and m edges. Let $A = (a_{ij})$ be the adjacency matrix of the graph. The energy E(G) of G is defined as $E(G) = \sum_{i=1}^{n} |\lambda_i|$[2,5]. In the year 2008, I.Gutman and others introduced the concept of graph distance energy [4]. Jieshan Vang, Lihuayou and I.Gutman introduced the distance Laplacian energy of a graph in the year 2013[8]. The monophonic number of a graph was introduced by A.P.Santhakumaran and others in 2014[10]. Let G be a connected graph with vertex set V(G)={ $v_1, v_2 \dots. v_p$} and size q. The distance matrix or D-matrix, D of G is defined as D = (d_{ij}) ,where d_{ij} is the distance between the vertices v_i and v_j in G. The eigen values $\mu_1, \mu_2 \dots. \mu_p$ of the D-matrix of G are said to be the D-eigen values of G and to form the D-spectrum of G, denoted by $Spec_D(G)$. The D-energy $E_D(G) = \sum_{i=1}^{n} |\mu_i|$[4] . Let G be a connected graph with vertex set $v_1, v_2 \dots. v_n$. The Monophonic Distance matrix G is defined as

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$M = M(G) = (d_{m_{ij}})_{n\times n}$, where $d_{m_{ij}} = \begin{cases} d_m(v_i, v_j) & if \quad i \neq j \\ 0 & otherwise \end{cases}$

Here $d_m(v_i, v_j)$ is the Monophonic Distance of v_i to v_j. The connected graph G and its Monophonic Distance Laplacian matrix defined as $M^L(G) = MT(G) - M(G)$. The eigen values of Monophonic Distance $M^L(G)$ are denoted by $\mu_1^L, \mu_2^L \ldots\ldots, \mu_n^L$ and are said to be $M^L -$ eigen values of G and to form the M^L −spectrum of G, denoted by $Spec_{M^L}(G)$. Since the Monophonic Distance Laplacian matrix is symmetric and its eigen values are real, it can be ordered as $\mu_1^L \leq \mu_2^L \ldots\ldots \leq \mu_n^L$.

The Monophonic Distance Laplacian energy of a graph is defined as

$$LE_M(G) = \sum_{i=1}^{n} \left| \mu_i^L - \frac{1}{n} \sum_{j=1}^{n} MT_G(v_j) \right| ,$$

where $MT_G(v_j)$ is the j th row sum of Monophonic Distance matrix $M(G)$.

2 Results of Some Product Graphs

Definition 2.1

The tensor product of two graphs G_1 and G_2 is the graph denoted by $G_1 \oplus G_2$,with vertex set V($G_1 \oplus G_2$) = V(G_1) × V(G_2), and any two of its vertices (u_1, v_1) and (u_2, v_2) are adjacent, whenever u_1 is adjacent to u_2 in G_1 and v_1 is adjacent to v_2 in G_2.

Example 2.2

The Monophonic Distance Laplacian energy of $K_4 \oplus K_4$ is $LE_M\ (K_n \oplus K_n) = 126$.

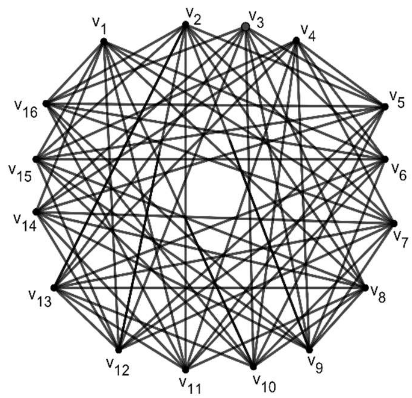

Fig:1

The Monophonic Distance matrix $M(K_n \oplus K_n)\ is$

$$\begin{pmatrix}
0 & 4 & 4 & 4 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 1\\
4 & 0 & 4 & 4 & 1 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 1 & 4 & 1 & 1\\
4 & 4 & 0 & 4 & 1 & 1 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 1 & 4 & 1\\
4 & 4 & 4 & 0 & 0 & 4 & 4 & 4 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 1\\
4 & 1 & 1 & 1 & 0 & 4 & 4 & 4 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 1\\
1 & 4 & 1 & 1 & 4 & 0 & 4 & 4 & 1 & 4 & 1 & 1 & 1 & 4 & 1 & 1\\
1 & 1 & 4 & 1 & 4 & 4 & 0 & 4 & 1 & 1 & 4 & 1 & 1 & 1 & 4 & 1\\
1 & 1 & 1 & 4 & 4 & 4 & 4 & 0 & 1 & 1 & 1 & 4 & 1 & 1 & 1 & 4\\
4 & 1 & 1 & 1 & 4 & 1 & 1 & 1 & 0 & 4 & 4 & 4 & 4 & 1 & 1 & 1\\
1 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 4 & 0 & 4 & 4 & 1 & 4 & 1 & 1\\
1 & 1 & 4 & 1 & 1 & 1 & 4 & 1 & 4 & 4 & 0 & 4 & 1 & 1 & 4 & 1\\
1 & 1 & 1 & 4 & 1 & 1 & 1 & 4 & 4 & 4 & 4 & 0 & 1 & 1 & 1 & 4\\
4 & 1 & 1 & 1 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 1 & 0 & 4 & 4 & 4\\
1 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 4 & 0 & 4 & 4\\
1 & 1 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 1 & 4 & 1 & 4 & 4 & 0 & 4\\
1 & 1 & 1 & 4 & 1 & 1 & 1 & 4 & 1 & 1 & 1 & 4 & 4 & 4 & 4 & 0
\end{pmatrix}$$

Theorem 2.3

Let K_n be the complete graph of order n^2 , then
$LE_M(K_n \oplus K_n\ ) = 14(n^2 - 2n + 1), for\ n \geq 3.$

Proof

The $M^L -$ spectrum of $K_n \oplus K_n$ is

$$Spec_{M^L}(\ K_n \oplus K_n) = \begin{pmatrix} 0 & n^2 + 3n & n^2 + 6n \\ 1 & 2n - 2 & n^2 - 2n + 1 \end{pmatrix}$$

The Monophonic Distance Laplacian energy of $K_n \oplus K_n$ is

$$LE_M\ (K_n \oplus K_n) = \sum_{i=1}^{n^2} \left|\mu_i^L - n^2 + 6n - 7\right|$$

$$= |0 - n^2 + 6n - 7| +$$

$$(2n - 2)|(n^2 + 3n) - (n^2 + 6n - 7)| +$$

$$n^2 - 2n + 1\ \ |(n^2 + 6n) - (n^2 + 6n - 7)|$$

$$LE_M(K_n \oplus K_n\ ) = \ 14[n^2 - 2n + 1]$$

Definition 2.4

The cartesian product of two graphs G and H denotes by G⊗H has the vertex set V(G)× V(H) and in which two vertices (g, h) and (g', h') are adjacent if and only if either $g = g'$ and h is adjacent to h'in H (or) $h = h'$ and g is adjacent to g' in G.[10]

Theorem 2.5

If $(P_2 \otimes K_{n,n})$ be the cartesian product of complete bipartite graph $K_{n,n}$ and path graph P_2
$LE_M(P_2 \otimes K_{n,n}) = LE_M(P_2\ ) + 5LE_M\left(K_{n,n}\ \right) + 6$, for $n \geq 3$.

proof

The $M^L -$ spectrum of $(P_2 \otimes K_{n,n})$ is

$$Spec_{M^L}(P_2 \otimes K_{n,n}) = \begin{pmatrix} 0 & 12n-8 & 10n & 14n-8 & 14n & 18n-8 \\ 1 & 1 & 1 & 2n-2 & 2n-2 & 1 \end{pmatrix}$$

The Monophonic Distance Laplacian energy of $(P_2 \otimes K_{n,n})$ is

$$LE_M(P_2 \otimes K_{n,n}) = \sum_{i=1}^{2} \left|\mu_i^L - (n-1)\right| + \sum_{i=1}^{2n} \left|\mu_i^L - (3n-2)\right| + 6$$
$$= 2 + 5[8(n-1)] + 6$$
$$= LE_M(2\,) + 5LE_M\left(K_{n,n}\,\right) + 6$$

Definition 2.6

The lexicographic product G•H of two graphs G and H has vertex set V(G)× V(H) and two vertices (u_1, v_1) and (u_2, v_2) are adjacent whenever $u_1u_2 \in E(G)$ or $u_1 = u_2$ and $v_1v_2 \in E(H)$.

Theorem 2.7

If $C_3 \bullet K_n$ be the lexicographic product of cycle graph C_3 and complete graph K_n.Then $LE_M(\,C_3 \bullet K_n)$= 2(3n-1).

Proof

Let $C_3 \bullet K_n$ be the lexicographic graph of order 3n.

Monophonic Distance matrix is written as

$$M(\,C_3 \bullet K_n) = \begin{pmatrix} J_n - I_n & J_n & J_n \\ J_n & J_n - I_n & J_n \\ J_n & J_n & J_n - I_n \end{pmatrix}$$, where J_n is the matrix with all entries 1's of order *n* and $I_n$is the identity matrix of order *n.*

Monophonic Distance Laplacian matrix is of the form

$$M^L(\,C_3 \bullet K_n) = \begin{pmatrix} J_n + (3n-2)I_n & -J_n & -J_n \\ -J_n & J_n + (3n-2)I_n & -J_n \\ -J_n & -J_n & J_n + (3n-2)I_n \end{pmatrix}$$

The $M^L -$ spectrum of $\mathrm{Spec}_{M^L}(\,C_3 \bullet K_n) = \begin{pmatrix} 0 & 3n \\ 1 & 3n-1 \end{pmatrix}$

Monophonic distance Laplacian energy is

$$LE_M\ (\,C_3 \bullet K_n) = \sum_{i=1}^{3} \left|\mu_i^L - 2\right| + 3\sum_{i=1}^{n} \left|\mu_i^L - (n-1)\right|$$

= 4+3(2n-2)

=2(3n-1).

Conclusion

In this paper we found Monophonic Distance Laplacian energy of cartesian, lexicographic and tensor product of graphs. We can extend the concepts to some new products of graphs also.

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