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N – GENERATED INTUITIONISTIC FUZZY TRANSLATION AND FUZZY MULTIPLICATION IN Z – ALGEBRA

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Abstract: In this paper, we evaluated the idea of N-Generated Fuzzy & Intuitionistic fuzzy Translation and Multiplication in Z-Algebra. For this, we examined some proportions of Fuzzy & Intuitionistic fuzzy Translation and Multiplication in Z-Algebra and we have achieved certain outcomes.

Keywords: Z-Algebras, Z–Ideal, Fuzzy Z–Ideal, Fuzzy Z-Sub algebra, n- generated Fuzzy Z-ideal, n- generated Fuzzy Z-subalgebra

Introduction: In 1965, the concept of fuzzy sets[8] inspired Zadeh L A. Several academics looked into the possibility of generalising a fuzzy subset. Fuzzy mathematics arose from the study of fuzzy subsets and their applications in many mathematical settings. Kyoung Ja Lee, Young Bae Jun and Myung Im Doh[4] established the concept of fuzzy translation and fuzzy multiplication of BCK-algebras in 2009. BCK-algebras and BCI-algebras are algebras derived from the BCK and BCI logics, respectively. Many new algebras have been produced since then. The Z-algebras, discovered by Chandramouleeswaran.M, Muralikrishna.P, Sujatha.K and Sabarinathan.S[2] in 2017, are one such class of algebra developed from propositional logic. Sowmiya.S and Jeyalakshmi.P[3] introduced the new concept on Fuzzy Z-Ideals in Z-Algebra in 2019. In 2014, Abu Ayub Ansari and Chandramouleeswaran.M [1] proposed the idea of fuzzy translation of fuzzy β -ideals of β-algebras. Priya and Ramachandran.T [5] introduced the new fuzzy translation and multiplication notation for PS-algebras in 2014. The notion of fuzzy translation and multiplication on B-algebras was introducedby Prasanna.A, Premkumar.M, and Ismail Mohideen.S[6] in 2018. In 2019, Prasanna.A, Premkumar.M, and Ismail Mohideen.S [7] proposed the new concept of fuzzy translation and fuzzy multiplication of BG-Algebra. In this paper, we explored the concepts of fuzzy translation and fuzzy multiplication on Z-algebras, as well as some of the Z-algebra's features.

This paper we deals with four sections. The first section covers the fundamental definitions of BCK&BCI – Algebras, Z-Algebra and theorems on Fuzzy Translation and Fuzzy Multiplication in Z-Algebra. The second section deals with the theorems in Intuitionistic Fuzzy Translation and Multiplication in Z-Algebra, as well as some additional definitions. In third section, we briefed about the definitions and theorems on N-Generated Fuzzy Translation and Multiplication in Z-Algebra. In fourth section, we covered the

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definitions and theorems on N-Generated Intuitionistic Fuzzy Translation and Multiplication in Z-Algebra.

Preliminaries

1. FUZZY TRANSLATION AND FUZZY MULTIPLICATION IN Z – ALGEBRA

Definition 1.1

Let $\mathbb{P}$ be a non-empty set with nullary operation 0 and let $*$ be a binary operation is known as BCK – Algebra. If it fulfils the following criteria for any $p, q, r \in \mathbb{P}$,

(A-1) $(p * q) * (p * r) \leq (r * q)$

(A-2) $p * (p * q) \leq q$

(A-3) $p \leq p$

(A-4) $p \leq q$ and $q \leq p \Rightarrow p = q$

(A-5) $0 \leq p \Rightarrow p = 0$, where $p \leq q$ is determined by $p * q = 0$

Definition 1.2

Let $\mathbb{P}$ be a non-empty set with nullary operation 0 and let $*$ be a binary operation is known as BCI – Algebra. If it fulfils (A-1), (A-2),(A-3), (A-4) and the following criteria for any $p, q, r \in \mathbb{P}$,

i. $p \leq 0 \Rightarrow p = 0$, where $p \leq q$ is determined by $p * q = 0$

Definition 1.3

Let $\mathbb{P}$ be a non-empty set with nullary operation 0 and $*$ be a binary operation is known as Z – Algebra. If it fulfils the following criteria for any $p, q \in \mathbb{P}$,

i. $p * 0 = 0$

ii. $0 * p = p$

iii. $p * p = p$

iv. $p * q = q * p$ when $p \neq 0$ & $q \neq 0$ for all $p, q \in \mathbb{P}$

Example 1.3.1

The following table is for the set $\mathbb{P} = \{0, a, b, c\}$

$*$	**0**	a	b	c
0	0	a	b	c
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Then $(\mathbb{P};*,0)$ is a Z – Algebra.

Example 1.3.2

The below table shows $\mathbb{P} = \{0, \mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}$ is a Z – algebra.

$*$	**0**	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{c}$
0	0	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{c}$
$\mathfrak{a}$	0	$\mathfrak{a}$	$\mathfrak{c}$	$\mathfrak{b}$
$\mathfrak{b}$	0	$\mathfrak{c}$	$\mathfrak{b}$	$\mathfrak{a}$

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c	0	b	a	c

Definition 1.4

Let ℚis as Z – sub-algebra of ℙ ⇔ ℚ be a non-empty subset of Z – Algebrafor all $p, q \in$ ℚ.

Definition 1.5

The subset of ℙ and ℝis known as Z – Ideal of the set ℙof Z -Algebra with the following conditions are fulfils,

i. $0 \in \mathbb{R}$

ii. $p * q \in \mathbb{R}$ and $q \in \mathbb{R} \Longrightarrow p \in \mathbb{R}$ for all $p, q \in \mathbb{R}$

Definition 1.6

Let ξ be a fuzzy set which belongs to ℙ. Then ξ is known as fuzzy Z – bi-ideal, if the following criteria is fulfilled. $\xi(p, \sigma, q) \geq \min\{\xi(p), \xi(q)\}$ for any $p, q, \sigma \in \mathbb{P}$.

Definition 1.7

If $\theta(p * q) \geq \min\{\theta(p), \theta(q)\} \forall\, p, q \in \mathbb{P}$ then θ is known as fuzzy sub-algebra on ℙ, on Z – Algebra.

Definition 1.8

Let θ be a fuzzy set in ℙ is termed as fuzzy BCK – Ideal of ℙ. If it satisfies the following criteria,

i. $\theta(0) \geq \theta(p)$

ii. $\theta(p) \geq \min\{\theta(p * q), \theta(q)\}$

Definition 1.9

Let a fuzzy set ξ in ℙ is known as fuzzy Z – Ideal of ℙ.If the following criteria are satisfied,

i. $\xi(0) \geq \xi(p)$

ii. $\xi(p) \geq \min\{\xi(p * q), \xi(q)\}$

Theorem 1.10

Letσ is a fuzzy subset of ℙ, then the $\gamma -$ Fuzzy Translation $\sigma_\gamma{}^U(p)$ of σbe a fuzzy Z – Ideal of ℙ , $\forall \gamma \in [0, U]$

Proof:

Let σ be a fuzzy Z – Ideal of ℙwith$\gamma \in [0, U]$

To Prove: σ is a fuzzy Z – Ideal of ℙ.

Claim (1):

$$\sigma_\gamma{}^U(0) \Leftrightarrow \sigma(0) + \gamma$$
$$\geq \sigma(p) + \gamma$$
$$= \sigma_\gamma{}^U(p)$$

Claim (2):

$$\sigma_\gamma{}^U(p) \Leftrightarrow \sigma(p) + \gamma$$
$$\geq \{\sigma(p * q) \wedge \sigma(q)\} + \gamma$$
$$\geq \{(\sigma(p * q) + \gamma) \wedge (\sigma(q) + \gamma)\}$$
$$\geq \{\sigma_\gamma{}^U(p * q) \wedge \sigma_\gamma{}^U(q)\}$$

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$\sigma_\gamma{}^U(\wp) \geq \{\sigma_\gamma{}^U(\wp * q) \wedge \sigma_\gamma{}^U(q)\}$

Hence σ is a fuzzy Z – Ideal of $\mathbb{P}$.

Theorem 1.11

Let a fuzzy Z – Ideal σ of $\mathbb{P}$ be a Z – algebra, then $\sigma_\gamma{}^U$ be the $\gamma -$ FT of σ is termed to be a fuzzy Z - sub-algebra of $\mathbb{P}$ and for any $\gamma \in [0, U]$.

Proof:

Let $\wp, q \in \mathbb{P}$

We have,

$\sigma_\gamma{}^U(\wp * q) = \sigma(\wp * q) + \gamma$

$\geq \{\sigma(q * (\wp * q)) \wedge \sigma(q)\} + \gamma$

$= \{\sigma(\wp * (q * q)) \wedge \sigma(q)\} + \gamma$

$\geq \{\sigma(\wp) \wedge \sigma(q)\} + \gamma$

$\geq \{(\sigma(\wp) + \gamma) \wedge (\sigma(q) + \gamma)\}$

$= \{\sigma_\gamma{}^U(\wp) \wedge \sigma_\gamma{}^U(q)\}$

Therefore $\sigma_\gamma{}^U$ is a fuzzy Z – algebra of $\mathbb{P}$.

Theorem 1.12

Let $\sigma_\gamma{}^U$ of σ be a $\gamma -$ Fuzzy Translation then for all $\gamma \in [0, U]$. Then σ is a fuzzy Z – sub-algebra of $\mathbb{P}$.

Proof:

We assume that $\sigma_\gamma{}^U$ of σ be a fuzzy Z – Ideal of $\mathbb{P}$. Then we have

$\sigma(\wp * q) + \gamma = \sigma_\gamma{}^U(\wp * q)$

$\geq \{\sigma_\gamma{}^U(\wp * (\wp * q)) \wedge \sigma_\gamma{}^U(q)\}$

$= \{\sigma_\gamma{}^U((\wp * \wp) * q) \wedge \sigma_\gamma{}^U(q)\}$

$\geq \{\sigma_\gamma{}^U(\wp) \wedge \sigma_\gamma{}^U(q)\}$

$= \{(\sigma(\wp) + \gamma) \wedge (\sigma(q) + \gamma)\}$

$= \{\sigma(\wp) \wedge \sigma(q)\} + \gamma$

Which implies $\sigma(\wp * q) \geq \{\sigma(\wp) \wedge \sigma(q)\}$

Therefore σ is fuzzy Z – sub-algebra of $\mathbb{P}$.

Theorem 1.13

Let afuzzy subsetσ of $\mathbb{P}$ such that the γ – Fuzzy Multiplication $\sigma_\gamma{}^V(\wp)$ of σ is said to be fuzzy Z – Ideal of $\mathbb{P}$ and $\gamma \in [0,1]$. Then σbe a fuzzy Z – Ideal of $\mathbb{P}$.

Proof:

Let $\sigma_\gamma{}^V$ is a fuzzy Z – Ideal of $\mathbb{P}$ for all $\gamma \in [0,1]$.Let $\wp, q \in \mathbb{P}$. We now have,

i. $\gamma\sigma(\wp) = \sigma_\gamma{}^V(0)$

$\geq \sigma_\gamma{}^V(\wp)$

$= \gamma\sigma(\wp)$ which implies $\sigma(0) \geq \sigma(\wp)$

ii. $\gamma\sigma(\wp) = \sigma_\gamma{}^V(\wp)$

$\geq \{\sigma_\gamma{}^V(\wp * q) \wedge \sigma_\gamma{}^V(q)\}$

$= \{(\gamma\sigma(\wp * q)) \wedge (\gamma\sigma(q))\}$

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$= \gamma\{\sigma(p * q) \wedge \sigma(q)\}$
which implies $\sigma(p) \geq \{\sigma(p * q) \wedge \sigma(q)\}$
Therefore σ is a fuzzy Z-Ideal of $\mathbb{P}$.

Theorem 1.14

A σ is a fuzzy Z – ideal of $\mathbb{P}$. Then the γ - FM$\sigma_\gamma{}^V(p)$ of σ is a fuzzy Z – Ideal of $\mathbb{P}$, for all γ∈ [0,1].

Proof:

If σ is a fuzzy Z – Ideal of $\mathbb{P}$ and γ∈ [0,1].
We now have
i. $\sigma_\gamma{}^V(0) = \gamma\sigma(p)$
$\geq \gamma\sigma(p)$
$= \sigma_\gamma{}^V(p)$
$\sigma_\gamma{}^V(0) \geq \sigma_\gamma{}^V(p)$
ii. $\sigma_\gamma{}^V(p) = \gamma\sigma(p)$
$\geq \gamma\{\sigma(p * q) \wedge \sigma(q)\}$
$= \gamma\{\sigma(p * q) \wedge \sigma(q)\}$
$= \{(\gamma\sigma(p * q)) \wedge (\gamma\sigma(q))\}$
which implies $\sigma_\gamma{}^V(p) \geq \{\sigma_\gamma{}^V(p * q) \wedge \sigma_\gamma{}^V(q)\}$
Therefore, $\sigma_\gamma{}^V$ of σ is a fuzzy Z – Ideal of $\mathbb{P}$ and then for all $p, q \in [0,1]$

Theorem 1.15

If γ - Fuzzy Multiplication $\sigma_\gamma{}^V(p)$ of a fuzzy Z – sub-algebra σ of $\mathbb{P}$. Then σ be a fuzzy Z – sub-algebra of $\mathbb{P}$ and let γ∈ [0,1].

Proof:

Let $p, q \in \mathbb{P}$&$\forall\gamma \in [0,1]$

$$\sigma(p * q) \geq \{\sigma(p) \wedge \sigma(q)\}$$

We have, $\sigma_\gamma{}^V(p * q) = \gamma\sigma(p * q)$

$$\geq \gamma\{\sigma(p) \wedge \sigma(q)\}$$
$$\geq \gamma\sigma(p) \wedge \gamma\sigma(q)$$

which implies $\sigma_\gamma{}^V(p * q) \geq \sigma_\gamma{}^V(p) \wedge \sigma_\gamma{}^V(q)$
Hence, $\sigma_\gamma{}^V$ is fuzzy Z – sub-algebra of $\mathbb{P}$.

Theorem 1.16

Let a fuzzy Z – sub-algebra of $\mathbb{P}$ be σ and if γ - FM $\sigma_\gamma{}^V(p)$ of σ, then for every fuzzy subset σ of $\mathbb{P}$ and for any γ∈ [0,1].

Proof:

Let we assume $\sigma_\gamma{}^V(p)$ of σ is fuzzy Z – sub-algebra.
We know that γ∈ [0,1].
$\gamma\sigma(p * q) = \sigma_\gamma{}^V(p * q)$
$\geq \sigma_\gamma{}^V(p) \wedge \sigma_\gamma{}^V(q)$
$= \{\gamma\sigma(p) \wedge \gamma\sigma(q)\}$

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$= \gamma\{\sigma(p) \wedge \sigma(q)\}$

$$\Rightarrow \sigma(p * q) \geq \sigma(p) \wedge \sigma(q)$$

$\therefore \sigma$ is fuzzy Z – sub-algebra of $\mathbb{P}$.

2. INTUITIONISTIC FUZZY TRANSLATION AND FUZZY MULTIPLICATION IN Z - ALGEBRA

In a non-empty set $\mathbb{P}$, an intuitionistic fuzzy set $\mathbb{P}$ is an object with, $\mathbb{P} = \{(p, \varphi_{\mathbb{A}}(p), \delta_{\mathbb{A}}(p) \mid p \in \mathbb{P}\}$ where the parameters $\varphi_{\mathbb{A}} : \mathbb{P} \longrightarrow [0,1]$ and $\delta_{\mathbb{A}}:\mathbb{P} \longrightarrow [0,1]$ represents the degree of membership and non-membership correspondingly, $0 \leq \varphi_{\mathbb{A}}(p), \delta_{\mathbb{A}}(p) \leq 1$ for all $p \in \mathbb{P}$.

An ordered pair $(\varphi_{\mathbb{A}}, \delta_{\mathbb{A}})$ in $I^X \times I^X$ defined to an Intuitionistic fuzzy set $\mathbb{A} = \{(p, \varphi_{\mathbb{A}}(p), \delta_{\mathbb{A}}(p) \mid p \in \mathbb{P}\}$ in $\mathbb{P}$. For the Intuitionistic Fuzzy Set $\mathbb{A} = \{(p, \varphi_{\mathbb{A}}(p), \delta_{\mathbb{A}}(p) \mid p \in \mathbb{P}\}$ we will use the symbol $\mathbb{A} = (\varphi_{\mathbb{A}}, \delta_{\mathbb{A}})$.

Definition 2.1

Let $\mathbb{A} = (\mathbb{P}, \varphi_{\mathbb{A}}, \delta_{\mathbb{A}})$ in $\mathbb{P}$ be an intuitionistic fuzzy set on Z - Algebra. If it fulfils the following criteria,

i. $\varphi_{\mathbb{A}}(0) \geq \varphi_{\mathbb{A}}(p)$
ii. $\delta_{\mathbb{A}}(0) \leq \delta_{\mathbb{A}}(p)$
iii. $\varphi_{\mathbb{A}}(p) \geq \min\{\varphi_{\mathbb{A}}(p * q), \varphi_{\mathbb{A}}(q)\}$
iv. $\delta_{\mathbb{A}}(p) \leq \max\{\delta_{\mathbb{A}}(p * q), \delta_{\mathbb{A}}(q)\}$

for all $p, q \in \mathbb{P}$

Then it is known as intuitionistic fuzzy Z – Ideal of $\mathbb{P}$.

Definition 2.2

Let the set $\mathbb{A} = (\varphi_{\mathbb{A}}, \delta_{\mathbb{A}})$ is known as intuitionistic fuzzy sub-algebra on $\mathbb{P}$.

$\varphi_{\mathbb{A}}(p * q) \geq \min\{\varphi_{\mathbb{A}}(p), \varphi_{\mathbb{A}}(q)\}$

$\delta_{\mathbb{A}}(p * q) \leq \max\{\delta_{\mathbb{A}}(p), \delta_{\mathbb{A}}(q)\}$

for all $p, q \in \mathbb{P}$.

where $\mathbb{A} = (\varphi_{\mathbb{A}}, \delta_{\mathbb{A}})$ in $\mathbb{P}$ be an intuitionistic fuzzy set on Z - Algebra.

Theorem 2.3

If $\mathbb{A} = (\varphi_{\mathbb{A}}, \delta_{\mathbb{A}})$ be an intuitionistic fuzzy subset of $\mathbb{P}$, then γ – Intuitionistic Fuzzy Translation $\mathbb{A}_{\gamma}^{U}(p) = ((\varphi_{\mathbb{A}})_{\gamma}^{U}(p), (\delta_{\mathbb{A}})_{\gamma}^{U}(p))$ of $\mathbb{A}$ is an intuitionistic fuzzy Z – Ideal of $\mathbb{P}$, $\forall \gamma \in [0, U]$

Proof:

Let $\mathbb{A}$ be an intuitionistic fuzzysubset of $\mathbb{P}$ with let $\gamma \in [0, U]$

i. $(\varphi_{\mathbb{A}})_{\gamma}^{U}(0) \Leftrightarrow \varphi_{\mathbb{A}}(0) + \gamma \geq \varphi_{\mathbb{A}}(p) + \gamma$

$= (\varphi_{\mathbb{A}})_{\gamma}^{U}(p)$

ii. $(\delta_{\mathbb{A}})_{\gamma}^{U}(0) \Leftrightarrow \delta_{\mathbb{A}}(0) - \gamma$

$$\leq \delta_{\mathbb{A}}(p) - \gamma$$

$= (\delta_{\mathbb{A}})_{\gamma}^{U}(p)$

iii. $(\varphi_{\mathbb{A}})_{\gamma}^{U}(p) \Leftrightarrow \varphi_{\mathbb{A}}(p) + \gamma$

$\geq \min\{\varphi_{\mathbb{A}}(p * q), \varphi_{\mathbb{A}}(q)\} + \gamma$

$= \min\{(\varphi_{\mathbb{A}}(p * q) + \gamma), (\varphi_{\mathbb{A}}(q) + \gamma)\}$

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$(\varphi_{\mathbb{A}})^{U}_{\gamma}(p) \geq \min\{(\varphi_{\mathbb{A}})^{U}_{\gamma}(p * q), (\varphi_{\mathbb{A}})^{U}_{\gamma}(q)\}$

iv. $(\delta_{\mathbb{A}})^{U}_{\gamma}(p) \Leftrightarrow \delta_{\mathbb{A}}(p) - \gamma$

$\leq \max\{\delta_{\mathbb{A}}(p * q), \delta_{\mathbb{A}}(q)\} - \gamma$

$= \max\{(\delta_{\mathbb{A}}(p * q) - \gamma), (\delta_{\mathbb{A}}(q) - \gamma)\}$

$(\delta_{\mathbb{A}})^{U}_{\gamma}(p) \leq \max\{(\delta_{\mathbb{A}})^{U}_{\gamma}(p * q), (\delta_{\mathbb{A}})^{U}_{\gamma}(q)\}$

Hence $\mathbb{A} = (\varphi_{\mathbb{A}}, \delta_{\mathbb{A}})$ is an intuitionistic fuzzy Z – Ideal of $\mathbb{P}$.

Theorem 2.4

If an intuitionistic fuzzy Z – Ideal of $\mathbb{P}$ is $\mathbb{A}$, then $\mathbb{A}^{U}_{\gamma} = ((\varphi_{\mathbb{A}})^{U}_{\gamma}, (\delta_{\mathbb{A}})^{U}_{\gamma})$ be γ –Intuitionistic FT of $\mathbb{A}$ is Intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$, then for any $\gamma \in [0, U]$.

Proof:

We have, $(\varphi_{\mathbb{A}})^{U}_{\gamma}(p * q) = \{(\varphi_{\mathbb{A}}(p * q) + \gamma)$

$$\begin{aligned}
&\geq \min\{\varphi_{\mathbb{A}}(q * (p * q)), \varphi_{\mathbb{A}}(q)\} + \gamma \\
&= \min\{\varphi_{\mathbb{A}}(p * (q * q)), \varphi_{\mathbb{A}}(q)\} + \gamma \\
&\geq \min\{\varphi_{\mathbb{A}}(p), \varphi_{\mathbb{A}}(q)\} + \gamma \\
&\geq \min\{(\varphi_{\mathbb{A}}(p) + \gamma), (\varphi_{\mathbb{A}}(q) + \gamma)\} \\
&= \min\{(\varphi_{\mathbb{A}})^{U}_{\gamma}(p), (\varphi_{\mathbb{A}})^{U}_{\gamma}(q)\}
\end{aligned}$$

$(\delta_{\mathbb{A}})^{U}_{\gamma}(p * q) = \{(\delta_{\mathbb{A}}(p * q) - \gamma)$

$$\begin{aligned}
&\leq \max\{\delta_{\mathbb{A}}(q * (p * q)), \delta_{\mathbb{A}}(q)\} - \gamma \\
&= \max\{\delta_{\mathbb{A}}(p * (q * q)), \delta_{\mathbb{A}}(q)\} - \gamma \\
&\leq \max\{\delta_{\mathbb{A}}(p), \delta_{\mathbb{A}}(q)\} - \gamma \\
&\leq \max\{(\delta_{\mathbb{A}}(p) - \gamma), (\delta_{\mathbb{A}}(q) - \gamma)\} \\
&= \max\{(\delta_{\mathbb{A}})^{U}_{\gamma}(p), (\delta_{\mathbb{A}})^{U}_{\gamma}(q)\}
\end{aligned}$$

Therefore $\mathbb{A}^{U}_{\gamma} = ((\varphi_{\mathbb{A}})^{U}_{\gamma}, (\delta_{\mathbb{A}})^{U}_{\gamma})$ is Intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$.

Theorem 2.5

Let $\mathbb{A} = (\varphi_{\mathbb{A}}, \delta_{\mathbb{A}})$ be an Intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$ and then let $\mathbb{A}^{U}_{\gamma}$ of a γ – Intuitionistic Fuzzy Translation $\mathbb{A}$, $\forall \gamma \in [0, U]$. Then $\mathbb{A}$ be an Intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$.

Proof:

$\varphi_{\mathbb{A}}(p * q) + \gamma = (\varphi_{\mathbb{A}})^{U}_{\gamma}(p * q)$

$$\begin{aligned}
&\geq \min\left\{(\varphi_{\mathbb{A}})^{U}_{\gamma}(p * (p * q)), (\varphi_{\mathbb{A}})^{U}_{\gamma}(q)\right\} \\
&= \min\left\{(\varphi_{\mathbb{A}})^{U}_{\gamma}((p * p) * q), (\varphi_{\mathbb{A}})^{U}_{\gamma}(q)\right\} \\
&\geq \min\left\{(\varphi_{\mathbb{A}})^{U}_{\gamma}(p), (\varphi_{\mathbb{A}})^{U}_{\gamma}(q)\right\} \\
&\geq \min\{(\varphi_{\mathbb{A}}(p) + \gamma), (\varphi_{\mathbb{A}}(q) + \gamma)\} \\
&= \min\{\varphi_{\mathbb{A}}(p), \varphi_{\mathbb{A}}(q)\} + \gamma
\end{aligned}$$

$$\varphi_{\mathbb{A}}(p * q) \geq \min\{\varphi_{\mathbb{A}}(p), \varphi_{\mathbb{A}}(q)\}$$

$\delta_{\mathbb{A}}(p * q) - \gamma = (\delta_{\mathbb{A}})^{U}_{\gamma}(p * q)$

$$\begin{aligned}
&\leq \max\left\{(\delta_{\mathbb{A}})^{U}_{\gamma}(p * (p * q)), (\delta_{\mathbb{A}})^{U}_{\gamma}(q)\right\} \\
&= \max\left\{(\delta_{\mathbb{A}})^{U}_{\gamma}((p * p) * q), (\delta_{\mathbb{A}})^{U}_{\gamma}(q)\right\} \\
&\leq \max\left\{(\delta_{\mathbb{A}})^{U}_{\gamma}(p), (\delta_{\mathbb{A}})^{U}_{\gamma}(q)\right\}
\end{aligned}$$

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$$\leq \max\{(\delta_{\mathbb{A}}(p) - \gamma), (\delta_{\mathbb{A}}(q) - \gamma)\}$$
$$= \max\{\delta_{\mathbb{A}}(p), \delta_{\mathbb{A}}(q)\} - \gamma$$
$$\delta_{\mathbb{A}}(p * q) \leq \max\{\delta_{\mathbb{A}}(p), \delta_{\mathbb{A}}(q)\}$$

Hence $\mathbb{A} = (\varphi_{\mathbb{A}}, \delta_{\mathbb{A}})$ is an Intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$.

Theorem 2.6

Let $\mathbb{A}$ is an intuitionistic fuzzy subset of $\mathbb{P}$ such that the Intuitionistic Fuzzy Multiplication - γ be$\mathbb{A}_{\gamma}^{V}(p)$. Then$\mathbb{A}$be an intuitionistic fuzzy Z – Ideal of $\mathbb{P}$, for all $\gamma \in [0,1]$.

Proof:

Let $\mathbb{A}_{\gamma}^{V}$ is an intuitionistic fuzzy Z – Ideal of $\mathbb{P}$ for all $\gamma \in [0,1]$.

Let $p, q \in \mathbb{P}$

i. $\gamma\varphi_{\mathbb{A}}(p) = (\varphi_{\mathbb{A}})_{\gamma}^{V}(0)$

$\geq (\varphi_{\mathbb{A}})_{\gamma}^{V}(p)$

$= \gamma\varphi_{\mathbb{A}}(p)$ which implies $\varphi_{\mathbb{A}}(0) \geq \varphi_{\mathbb{A}}(p)$

ii. $\gamma\delta_{\mathbb{A}}(p) = (\delta_{\mathbb{A}})_{\gamma}^{V}(0)$

$\leq (\delta_{\mathbb{A}})_{\gamma}^{V}(p)$

$= \gamma\delta_{\mathbb{A}}(p)$ which implies $\delta_{\mathbb{A}}(0) \leq \delta_{\mathbb{A}}(p)$

iii. $\gamma\varphi_{\mathbb{A}}(p) = (\varphi_{\mathbb{A}})_{\gamma}^{V}(p)$

$\geq \min\{(\varphi_{\mathbb{A}})_{\gamma}^{V}(p * q), (\varphi_{\mathbb{A}})_{\gamma}^{V}(q)\}$

$= \min\{(\gamma\varphi_{\mathbb{A}}(p * q)), (\gamma\varphi_{\mathbb{A}}(q))\}$

$= \min \gamma\{\varphi_{\mathbb{A}}(p * q), \varphi_{\mathbb{A}}(q)\}$

which implies $\varphi_{\mathbb{A}}(p) \geq \min\{(p * q), \varphi_{\mathbb{A}}(q)\}$

iv. $\gamma\delta_{\mathbb{A}}(p) = (\delta_{\mathbb{A}})_{\gamma}^{V}(p)$

$\leq \max\{(\delta_{\mathbb{A}})_{\gamma}^{V}(p * q), (\delta_{\mathbb{A}})_{\gamma}^{V}(q)\}$

$= \max\{(\gamma\delta_{\mathbb{A}}(p * q)), (\gamma\delta_{\mathbb{A}}(q))\}$

$= \max \gamma\{\delta_{\mathbb{A}}(p * q), \delta_{\mathbb{A}}(q)\}$

which implies $\delta_{\mathbb{A}}(p) \leq \max\{(p * q), \delta_{\mathbb{A}}(q)\}$

Therefore $\mathbb{A} = (\varphi_{\mathbb{A}}, \delta_{\mathbb{A}})$ be an intuitionistic fuzzy Z – Ideal of $\mathbb{P}$.

Theorem 2.7

If $\mathbb{A}$be an intuitionistic fuzzy Z – Ideal of $\mathbb{P}$. Then the Intuitionistic FM-γ be$\mathbb{A}_{\gamma}^{V}(p)$ of $\mathbb{A} = (\varphi_{\mathbb{A}}, \delta_{\mathbb{A}})$ is an intuitionistic fuzzy Z – Ideal of $\mathbb{P}$, for all $\gamma \in [0,1]$.

Proof:

If $\mathbb{A}$ is an intuitionistic fuzzy Z – Ideal of $\mathbb{P}$, for all $\gamma \in [0,1]$.

i. $(\varphi_{\mathbb{A}})_{\gamma}^{V}(0) = \gamma\varphi_{\mathbb{A}}(p) \geq \gamma\varphi_{\mathbb{A}}(p)$

$= (\varphi_{\mathbb{A}})_{\gamma}^{V}(p)$

which implies $(\varphi_{\mathbb{A}})_{\gamma}^{V}(0) \geq (\varphi_{\mathbb{A}})_{\gamma}^{V}(p)$

ii. $(\delta_{\mathbb{A}})_{\gamma}^{V}(0) = \gamma\delta_{\mathbb{A}}(p) \leq \gamma\delta_{\mathbb{A}}(p)$

$= (\delta_{\mathbb{A}})_{\gamma}^{V}(p)$

which implies $(\delta_{\mathbb{A}})_{\gamma}^{V}(0) \leq (\delta_{\mathbb{A}})_{\gamma}^{V}(p)$

iii. $(\varphi_{\mathbb{A}})_{\gamma}^{V}(p) = \gamma\varphi_{\mathbb{A}}(p)$

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$\geq \min \gamma\{\varphi_{\mathbb{A}}(p * q), \varphi_{\mathbb{A}}(q)\}$

$= \min \gamma\{\varphi_{\mathbb{A}}(p * q), \varphi_{\mathbb{A}}(q)\}$

$= \min \{(\gamma\varphi_{\mathbb{A}}(p * q)), (\gamma\varphi_{\mathbb{A}}(q))\}$

which implies $(\varphi_{\mathbb{A}})_{\gamma}^{V}(p) \geq \min \{(\varphi_{\mathbb{A}})_{\gamma}^{V}(p * q), (\varphi_{\mathbb{A}})_{\gamma}^{V}(q)\}$

iv. $(\delta_{\mathbb{A}})_{\gamma}^{V}(p) = \gamma\delta_{\mathbb{A}}(p)$

$\leq \max \gamma\{\delta_{\mathbb{A}}(p * q), \delta_{\mathbb{A}}(q)\}$

$= \max \gamma\{\delta_{\mathbb{A}}(p * q), \delta_{\mathbb{A}}(q)\}$

$= \max \{(\gamma\delta_{\mathbb{A}}(p * q)), (\gamma\delta_{\mathbb{A}}(q))\}$

which implies $(\delta_{\mathbb{A}})_{\gamma}^{V}(p) \leq \max \{(\delta_{\mathbb{A}})_{\gamma}^{V}(p * q), (\delta_{\mathbb{A}})_{\gamma}^{V}(q)\}$

Therefore $\mathbb{A}_{\gamma}^{V} = ((\varphi_{\mathbb{A}})_{\gamma}^{V}, (\delta_{\mathbb{A}})_{\gamma}^{V})$ of $\mathbb{A}$ is an intuitionistic fuzzy Z – Ideal of $\mathbb{P}$, and then for all $\gamma \in [0,1]$.

Theorem 2.8

For any Intuitionistic fuzzy Z – sub-algebra $\mathbb{A}$ of $\mathbb{P}$, $\forall \gamma \in [0,1]$. If γ - Intuitionistic Fuzzy Multiplication be$\mathbb{A}_{\gamma}^{V}$ of $\mathbb{A}$ be Intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$.

Proof:

Let $\gamma \in [0,1]$& let $p, q \in \mathbb{P}$.

Now, $(\varphi_{\mathbb{A}})_{\gamma}^{V}(p * q) = \gamma\varphi_{\mathbb{A}}(p * q) \geq \min \{\gamma(\varphi_{\mathbb{A}}(p), \varphi_{\mathbb{A}}(q))\}$

$\geq \min\{\gamma\varphi_{\mathbb{A}}(p), \gamma\varphi_{\mathbb{A}}(q)\}$

$(\varphi_{\mathbb{A}})_{\gamma}^{V}(p * q) \geq \min\{(\varphi_{\mathbb{A}})_{\gamma}^{V}(p), (\varphi_{\mathbb{A}})_{\gamma}^{V}(q)\}$

$(\delta_{\mathbb{A}})_{\gamma}^{V}(p * q) = \gamma\delta_{\mathbb{A}}(p * q)$

$\leq \max\{\gamma(\delta_{\mathbb{A}}(p), \delta_{\mathbb{A}}(q))\}$

$\leq \max\{\gamma\delta_{\mathbb{A}}(p), \gamma\delta_{\mathbb{A}}(q)\}$

$(\delta_{\mathbb{A}})_{\gamma}^{V}(p * q) \leq \max\{(\delta_{\mathbb{A}})_{\gamma}^{V}(p), (\delta_{\mathbb{A}})_{\gamma}^{V}(q)\}$

Hence an intuitionistic fuzzy Z – sub-algebra $\mathbb{A}_{\gamma}^{V}(p)$ of $\mathbb{P}$.

Theorem 2.9

If$\mathbb{A}_{\gamma}^{V}(p)$ be aγ - Intuitionistic FM of $\mathbb{A}$ be an Intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$. Then for any an Intuitionistic fuzzy Z – sub-algebra $\mathbb{A}$ of $\mathbb{P}$.

Proof:

Let $\mathbb{A}_{\gamma}^{V}(p)$ of $\mathbb{A}$ in $\mathbb{P}$, for any $\gamma \in [0,1]$.

$\gamma\varphi_{\mathbb{A}}(p * q) = (\varphi_{\mathbb{A}})_{\gamma}^{V}(p * q)$

$\geq \min\{(\varphi_{\mathbb{A}})_{\gamma}^{V}(p), (\varphi_{\mathbb{A}})_{\gamma}^{V}(q)\}$

$= \min\{\gamma\varphi_{\mathbb{A}}(p), \gamma\varphi_{\mathbb{A}}(q)\}$

$= \min \{\gamma(\varphi_{\mathbb{A}}(p), \varphi_{\mathbb{A}}(q))\}$

Implies that $\varphi_{\mathbb{A}}(p * q) \geq \min\{\varphi_{\mathbb{A}}(p), \varphi_{\mathbb{A}}(q)\}$

$\gamma\delta_{\mathbb{A}}(p * q) = (\delta_{\mathbb{A}})_{\gamma}^{V}(p * q)$

$\leq \max\{(\delta_{\mathbb{A}})_{\gamma}^{V}(p), (\delta_{\mathbb{A}})_{\gamma}^{V}(q)\}$

$= \max\{\gamma\delta_{\mathbb{A}}(p), \gamma\delta_{\mathbb{A}}(q)\}$

$= \max\{\gamma(\delta_{\mathbb{A}}(p), \delta_{\mathbb{A}}(q))\}$

which implies $\delta_{\mathbb{A}}(p * q) \leq \max\{\delta_{\mathbb{A}}(p), \delta_{\mathbb{A}}(q)\}$

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Hence $\mathbb{A}$ be intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$.

3. N – GENERATED FUZZY TRANSLATION AND FUZZY MULTIPLICATION IN Z – ALGEBRA

Definition 3.1

Let ξ_n in $\mathbb{P}$ is termed as N-generated fuzzy subalgebra on $\mathbb{P}$. The following criteria is satisfied, $\xi_n(\wp_n * q_n) \geq \min\{\xi_n(\wp_n), \xi_n(q_n)\}$

Definition 3.2

(A1) $\xi_n(0) \geq \xi_n(\wp_n)$

(A2) $\xi_n(\wp_n) \geq \min\{\xi_n(\wp_n * q_n), \xi_n(q_n)\}$

The following criteria are satisfied, If a fuzzy set ξ_n in $\mathbb{P}$ is termed as N-generated fuzzy BCK – Ideal of $\mathbb{P}$.

Definition 3.3

Let ξ_n be a fuzzy set in $\mathbb{P}$ is defined as N-generated fuzzy Z – Ideal of $\mathbb{P}$. If it fulfils the following criteria,

i. $\xi_n(0) \geq \xi_n(\wp_n)$

ii. $\xi_n(\wp_n) \geq \min\{\xi_n(\wp_n * q_n), \xi_n(q_n)\}$

Theorem 3.4

If σ is an N-generated fuzzy Z – Ideal of $\mathbb{P}$ and γ is an N-generated Fuzzy Translation $\sum_{n=1}^{N} \sigma_{\gamma_n}^{U_n}(\wp_n)$ is an N-generated fuzzy Z – Ideal of $\mathbb{P}$, $\forall \gamma_n \in [0, U]$

Proof:

Let σ be a N-generated fuzzy Z – Ideal of $\mathbb{P}$ and $\gamma_n \in [0, U]$

We now have

i. $\sum_{n=1}^{N} \sigma_{\gamma_n}^{U_n}(0) = \sum_{n=1}^{N} \sigma(0) + \gamma_n$

$$\geq \sum_{n=1}^{N} \sigma(\wp_n) + \gamma_n$$

$$= \sum_{n=1}^{N} \sigma_{\gamma_n}^{U_n}(\wp_n)$$

ii. $\sum_{n=1}^{N} \sigma_{\gamma_n}^{U_n}(\wp_n) = \sum_{n=1}^{N} \sigma(\wp_n) + \gamma_n$

$$\geq \sum_{n=1}^{N} \{\sigma(\wp_n * q_n) \wedge \sigma(q_n)\} + \gamma_n$$

$$= \sum_{n=1}^{N} \{(\sigma(\wp_n * q_n) + \gamma_n) \wedge (\sigma(q_n) + \gamma_n)\}$$

$$\sum_{n=1}^{N} \sigma_{\gamma_n}^{U_n}(\wp_n) \geq \sum_{n=1}^{N} \{\sigma_{\gamma_n}^{U_n}(\wp_n * q_n) \wedge \sigma_{\gamma_n}^{U_n}(q_n)\}$$

Hence σ is a N-generated fuzzy Z – Ideal of $\mathbb{P}$.

Theorem 3.5

Let σ is a N-generated fuzzy subset of $\mathbb{P}$ such that the γ – N-Generated Fuzzy Translation $\sum_{n=1}^{N} \sigma_{\gamma_n}^{U_n}(\wp_n)$. Then σ is a N-generated fuzzy Z – Ideal of $\mathbb{P}$ and $\gamma_n \in [0, U]$.

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Proof:

Let $\sigma_{\gamma_n}^{U_n}$ be a N-generated fuzzy Z – Ideal of $\mathbb{P}$ and let $\gamma_n \in [0, U]$

Let $\wp_n, q_n \in \mathbb{P}$

i. $\sum_{n=1}^{N} \sigma(0) + \gamma_n = \sum_{n=1}^{N} \sigma_{\gamma_n}^{U_n}(0)$

$\geq \sum_{n=1}^{N} \sigma_{\gamma_n}^{U_n}(\wp_n)$

$= \sum_{n=1}^{N} \sigma(\wp_n) + \gamma_n$

which implies $\sum_{n=1}^{N} \sigma(0) \geq \sum_{n=1}^{N} \sigma(\wp_n)$

ii. $\sum_{n=1}^{N} \sigma(\wp_n) + \gamma_n = \sum_{n=1}^{N} \sigma_{\gamma_n}^{U_n}(\wp_n)$

$$\geq \sum_{n=1}^{N} \{\sigma_{\gamma_n}^{U_n}(\wp_n * q_n) \wedge \sigma_{\gamma_n}^{U_n}(q_n)\}$$

$= \sum_{n=1}^{N} \{(\sigma(\wp_n * q_n) + \gamma_n) \wedge (\sigma(q_n) + \gamma_n)\}$

$= \sum_{n=1}^{N} \{\sigma(\wp_n * q_n) \wedge \sigma(q_n)\} + \gamma_n$

which implies $\sum_{n=1}^{N} \sigma(\wp_n) \geq \sum_{n=1}^{N} \{(\wp_n * q_n) \wedge \sigma(q_n)\}$

Therefore $\sigma_{\gamma_n}^{U_n}$ of σ is a N-generated fuzzy Z – Ideal of $\mathbb{P}$.

Theorem 3.6

Let a N-generated fuzzy Z – Ideal σ of $\mathbb{P}$ and $\gamma_n \in [0, U]$. Let $\mathbb{P}$ be a Z – algebra, then $\sigma_{\gamma_n}^{U_n}$ be γ –N-generated Fuzzy Translation of σ is N-generated fuzzy Z - sub-algebra of $\mathbb{P}$.

Proof:

Let $\gamma_n \in [0, U]$ & $\wp_n, q_n \in \mathbb{P}$

We have,

$\sum_{n=1}^{N} \sigma_{\gamma_n}^{U_n}(\wp_n * q_n) = \sum_{n=1}^{N} \sigma(\wp_n * q_n) + \gamma_n$

$$\geq \sum_{n=1}^{N} \{\sigma(q_n * (\wp_n * q_n)) \wedge \sigma(q_n)\} + \gamma_n$$

$= \sum_{n=1}^{N} \{\sigma(\wp_n * (q_n * q_n)) \wedge \sigma(q_n)\} + \gamma_n$

$$\geq \sum_{n=1}^{N} \{\sigma(\wp_n) \wedge \sigma(q_n)\} + \gamma_n$$

$$\geq \sum_{n=1}^{N} \{(\sigma(\wp_n) + \gamma_n) \wedge (\sigma(q_n) + \gamma_n)\}$$

$= \sum_{n=1}^{N} \{\sigma_{\gamma_n}^{U_n}(\wp_n) \wedge \sigma_{\gamma_n}^{U_n}(q_n)\}$

$\therefore \sigma_{\gamma_n}^{U_n}$ is N-generated fuzzy Z – algebra of $\mathbb{P}$.

Theorem 3.7

Let σ be N-generated fuzzy Z – sub-algebra of $\mathbb{P}$ and let $\sigma_{\gamma_n}^{U_n}$ of σ be a N-generated Fuzzy Translation - γ and then $\forall \gamma_n \in [0, U]$ and then σ is N-generated fuzzy Z – sub-algebra of $\mathbb{P}$.

Proof:

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We assume that $\sigma_{\gamma_n}^{U_n}$ of σ be a N-generated fuzzy Z – Ideal of $\mathbb{P}$.

Then we have

$$\sum_{n=1}^{N}\sigma(\wp_n * q_n)+\gamma_n = \sum_{n=1}^{N}\sigma_{\gamma_n}^{U_n}(\wp_n * q_n)$$

$$\geq \sum_{n=1}^{N}\{\sigma_{\gamma_n}^{U_n}(\wp_n *(\wp_n * q_n)) \wedge \sigma_{\gamma_n}^{U_n}(q_n)\}$$

$$= \sum_{n=1}^{N}\{\sigma_{\gamma_n}^{U_n}(\wp_n * \wp_n) * q_n) \wedge \sigma_{\gamma_n}^{U_n}(q_n)\}$$

$$\geq \sum_{n=1}^{N}\{\sigma_{\gamma_n}^{U_n}(\wp_n) \wedge \sigma_{\gamma_n}^{U_n}(q_n)\}$$

$$= \sum_{n=1}^{N}\{(\sigma(\wp_n)+\gamma_n) \wedge (\sigma(q_n)+\gamma_n)\}$$

$$= \sum_{n=1}^{N}\{\sigma(\wp_n) \wedge \sigma(q_n)\}+\gamma_n$$

$$\Rightarrow \sum_{n=1}^{N}\sigma(\wp_n * q_n) \geq \sum_{n=1}^{N}\{\sigma(\wp_n) \wedge \sigma(q_n)\}$$

$\therefore$ σ is N-generated fuzzy Z – sub-algebra of $\mathbb{P}$.

Theorem 3.8

Let σ is a N-generated fuzzy subset of $\mathbb{P}$ such that the γ is a N-generated Fuzzy Multiplication $\sum_{n=1}^{N}\sigma_{\gamma_n}^{V_n}(\wp_n)$. Then σ is a N-generated fuzzy Z – Ideal of $\mathbb{P}$ and for any $\gamma_n \in [0,1]$.

Proof:

Let $\sum_{n=1}^{N}\sigma_{\gamma_n}^{V_n}$ is a N-generated fuzzy Z – Ideal of $\mathbb{P}$ for all $\gamma_n \in [0,1]$.

Let $\wp_n, q_n \in \mathbb{P}$

We now have,

i. $\sum_{n=1}^{N}\gamma_n\sigma(\wp_n) = \sum_{n=1}^{N}\sigma_{\gamma_n}^{V_n}(0)$

$\geq \sum_{n=1}^{N}\sigma_{\gamma_n}^{V_n}(\wp_n)$

$= \sum_{n=1}^{N}\gamma_n\sigma(\wp_n)$

which implies $\sum_{n=1}^{N}\sigma(0) \geq \sum_{n=1}^{N}\sigma(\wp_n)$

ii. $\sum_{n=1}^{N}\gamma_n\sigma(\wp_n) = \sum_{n=1}^{N}\sigma_{\gamma_n}^{V_n}(\wp_n)$

$$\geq \sum_{n=1}^{N}\{\sigma_{\gamma_n}^{V_n}(\wp_n * q_n) \wedge \sigma_{\gamma_n}^{V_n}(q_n)\}$$

$$= \sum_{n=1}^{N}\{(\gamma_n\sigma(\wp_n * q_n)) \wedge (\gamma_n\sigma(q_n))\}$$

$$= \sum_{n=1}^{N}\gamma_n\{\sigma(\wp_n * q_n) \wedge \sigma(q_n)\}$$

which implies $\sum_{n=1}^{N}\sigma(\wp_n) \geq \sum_{n=1}^{N}\{(\wp_n * q_n) \wedge \sigma(q_n)\}$

Therefore σ is a N-generated fuzzy Z-Ideal of $\mathbb{P}$.

Theorem 3.9

A σ is a N-generated fuzzy Z – ideal of $\mathbb{P}$. Then the γ be an N-generated Fuzzy Multiplication $-\gamma$, $\sum_{n=1}^{N}\sigma_{\gamma_n}^{V_n}(\wp_n)$ of N-generated fuzzy Z – Ideal σ of $\mathbb{P}$, for any $\gamma_n \in [0,1]$.

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Proof:

If σ is a N-generated fuzzy Z – Ideal of $\mathbb{P}$ and $\gamma_n \in [0,1]$.

We now have

i. $\sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}(0) = \sum_{n=1}^{N} \gamma_n \sigma(\wp_n)$
$\geq \sum_{n=1}^{N} \gamma_n \sigma(\wp_n)$
$= \sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}(\wp_n)$

which implies $\sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}(0) \geq \sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}(\wp_n)$

ii. $\sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}(\wp_n) = \sum_{n=1}^{N} \gamma_n \sigma(\wp_n)$
$\geq \sum_{n=1}^{N} \gamma_n \{\sigma(\wp_n * q_n) \wedge \sigma(q_n)\}$
$= \sum_{n=1}^{N} \gamma_n \{\sigma(\wp_n * q_n) \wedge \sigma(q_n)\}$
$= \sum_{n=1}^{N} \{(\gamma_n \sigma(\wp_n * q_n)) \wedge (\gamma_n \sigma(q_n))\}$

which implies $\sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}(\wp_n) \geq \sum_{n=1}^{N} \{\sigma_{\gamma_n}^{V_n}(\wp_n * q_n) \wedge \sigma_{\gamma_n}^{V_n}(q_n)\}$

Therefore, $\sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}$ of σ is a N-generated fuzzy Z – Ideal of $\mathbb{P}$ and then for every $\wp_n, q_n \in [0,1]$.

Theorem 3.10

Let σ be N-generated fuzzy Z – sub-algebra of $\mathbb{P}$, $\gamma_n \in [0,1]$. If N-generated Fuzzy Multiplication - γ be $\sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}(\wp_n)$ of σ is N-generated fuzzy Z – sub-algebra of $\mathbb{P}$.

Proof:

Let $\gamma_n \in [0,1]$ & $\wp_n, q_n \in \mathbb{P}$

Then $\sum_{n=1}^{N} \sigma(\wp_n * q_n) \geq \sum_{n=1}^{N} \{\sigma(\wp_n) \wedge \sigma(q_n)\}$

We have,

$$\sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}(\wp_n * q_n) = \sum_{n=1}^{N} \gamma_n \sigma(\wp_n * q_n)$$
$$\geq \sum_{n=1}^{N} \gamma_n \{\sigma(\wp_n) \wedge \sigma(q_n)\}$$
$$\geq \sum_{n=1}^{N} \gamma_n \sigma(\wp_n) \wedge \gamma_n \sigma(q_n)$$

which implies $\sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}(\wp_n * q_n) \geq \sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}(\wp_n) \wedge \sigma_{\gamma_n}^{V_n}(q_n)$

Hence, $\sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}$ is N-generated fuzzy Z – sub-algebra of $\mathbb{P}$.

Theorem 3.11

Let σ be N-generated fuzzy Z – sub-algebra of $\mathbb{P}$. If γ is an N-generated Fuzzy Multiplication $\sum_{n=1}^{N} \sigma_{\gamma_n}^{V_n}(\wp_n)$ and then σ be N-generated fuzzy subset of $\mathbb{P}$, $\forall \gamma_n \in [0,1]$.

Proof:

Let $\sigma_{\gamma_n}^{V_n}(\wp_n)$ of σ is N-generated fuzzy Z – sub-algebra and $\gamma_n \in [0,1]$.

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$$\sum_{n=1}^{N}\gamma_n\sigma(\wp_n * q_n) = \sum_{n=1}^{N}\sigma_{\gamma_n}^{V_n}(\wp_n * q_n)$$
$$\geq \sum_{n=1}^{N}\sigma_{\gamma_n}^{V_n}(\wp_n) \wedge \sigma_{\gamma_n}^{V_n}(q_n)$$
$$= \sum_{n=1}^{N}\{\gamma_n\sigma(\wp_n) \wedge \gamma_n\sigma(q_n)\}$$
$$= \sum_{n=1}^{N}\gamma_n\{\sigma(\wp_n) \wedge \sigma(q_n)\}$$
$$\Rightarrow \sum_{n=1}^{N}\sigma(\wp_n * q_n) \geq \sum_{n=1}^{N}\sigma(\wp_n) \wedge \sigma(q_n)$$

$\therefore \sigma$ is a N-generated fuzzy Z – sub-algebra of $\mathbb{P}$.

4. N – GENERATED INTUITIONISTIC FUZZY TRANSLATION AND FUZZY MULTIPLICATION IN Z – ALGEBRA

Definition 4.1

An N-generated intuitionistic fuzzy set on Z – Algebra is $\sum_{n=1}^{N}\mathbb{A}= (\mathbb{P}, \varphi_{\mathbb{A}_n}, \delta_{\mathbb{A}_n})$ is in $\mathbb{P}$ is termed as N-generated intuitionistic fuzzy Z – Ideal of $\mathbb{P}$, if it fulfils the following criteria,

i. $\sum_{n=1}^{N}\varphi_{\mathbb{A}_n}(0) \geq \sum_{n=1}^{N}\varphi_{\mathbb{A}_n}(\wp_n)$

ii. $\sum_{n=1}^{N}\delta_{\mathbb{A}_n}(0) \leq \sum_{n=1}^{N}\delta_{\mathbb{A}_n}(\wp_n)$

iii. $\sum_{n=1}^{N}\varphi_{\mathbb{A}_n}(\wp_n) \geq \sum_{n=1}^{N}\min\{\varphi_{\mathbb{A}_n}(\wp_n * q_n), \varphi_{\mathbb{A}_n}(q_n)\}$

iv. $\sum_{n=1}^{N}\delta_{\mathbb{A}_n}(\wp_n) \leq \sum_{n=1}^{N}\max\{\delta_{\mathbb{A}_n}(\wp_n * q_n), \delta_{\mathbb{A}_n}(q_n)\}$

for all $\wp_n, q_n \in \mathbb{P}$

Definition 4.2

Let $\sum_{n=1}^{N}\mathbb{A}= (\varphi_{\mathbb{A}_n}, \delta_{\mathbb{A}_n})$ be the N – Generated intuitionistic fuzzy set in $\mathbb{P}$. If the below criteria are satisfied,

$\varphi_{\mathbb{A}_n}(\wp_n * q_n) \geq \min\{\varphi_{\mathbb{A}_n}(\wp_n), \varphi_{\mathbb{A}_n}(q_n)\}$

$\delta_{\mathbb{A}_n}(\wp_n * q_n) \leq \max\{\delta_{\mathbb{A}_n}(\wp_n), \delta_{\mathbb{A}_n}(q_n)\}$

for all $\wp_n, q_n \in \mathbb{P}$

Then it is termed as N – Generated Intuitionistic sub – algebra on $\mathbb{P}$.

Theorem 4.3

If $\sum_{n=1}^{N}\mathbb{A}= (\varphi_{\mathbb{A}_n}, \delta_{\mathbb{A}_n}) \subset \mathbb{P}$ be an N-generated intuitionistic fuzzy Z – Ideal, then N-generated Intuitionistic Fuzzy γ - Translation $\sum_{n=1}^{N}\mathbb{A}_{\gamma_n}^{U_n}(\wp_n)$of $\sum_{n=1}^{N}\mathbb{A}$is an N-generated intuitionistic fuzzy Z – Ideal of $\mathbb{P}$, $\forall\gamma_n \in [0, U]$

Proof:

Let N-generated intuitionistic fuzzy Z – Ideal be $\sum_{n=1}^{N}\mathbb{A}$ of $\mathbb{P}$ and $\gamma_n \in [0, U]$

i. $\sum_{n=1}^{N}(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(0) = \sum_{n=1}^{N}\varphi_{\mathbb{A}_n}(0) + \gamma_n$

$$\geq \sum_{n=1}^{N}\varphi_{\mathbb{A}_n}(\wp_n) + \gamma_n$$
$$= \sum_{n=1}^{N}(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n)$$

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ii. $\sum_{n=1}^{N}(\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(0) = \sum_{n=1}^{N}\delta_{\mathbb{A}_n}(0) - \gamma_n$

$$\leq \sum_{n=1}^{N}\delta_{\mathbb{A}_n}(\wp_n) - \gamma_n$$

$$= \sum_{n=1}^{N}(\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n)$$

iii. $\sum_{n=1}^{N}(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n) = \sum_{n=1}^{N}\varphi_{\mathbb{A}_n}(\wp_n) + \gamma_n$

$$\geq \sum_{n=1}^{N}\min\{\varphi_{\mathbb{A}_n}(\wp_n * q_n), \varphi_{\mathbb{A}_n}(q_n)\} + \gamma_n$$

$$= \sum_{n=1}^{N}\min\{(\varphi_{\mathbb{A}_n}(\wp_n * q_n) + \gamma_n), (\varphi_{\mathbb{A}_n}(q_n) + \gamma_n)\}$$

$$\sum_{n=1}^{N}(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n) \geq \sum_{n=1}^{N}\min\{(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n * q_n), (\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

iv. $\sum_{n=1}^{N}(\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n) = \sum_{n=1}^{N}\delta_{\mathbb{A}_n}(\wp_n) - \gamma_n$

$$\leq \sum_{n=1}^{N}\max\{\delta_{\mathbb{A}_n}(\wp_n * q_n), \delta_{\mathbb{A}_n}(q_n)\} - \gamma_n$$

$$= \sum_{n=1}^{N}\max\{(\delta_{\mathbb{A}_n}((\wp_n * q_n)) - \gamma_n), (\delta_{\mathbb{A}_n}(q_n) - \gamma_n)\}$$

$$\sum_{n=1}^{N}(\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n) \leq \sum_{n=1}^{N}\max\{(\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n * q_n), (\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

Hence $\sum_{n=1}^{N}\mathbb{A}$ is known as N-generated intuitionistic fuzzy Z – Ideal of $\mathbb{P}$.

Theorem 4.4

Let $\sum_{n=1}^{N}\mathbb{A}$ is an N-generated intuitionistic fuzzy subset and Z – Ideal of $\mathbb{P}$ such that γ be the N-generated Intuitionistic fuzzy translation $\sum_{n=1}^{N}\mathbb{A}_{\gamma_n}^{U_n}(\wp_n)$, $\forall\gamma_n \in [0, U]$. Then $\sum_{n=1}^{N}\mathbb{A}$ be a N-generated intuitionistic fuzzy Z – Ideal of $\mathbb{P}$.

Proof:

Let N-generated intuitionistic fuzzy Z – Ideal be $\sum_{n=1}^{N}\mathbb{A}_{\gamma_n}^{U_n}(\wp_n)$ of $\mathbb{P}$, for any $\gamma_n \in [0, U]$.

Let $\wp_n, q_n \in \mathbb{P}$

i. $\sum_{n=1}^{N}\varphi_{\mathbb{A}_n}(0) + \gamma_n = \sum_{n=1}^{N}(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(0)$

$$\geq \sum_{n=1}^{N}(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n)$$

$$= \sum_{n=1}^{N}\varphi_{\mathbb{A}_n}(0) + \gamma_n$$

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which implies $\sum_{n=1}^{N} \varphi_{\mathbb{A}_n}(0) \geq \sum_{n=1}^{N} \varphi_{\mathbb{A}_n}(\wp_n)$

ii.$\sum_{n=1}^{N} \delta_{\mathbb{A}_n}(0) - \gamma_n = \sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(0)$

$$\leq \sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n)$$

$= \sum_{n=1}^{N} \delta_{\mathbb{A}_n}(0) - \gamma_n$

which implies $\sum_{n=1}^{N} \delta_{\mathbb{A}_n}(0) \leq \sum_{n=1}^{N} \delta_{\mathbb{A}_n}(\wp_n)$

iii.$\sum_{n=1}^{N} \varphi_{\mathbb{A}_n}(\wp_n) + \gamma_n = \sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n)$

$$\geq \sum_{n=1}^{N} \min\{(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n * q_n), (\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

$=\sum_{n=1}^{N} \min \; \{(\varphi_{\mathbb{A}_n}(\wp_n * q_n) + \gamma_n),$

$(\varphi_{\mathbb{A}_n}(q_n) + \gamma_n)\}$

$= \sum_{n=1}^{N} \min\{\varphi_{\mathbb{A}_n}(\wp_n * q_n), \varphi_{\mathbb{A}_n}(q_n)\} + \gamma_n$

which implies $\sum_{n=1}^{N} \varphi_{\mathbb{A}_n}(\wp_n) \geq \sum_{n=1}^{N} \min\{(\wp_n * q_n), \varphi_{\mathbb{A}_n}(q_n)\}$

iv.$\sum_{n=1}^{N} \delta_{\mathbb{A}_n}(\wp_n) - \gamma_n = \sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n)$

$$\leq \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n * q_n), (\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

$= \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}_n}((\wp_n * q_n)) - \gamma_n),$

$(\delta_{\mathbb{A}_n}(q_n) - \gamma_n)\}$

$= \sum_{n=1}^{N} \max\{\delta_{\mathbb{A}_n}(\wp_n * q_n), \delta_{\mathbb{A}_n}(q_n)\} - \gamma_n$

which implies $\sum_{n=1}^{N} \delta_{\mathbb{A}_n}(\wp_n) \leq \sum_{n=1}^{N} \max\{(\wp_n * q_n), \delta_{\mathbb{A}_n}(q_n)\}$

Therefore $\sum_{n=1}^{N} \mathbb{A}_{\gamma_n}^{U_n}(\wp_n) = \sum_{n=1}^{N} ((\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n), (\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n))$of $\sum_{n=1}^{N} \mathbb{A}$ is said to be N-generated intuitionistic fuzzy Z – Ideal of $\mathbb{P}$.

Theorem 4.5

If N-generated intuitionistic fuzzy Z – Ideal of $\mathbb{P}$ is $\sum_{n=1}^{N} \mathbb{A}$, then$\sum_{n=1}^{N} \mathbb{A}_{\gamma_n}^{U_n}$ beN-generated Intuitionistic Fuzzy γ -Translationis N-generated intuitionistic fuzzy Z – sub-algebraf $\sum_{n=1}^{N} \mathbb{A}$of $\mathbb{P}$, then for any $\gamma_n \in [0, U]$.

Proof:

We have,

$\sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n * q_n) = \sum_{n=1}^{N} \{(\varphi_{\mathbb{A}_n}(\wp_n * q_n) + \gamma_n)$

$$\geq \sum_{n=1}^{N} \min\{\varphi_{\mathbb{A}_n}(q_n * (\wp_n * q_n)), \varphi_{\mathbb{A}_n}(q_n)\} + \gamma_n$$

$= \sum_{n=1}^{N} \min\{\varphi_{\mathbb{A}_n}(\wp_n * (q_n * q_n)), \varphi_{\mathbb{A}_n}(q_n)\} + \gamma_n$

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$$\geq \sum_{n=1}^{N} \min\{\varphi_{\mathbb{A}_n}(p_n), \varphi_{\mathbb{A}_n}(q_n)\} + \gamma_n$$

$$\geq \sum_{n=1}^{N} \min\{(\varphi_{\mathbb{A}_n}(p_n) + \gamma_n), (\varphi_{\mathbb{A}_n}(q_n) + \gamma_n)\}$$

$$= \sum_{n=1}^{N} \min\{(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(p_n), (\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

$$\sum_{n=1}^{N}(\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(p_n * q_n) = \sum_{n=1}^{N}\{(\delta_{\mathbb{A}_n}(p_n * q_n) - \gamma_n)$$

$$\leq \sum_{n=1}^{N} \max\{\delta_{\mathbb{A}_n}(q_n * (p_n * q_n)), \delta_{\mathbb{A}_n}(q_n)\} - \gamma_n$$

$$= \sum_{n=1}^{N} \max\{\delta_{\mathbb{A}_n}(p_n * (q_n * q_n)), \delta_{\mathbb{A}_n}(q_n)\} - \gamma_n$$

$$\leq \sum_{n=1}^{N} \max\{\delta_{\mathbb{A}_n}(p_n), \delta_{\mathbb{A}_n}(q_n)\} - \gamma_n$$

$$\leq \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}_n}(p_n) - \gamma_n), (\delta_{\mathbb{A}_n}(q_n) - \gamma_n)\}$$

$$= \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(p_n), (\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

Therefore $\sum_{n=1}^{N} \mathbb{A}_{\gamma_n}^{U_n}$ is a N-generated Intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$.

Theorem 4.6

Let$\sum_{n=1}^{N} \mathbb{A}$ is a N-generated Intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$ and then let $\sum_{n=1}^{N} \mathbb{A}_{\gamma_n}^{U_n}$of $\sum_{n=1}^{N} \mathbb{A} = (\varphi_{\mathbb{A}_n}, \delta_{\mathbb{A}_n})$ is a N-generated Intuitionistic γ - FT for any $\gamma_n \in [0, U]$. Then an N-generated Intuitionistic fuzzy Z – sub-algebra $\sum_{n=1}^{N} \mathbb{A}$ of $\mathbb{P}$.

Proof:

$$\sum_{n=1}^{N} \varphi_{\mathbb{A}_n}(p_n * q_n) + \gamma_n = \sum_{n=1}^{N}(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(p_n * q_n)$$

$$\geq \sum_{n=1}^{N} \min(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(p_n * (p_n * q_n)), (\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

$$= \sum_{n=1}^{N} \min\{(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(p_n * p_n) * q_n), (\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

$$\geq \sum_{n=1}^{N} \min\{(\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(p_n), (\varphi_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

$$\geq \sum_{n=1}^{N} \min\{(\varphi_{\mathbb{A}_n}(p_n) + \gamma_n), (\varphi_{\mathbb{A}_n}(q_n) + \gamma_n)\}$$

$$= \sum_{n=1}^{N} \min\{\varphi_{\mathbb{A}_n}(p_n), \varphi_{\mathbb{A}_n}(q_n)\} + \gamma_n$$

$$\sum_{n=1}^{N} \varphi_{\mathbb{A}_n}(p_n * q_n) \geq \sum_{n=1}^{N} \min\{\varphi_{\mathbb{A}_n}(p_n), \varphi_{\mathbb{A}_n}(q_n)\}$$

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$\sum_{n=1}^{N} \delta_{\mathbb{A}_n}(\wp_n * q_n) - \gamma_n = \sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n * q_n)$

$$\leq \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n * (\wp_n * q_n)), (\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

$$= \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n * \wp_n) * q_n), (\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

$$\leq \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(\wp_n), (\delta_{\mathbb{A}})_{\gamma_n}^{U_n}(q_n)\}$$

$$\leq \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}_n}(\wp_n) - \gamma_n), (\delta_{\mathbb{A}_n}(q_n) - \gamma_n)\}$$

$$= \sum_{n=1}^{N} \max\{\delta_{\mathbb{A}_n}(\wp_n), \delta_{\mathbb{A}_n}(q_n)\} - \gamma_n$$

$$\sum_{n=1}^{N} \delta_{\mathbb{A}_n}(\wp_n * q_n) \geq \sum_{n=1}^{N} \max\{\delta_{\mathbb{A}_n}(\wp_n), \delta_{\mathbb{A}_n}(q_n)\}$$

Hence $\sum_{n=1}^{N} \mathbb{A} = (\varphi_{\mathbb{A}_n}, \delta_{\mathbb{A}_n})$ is N-generated intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$.

Theorem 4.7

Let $\sum_{n=1}^{N} \mathbb{A}$is an N-generated intuitionistic fuzzy subset of $\mathbb{P}$ such that the N-generated Intuitionistic γ - FM$\sum_{n=1}^{N} \mathbb{A}_{\gamma_n}^{V_n}(\wp) \subset \sum_{n=1}^{N} \mathbb{A}$ is a N-generated intuitionistic fuzzy Z – Ideal of $\mathbb{P}$, for all $\gamma_n \in [0,1]$. Then an N-generated intuitionistic fuzzy Z – Ideal $\sum_{n=1}^{N} \mathbb{A}$ of $\mathbb{P}$.

Proof:

Let$\sum_{n=1}^{N} \mathbb{A}_{\gamma_n}^{V_n}$ is an N- generated intuitionistic fuzzy Z – Ideal of $\mathbb{P}$ for all $\gamma_n \in [0,1]$.

Let $\wp_n, q \in \mathbb{P}$

i. $\sum_{n=1}^{N} \gamma_n \varphi_{\mathbb{A}_n}(\wp_n) = \sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(0) \geq \sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n)$

$$= \sum_{n=1}^{N} \gamma_n \varphi_{\mathbb{A}_n}(\wp_n)$$

which implies $\sum_{n=1}^{N} \varphi_{\mathbb{A}_n}(0) \geq \sum_{n=1}^{N} \varphi_{\mathbb{A}_n}(\wp_n)$

ii. $\sum_{n=1}^{N} \gamma_n \delta_{\mathbb{A}_n}(\wp_n) = \sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(0) \leq \sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n)$

$$= \sum_{n=1}^{N} \gamma_n \delta_{\mathbb{A}_n}(\wp_n)$$

which implies $\sum_{n=1}^{N} \delta_{\mathbb{A}_n}(0) \leq \sum_{n=1}^{N} \delta_{\mathbb{A}_n}(\wp_n)$

iii. $\sum_{n=1}^{N} \gamma_n \varphi_{\mathbb{A}_n}(\wp_n) = \sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n)$

$$\geq \sum_{n=1}^{N} \min\{(\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n * q_n), (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(q_n)\}$$

$$= \sum_{n=1}^{N} \min\{(\gamma_n \varphi_{\mathbb{A}_n}(\wp_n * q_n)), (\gamma_n \varphi_{\mathbb{A}_n}(q_n))\}$$

$$= \sum_{n=1}^{N} \min \gamma_n\{\varphi_{\mathbb{A}_n}(\wp_n * q_n), \varphi_{\mathbb{A}_n}(q_n)\}$$

which implies $\sum_{n=1}^{N} \varphi_{\mathbb{A}_n}(\wp_n) \geq \sum_{n=1}^{N} \min\{(\wp_n * q_n), \varphi_{\mathbb{A}_n}(q_n)\}$

iv. $\sum_{n=1}^{N} \gamma_n \delta_{\mathbb{A}_n}(\wp_n) = \sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n)$

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$$\leq \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n * q_n), (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(q_n)\}$$
$$= \sum_{n=1}^{N} \max\{(\gamma_n \delta_{\mathbb{A}_n}(\wp_n * q_n)), (\gamma_n \delta_{\mathbb{A}_n}(q_n))\}$$
$$= \sum_{n=1}^{N} \max \gamma_n\{\delta_{\mathbb{A}_n}(\wp_n * q_n), \delta_{\mathbb{A}_n}(q_n)\}$$

which implies $\sum_{n=1}^{N} \delta_{\mathbb{A}_n}(\wp_n) \leq \sum_{n=1}^{N} \max\{(\wp_n * q_n), \delta_{\mathbb{A}_n}(q_n)\}$

Theorem 4.8

An N-generated intuitionistic fuzzy Z – Ideal of $\mathbb{P}$ be $\sum_{n=1}^{N} \mathbb{A}$. Then $\sum_{n=1}^{N} \mathbb{A}_{\gamma_n}^{V_n}(\wp_n)$ is γ – N-generated Intuitionistic FM of $\sum_{n=1}^{N} \mathbb{A}$is N-generated intuitionistic fuzzy Z – Ideal of $\mathbb{P}$, for all $\gamma_n \in [0,1]$.

Proof:

If $\sum_{n=1}^{N} \mathbb{A}$ is an N-generated intuitionistic fuzzy Z – Ideal of $\mathbb{P}$, for all $\gamma_n \in [0,1]$.

i. $\sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(0) = \sum_{n=1}^{N} \gamma_n \varphi_{\mathbb{A}_n}(\wp_n) \geq \sum_{n=1}^{N} \gamma_n \varphi_{\mathbb{A}_n}(\wp_n)$

$$= \sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n)$$

which implies $\sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(0) \geq \sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n)$

ii. $\sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(0) = \sum_{n=1}^{N} \gamma_n \delta_{\mathbb{A}_n}(\wp_n) \leq \sum_{n=1}^{N} \gamma_n \delta_{\mathbb{A}_n}(\wp_n)$

$$= \sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n)$$

which implies $\sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(0) \leq \sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n)$

iii. $\sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n) = \sum_{n=1}^{N} \gamma_n \varphi_{\mathbb{A}_n}(\wp_n)$

$$\geq \sum_{n=1}^{N} \min \gamma_n\{\varphi_{\mathbb{A}_n}(\wp_n * q_n), \varphi_{\mathbb{A}_n}(q_n)\}$$
$$= \sum_{n=1}^{N} \min\{(\gamma_n \varphi_{\mathbb{A}_n}(\wp_n * q_n)), (\gamma_n \varphi_{\mathbb{A}_n}(q_n))\}$$
$$\Rightarrow \sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n) \geq \sum_{n=1}^{N} \min\{(\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n * q_n), (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(q_n)\}$$

iv. $\sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n) = \sum_{n=1}^{N} \gamma_n \delta_{\mathbb{A}_n}(\wp_n)$

$$\leq \sum_{n=1}^{N} \max \gamma_n\{\delta_{\mathbb{A}_n}(\wp_n * q_n), \delta_{\mathbb{A}_n}(q_n)\}$$
$$= \sum_{n=1}^{N} \max\{(\gamma_n \delta_{\mathbb{A}_n}(\wp_n * q_n)), (\gamma_n \delta_{\mathbb{A}_n}(q_n))\}$$
$$\Rightarrow \sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n) \leq \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n * q_n), (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(q_n)\}$$

Therefore $\sum_{n=1}^{N} \mathbb{A}_{\gamma_n}^{V_n} = \sum_{n=1}^{N} \left((\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}, (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}\right)$of $\sum_{n=1}^{N} \mathbb{A} = \left(\varphi_{\mathbb{A}_n}, \delta_{\mathbb{A}_n}\right) \subset \mathbb{P}$, and then for all $\gamma_n \in [0,1]$.

Theorem 4.9

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For any N-generated intuitionistic fuzzy Z – sub-algebra $\sum_{n=1}^{N} \mathbb{A}$ of $\mathbb{P}$, $\forall \gamma_n \in [0,1]$. If γ - N-generated Intuitionistic Fuzzy Multiplication be$\sum_{n=1}^{N} \mathbb{A}_{\gamma_n}^{V_n}(\wp_n)$of $\sum_{n=1}^{N} \mathbb{A}$be N-generated intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$.

Proof:

Let $\gamma_n \in [0,1]$ and let $\wp_n, q_n \in \mathbb{P}$.

Now,

$$\sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n * q_n) = \sum_{n=1}^{N} \gamma_n \varphi_{\mathbb{A}_n}(\wp_n * q_n)$$

$$\geq \sum_{n=1}^{N} \min\{\gamma_n(\varphi_{\mathbb{A}_n}(\wp_n), \varphi_{\mathbb{A}_n}(q_n))\}$$

$$\geq \sum_{n=1}^{N} \min\{\gamma_n \varphi_{\mathbb{A}_n}(\wp_n), \gamma_n \varphi_{\mathbb{A}_n}(q_n)\}$$

$$\sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n * q_n) \geq \sum_{n=1}^{N} \min\{(\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n), (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(q_n)\}$$

$$\sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n * q_n) = \sum_{n=1}^{N} \gamma_n \delta_{\mathbb{A}_n}(\wp_n * q_n)$$

$$\leq \sum_{n=1}^{N} \max\{\gamma_n(\delta_{\mathbb{A}_n}(\wp_n), \delta_{\mathbb{A}_n}(q_n))\}$$

$$\leq \sum_{n=1}^{N} \max\{\gamma_n \delta_{\mathbb{A}_n}(\wp_n), \gamma_n \delta_{\mathbb{A}_n}(q_n)\}$$

$$\sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n * q_n) \leq \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n), (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(q_n)\}$$

Theorem 4.10

If γ - N-generated Intuitionistic Fuzzy Multiplication be $\sum_{n=1}^{N} \mathbb{A}_{\gamma_n}^{V_n}(\wp) = \sum_{n=1}^{N}((\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp), (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp))$ of $\sum_{n=1}^{N} \mathbb{A} = (\varphi_{\mathbb{A}_n}, \delta_{\mathbb{A}_n})$ be N-generated intuitionistic fuzzy Z – sub-algebra of $\mathbb{P}$. Then for all N-generated intuitionistic fuzzy Z – sub-algebra $\sum_{n=1}^{N} \mathbb{A}$of $\mathbb{P}$.

Proof:

We assume that $\sum_{n=1}^{N} \mathbb{A}_{\gamma_n}^{V_n}(\wp_n)$of $\sum_{n=1}^{N} \mathbb{A}$ in $\mathbb{P}$, $\forall \gamma_n \in [0,1]$.

$$\sum_{n=1}^{N} \gamma_n \varphi_{\mathbb{A}_n}(\wp_n * q_n) = \sum_{n=1}^{N} (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n * q_n)$$

$$\geq \sum_{n=1}^{N} \min\{(\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n), (\varphi_{\mathbb{A}})_{\gamma_n}^{V_n}(q_n)\}$$

$$= \sum_{n=1}^{N} \min\{\gamma_n \varphi_{\mathbb{A}_n}(\wp_n), \gamma_n \varphi_{\mathbb{A}_n}(q_n)\}$$

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$= \sum_{n=1}^{N} \min\{\gamma_n(\varphi_{\mathbb{A}_n}(\wp_n), \varphi_{\mathbb{A}_n}(q_n))\}$

Implies that $\sum_{n=1}^{N} \varphi_{\mathbb{A}_n}(\wp_n * q_n) \geq \sum_{n=1}^{N} \min\{\varphi_{\mathbb{A}_n}(\wp_n), \varphi_{\mathbb{A}_n}(q_n)\}$

$$\sum_{n=1}^{N} \gamma_n \delta_{\mathbb{A}_n}(\wp_n * q_n) = \sum_{n=1}^{N} (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n * q_n)$$

$$\leq \sum_{n=1}^{N} \max\{(\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(\wp_n), (\delta_{\mathbb{A}})_{\gamma_n}^{V_n}(q_n)\}$$

$$= \sum_{n=1}^{N} \max\{\gamma_n \delta_{\mathbb{A}_n}(\wp_n), \gamma_n \delta_{\mathbb{A}_n}(q_n)\}$$

$$= \sum_{n=1}^{N} \max\{\gamma_n(\delta_{\mathbb{A}_n}(\wp_n), \delta_{\mathbb{A}_n}(q_n))\}$$

Implies that $\sum_{n=1}^{N} \delta_{\mathbb{A}_n}(\wp_n * q_n) \leq \sum_{n=1}^{N} \max\{\delta_{\mathbb{A}_n}(\wp_n), \delta_{\mathbb{A}_n}(q_n)\}$

Hence $\sum_{n=1}^{N} \mathbb{A} \subset \mathbb{P}$ be a N-generated intuitionistic fuzzy Z – sub-algebra.

Conclusion: In this paper, the new concept of a n-generated Fuzzy and Intuitionistic Fuzzy Translation and Multiplication in Z-Algebra has been define through concept of Fuzzy Translation and Fuzzy Multiplication were discussed using Z-sub algebra and Z-Ideals, as well as various algebraic properties. Z-Algebras are seen as another generalisation of BCK/BCI–algebras. Fuzzy extensions of Z-Ideals in Z-Algebras have been investigated, which adds a new dimension to the previously defined Z-Algebra. They were also used to prove various theorem.

References:

[1] Abu Ayub Ansari and Chandramouleeswaran M: Fuzzy Translation of Fuzzy ideals of algebras, International Journal of Pure and Applied Mathematics, Vol.92 No. 5, (2014), 657-667.

[2] Chandramouleeswaran M,Muralikrishna P, Sujatha K, Sabarinathan S: A note on Z-algebra, Italian Journal of Pure and Applied Mathematics, 38 (2017), 707–714.

[3] Sowmiya S,Jeyalakshmi P:On Fuzzy Z-ideals in Z-algebras, Global Journal of Pure and Applied Mathematics, 15(4) (2019), 505–516.

[4] Kyoung Ja Lee, Young Bae Jun and Myung Im Doh: Fuzzy Translations and Fuzzy Multiplication of BCK/BCI-Algebras, Commun. Korean Math. Soc. 24 (2009), No. 3, 353-360.

[5] Priya and Ramachandran T: Fuzzy Translation and Multiplication on PS-Algebras, International Journal of Innovation in Science and Mathematics, Vol.2, No.5, (2014), 485-489.

[6] Prasanna A, Premkumar M and Ismail Mohideen S: Fuzzy Translation and Multiplication on B-Algebras, International Journal for Science and Advance Research In Technology, Vol.4, No.4, (2018), 2898-2901.

[7] Prasanna, PremkumarM and Ismail Mohideen S: Fuzzy Translations and fuzzy Multiplications on BG-Algebras, Proceedings in 4th AltermanInternationalConference-Cum on Computational & Geometric Algebra, (July 2019).

[8] Zadeh L A: Fuzzy Sets, Information and Control, Vol.8, (1965), 338-353.

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