# INVERSE OF TILDE(T) AND MINUS PARTIAL ORDERING ON INTUITIONISTIC FUZZY MATRICES

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**Abstract:** Aim of this article is to characterize the inverse or reverse T and minus orderings on Intuitionistic fuzzy matrices(IFM). Also, using the g- inverses, we discuss some Theorems and examples for the inverse or reverse T and minus ordering on IFM.
**Keywords**: Intuitionistic fuzzy matrices (IFM), Inverse or reverse T−ordering, Inverse or reverse minus ordering, g-inverse, Moore-penrose inverses.

**1. INTRODUCTION** : Let the IFM A of order m rows and n columns is in the form of $A=[y_{ij}, < a_{ij\alpha}, a_{ij\beta} >]$ ,where $a_{ij\alpha}$ and $a_{ij\beta}$ are called the degree of membership and also the non-membership of $y_{ij}$ in A , it preserving the condition $0 \le a_{ij\alpha} + a_{ij\beta} \le 1$ .In intuitionistic fuzzy matrices, partial ordering is significant. The idea of fuzzy matrix was first presented by Thomosan [2] in 1977 and it has further developments by various researchers. Jian Miao Chen pioneered the partial orderings on fuzzy matrices, which are comparable to the star ordering on complex matrices [3].After that, a lot of works have been done using this notion. A.R. Meenachi [1] characterizes the minus ordering on matrices in terms of their generalized inverses. Another novelty is the way she defines space ordering [6] on fuzzy matrices as a partial order on the set of all idempotent matrices in $F_n$. Partial ordering is a reflexive, anti-symmetric, transitive crisp binary relation R(X, X) [5]. The properties of this class of relations are denoted by the common symbol ≤ . Therefore, <x ,y> represents <x,y> ∈ R and indicates that x comes before y. The symbol ≥ [9] denotes the inverse partial ordering $R^{-1}$ (X, X). We say that y succeeds x if y ≤ x implying that <x,y> ∈ $R^{-1}$. The symbols $\le^{P}$ , $\le^{Q}$ and $\le^{R}$ are used to denote the various partial orderings P, Q, and R, respectively. In this Section I , As an analogue to the star ordering on complex matrices, we start with the T inverse or reverse ordering on IFM. We explore different ordering on the IFM using a variety of generalized inverses, including g-inverse, group inverse, and Moore-penrose inverses, and we analyses how these ordering relate to T ordering[8]. We derive some equivalent conditions for each ordering by using generalized inverses[10]. Additionally, we demonstrate that these orderings are the same for a particular class of IFM. In section II we study the minus ordering for IFM as an analogue of minus ordering for complex matrix studied in [7] and as a generalization of T-ordering for IFM introduced . We show that the minus ordering is only a partial ordering in the

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set of all regular fuzzy matrices. Finally, we characterize the minus ordering on matrix in terms of their generalized inverses.

## 2. INVERSE OR REVERSE T-ORDERING ON INTUITIONISTIC FUZZY MATRICES

**Definition:2.1** For $A, B$ belongs to $(IF)_{m\times n}$ the T−ordering $P \leq^T Q$ is well-defined as $P \leq^T Q$ $\Leftrightarrow$ $P^tP = P^tQ$ and $PP^t = QP^t$.

**Definition:2.2** For $A, B$ belongs to $(IF)_{m\times n}$ the T− Reverse (or) inverse ordering $P \overset{T}{\geq} Q$ is defined as $P \overset{T}{\geq} Q \Leftrightarrow Q^tQ = Q^tP$ and $QQ^t = PQ^t$.

**Example:2.1** Let us consider, $P = \begin{bmatrix} <1,0> & <0,1> \\ <1,0> & <1,0> \end{bmatrix}, Q = \begin{bmatrix} <1,0> & <1,0> \\ <1,0> & <1,0> \end{bmatrix}$.

**Theorem 2.1** . Let $P, Q \in (IF)_{m\times n}$ and $Q^+$ exists. Then the given conditions are equivalent.

(i) $P \overset{T}{\geq} Q$
(ii) $Q^+Q = Q^+P$ and $QQ^+ = PQ^+$
(iii) $QQ^+P = Q = PQ^+Q$

**Proof:** (i) $\Rightarrow$ (ii) ,By (i) We have $P \overset{T}{\geq} Q \Leftrightarrow Q^t Q = Q^t P$ and $QQ^t = PQ^t$
Then $Q^+Q = Q^+QQ^+Q = Q^+(Q^+)^t Q^t Q = Q^+(Q^+)^t Q^t P = Q^+Q\, Q^+P = Q^+P$
Similarly, $QQ^+ = PQ^+$
(ii) $\Rightarrow$ (iii) $Q^+Q = Q^+P$ implies $Q = QQ^+Q = QQ^+P$ and $QQ^+ = PQ^+$ implies $Q = QQ^+Q = PQ^+Q$
(iii) $\Rightarrow$ (i) By $Q = QQ^+P$, $(QQ^+)^t Q = (QQ^+)^t P$
Then, $Q^t (Q^t)^t Q^t Q = Q^t (Q^+)^t Q^t P$. Hence $Q^t Q = Q^t P$
Similarly, $QQ^t = PQ^t$ by $Q = PQ^+Q$

**Theorem 2.2** Let $P, Q \in (IF)_{m\times n}$ If $P^+$ and $Q^+$ both exists. Then the given conditions are equivalent.

(i) $P \overset{T}{\geq} Q$
(ii) $Q^+Q = P^+Q$ and $QQ^+ = QP^+$
(iii) $P^+QQ^+ = Q^+ = Q^+QP^+$
(iv) $Q^t Q P^+ = Q^t = P^+ Q Q^t$

**Proof:** (i) $\Rightarrow$ (iv) $Q^t Q = Q^t P$ implies $Q^t Q = Q^t PP^+P$
Then $Q^tQ = (Q^tQ)^t = (P^+P)^t(Q^tP)^t = P^+PQ^tQ$
Hence, $Q^tQQ^+ = P^+PQ^tQQ^+$ and $Q^t (QQ^+)^t = P^+PQ^t (QQ^+)^t$
Therefore, $Q^t = P^+PQ^t = P^+QQ^t$
Similarly, $Q^t = Q^tQP^+$ by $QQ^t = PQ^t$
(iv) $\Rightarrow$ (ii) By $Q^t = P^+QQ^t$, $Q^t (Q^+)^t = P^+QQ^t(Q^+)^t$
Then, $Q^+Q = P^+QQ^+Q = P^+Q$

Similarly, $QQ^+ = QP^+$ and $Q^t = Q^tQP^+$

(ii) $\Rightarrow$ (i) $Q^+Q = (Q^+Q)^t = (P^+Q)^t = (P^+PP^+Q)^t = (P^+Q)^t(P^+P)^t = (Q^+Q)^tP^+P = Q^+QP^+P = Q^+QQ^+P = Q^+P$

Similarly, we have $QQ^+ = PQ^+$ .Thus (i) holds by Theorem 2.1 (ii)

(ii) $\Rightarrow$ (iii) By $Q^+Q = P^+Q$, $Q^+= Q^+QQ^+= P^+QQ^+$

Similarly , $QQ^+ = QP^+$ implies $Q^+= Q^+QP^+$

(iii) $\Rightarrow$ (ii) $P^+QQ^+ = Q^+ =Q^+QP^+$ implies $Q^+Q = P^+QQ^+Q= P^+Q$ and $QQ^+ = QQ^+QP^+=QP^+$

**Theorem 2.3** In $(IF)^+_{m\times n}$, the set of all IFM $P \in (IF)_{m\times n}$ for which $P^+$ exists $\overset{T}{\geq}$ is a partial ordering.

**Proof:** $R \overset{T}{\geq} R$ obvious. If $R \overset{T}{\geq} Q$, $Q \overset{T}{\geq} R$ then $R = QR^+R$, $P=PP^+R$ by theorem 2.1 (iii). Thus, by Theorem 2.2(ii) $P=PP^+Q= PQ^+Q = Q$

If $R \overset{T}{\geq} Q$ ,$Q \overset{T}{\geq} P$ then $R = QR^+R$ and $Q = PQ^+Q$ by Theorem 2.1 (iii) and Theorem 2.2(ii) , we have $R=QR^+R= PQ^+QR^+R= PQ^+R= PR^+R$

Similarly, we have $R=RR^+P$ . Thus , $R \overset{T}{\geq} P$ by Theorem 2.1 (iii)

**Example:2.2** Let $Q = \begin{bmatrix} <1,0> & <0,1> \\ <1,0> & <1,0> \end{bmatrix}, P = \begin{bmatrix} <1,0> & <1,0> \\ <1,0> & <1,0> \end{bmatrix}$.For Q, $QQ^tQ \neq Q$. Therefore $Q^+$ does not exists. Here $Q \overset{T}{\geq} P$ and $P \overset{T}{\geq} Q$ but, $Q \neq P$. Thus $\overset{T}{\geq}$ is not a partial ordering in $(IF)_{m\times n}$.

**Theorem 2.4** If $P \overset{T}{\geq} Q$ then we have

(i) $P^+Q = Q^+P$ and $QP^+ = PQ^+$

(ii) $P^tQ = Q^tP$ and $PQ^t = QP^+$ (i.e) $P^tQ$ and $PQ^t$ are symmetric,

(iii) $QP^+Q = Q = PP^+Q = PQP^+ = P^+QP$, $PQ^+Q = Q = QQ^+P = QPQ^+ = Q^+PQ$

**(iv)** $PQ^tQ = QQ^tP = Q^tPQ = QPQ^t$, $QP^tP = PP^tQ = P^tQP = PQQ^t$

**Theorem 2.5** If $P \overset{T}{\geq} Q$ ,then we have

(i) $P^t \overset{T}{\geq} Q^t$

(ii) $P^+ \overset{T}{\geq} Q^+$

(iii) $P^tQ \overset{T}{\geq} P^tP, QP^t \overset{T}{\geq} QQ^t$

(iv) $P^+Q \overset{T}{\geq} P^+P, QP^+ \overset{T}{\geq} PP^+$

(v) $P^tP \overset{T}{\geq} Q^tQ, PP^t \overset{T}{\geq} QQ^t$

(vi) $P^+P \overset{T}{\geq} Q^+Q, PP^+ \overset{T}{\geq} QQ^+$

(vii) If $P^tP^+ = P^+P^t$ then $Q^tQ^+ = Q^+Q^t$

(viii) if $P^+ = P^t$ then $Q^+ = Q^t$
(ix) if $P^2=0$ then $Q^2=0$
(x) if $P=P^2$ then $Q=Q^2$
(xi) if $P = PP^T$ then $Q = QQ^t$
(xii) if $P = P^T = P^3$ and $Q=Q^t$ then $Q = Q^3$

**Proof:** (i) and (ii) hold clearly.

**iii.** $\left(P^tQ\right)^t P^tQ = Q^tPP^tQ = Q^tQQ^tQ = Q^tQQ^tP = Q^tQP^tP = Q^tPP^tP$

$Similarly, P^tQ\left(Q^tQ\right)^t = P^tP\left(P^tQ\right)^t$, $Thus\ P^tQ \overset{T}{\geq} P^tP.$

Similarly, we have, $QP^t \overset{T}{\geq} PP^T$

iv. $\left(P^+Q\right)^t P^+Q = \left(Q^+Q\right)^t Q^+Q = Q^t\left(Q^+\right)^t Q^+Q = Q^t\left(Q^+\right)^t Q^+P = Q^t\left(Q^+\right)^t P^+P = \left(Q^+Q\right)^t P^+P$

$= \left(P^+Q\right)^t P^+P\ and\ P^+Q\left(P^+Q\right)^T = P^+P\left(P^+Q\right)^T. Thus\ P^+Q \overset{T}{\geq} P^+P$

Similarly we have $QP^+ \overset{T}{\geq} PP^+$

v. $P^tP \overset{T}{\geq} Q^tQ, PP^t \overset{T}{\geq} QQ^t$

$\left(Q^tQ\right)^t Q^tQ = Q^tQQ^tP = Q^tQP^tP = \left(Q^tQ\right)^t P^tP$ and $Q^tQ\left(Q^tQ\right)^t = P^tP\left(Q^tQ\right)^t$

$P^tP \overset{T}{\geq} Q^tQ$

Similarly, $PP^t \overset{T}{\geq} QQ^t$

vi. $P^+P \overset{T}{\geq} Q^+Q, PP^+ \overset{T}{\geq} QQ^+$

$\left(Q^+Q\right)^t P^+P = Q^t\left(Q^+\right)^t P^+P = Q^t\left(Q^+\right)^t Q^+P = Q^t\left(Q^+\right)^t Q^+Q = \left(Q^+Q\right)^t Q^+Q$

and $P^+P\left(Q^+Q\right)^t = Q^+Q\left(Q^+Q\right)^t$ ( $Q^tQ = Q^tP$ and $QQ^t = PQ^t$)

vii. If $P^tP^+ = P^+P^t$ then $Q^tQ^+ = Q^+Q^t$
$Q^tQ^+ = Q^+QP^tP^+QQ^+ = Q^+QP^+P^tQQ^+ = Q^+Q^t$
viii. if $P^+ = P^t$ then $Q^+ = Q^t$
$Q^+ = Q^+ Q Q^+ = P^+ QQ^+ = P^+ QP^+ = P^+QP^t = P^+QQ^t = Q^t$
ix.if $P^2=0$ then $Q^2=0$
$Q^2 = QP^+PPP^+Q = QP^+P^2 P^+Q = 0$
x.if $P = P^2$ then $Q = Q^2$
$Q^2 = QP^+PPP^+Q = QP^+PP^+Q = QP^+Q = QQ^+Q = Q$
xi.if $P = PP^t$ then $Q = QQ^t$
By $P = PP^t$, we have $P^t=P$ and $P^+=P$
Then $QQ^t = PQ^t = PP^tQ = PPQ^t = PQQ^t = P^+QQ^t = Q^t = Q$
xii.$Q^3 = QQ^tQ = PQ^tQQ^+P = PP^tQQ^+P = PP^tPQ^+P = PQ^+P = PP^+Q = Q$

## 3.INVERSE OR REVERSE MINUS ORDERING ON IFM

**Definition:3.1** For $P\in(IF)^{-}_{m,n}$ and $Q\in(IF)_{m\times n}$ the inverse or Reverse minus ordering as $\geq$ is defined as $P\geq Q \Leftrightarrow Q^{-}Q=Q^{-}P$ and $QQ^{-}=PQ^{-}$ for some $Q^{-}\in Q\{1\}$

To specify the minus ordering with respect to particular g-inverse of P, let us write $P\geq Q$ with respect to $X \Leftrightarrow XQ=XP$ *and* $QX=PX$ *for* $X\in Q\{1\}$.

**Remark :3.1** For $Q\in(IF)^{-}_{m,n}$ and $P\in(IF)_{m\times n}$ if $Q^{+}$ exists, then $Q^{+}$ is unique and $Q^{+}=Q^{T}$ we have, $P\overset{T}{\geq}Q \Leftrightarrow P\geq Q$ with respect to $Q^{+}\Leftrightarrow Q^{t}Q=Q^{t}P$ *and* $QQ^{t}=PQ^{t}$,which is precisely Definition 2.1 of T-ordering. Thus T-ordering is a special case of minus ordering. However the converse $P\geq Q \Rightarrow P\overset{T}{\geq}Q$ need not be true.

**Example:3.1** Let us consider, $P=\begin{bmatrix} <1,0> & <1,0> \\ <0,0> & <0,0> \end{bmatrix}, Q=\begin{bmatrix} <1,0> & <1,0> \\ <0,0> & <0,0> \end{bmatrix}$. Since $Q^{t}$ is a g-inverse of Q, $Q^{+}$ exist and $Q^{+}=Q^{t}$ also Q is idempotent , Q itself is a g-inverse of Q, Q = QP=PQ implies $P\geq Q$ with respect to Q. $Q^{t}Q\neq Q^{t}P$ *and* $QQ^{t}\neq PQ^{t}$ .*Hence* $P\geq Q$ *not implies* $P\overset{T}{\geq}Q$.

**Theorem:3.1** For $Q\in(IF)^{-}_{m,n}$ and $P\in(IF)_{m\times n}$ the given conditions are equivalent

(i) $P\geq Q$

(ii) $Q=QQ^{-}P=PQ^{-}Q=PQ^{-}P$

**Proof:** (i) $\Rightarrow$(ii)

$P\geq Q \Leftrightarrow Q^{-}Q=Q^{-}P$ and $QQ^{-}=PQ^{-}$ for some $Q^{-}\in Q\{1\}$

Now, $Q=Q\left(Q^{-}Q\right)=QQ^{-}P$

$Q=\left(QQ^{-}\right)Q=PQ^{-}Q$

$Q=P\left(Q^{-}Q\right)=PQ^{-}P$

(ii) $\Rightarrow$(i)

Let $X=Q^{-}QQ^{-}$

$QXQ=Q(Q^{-}QQ^{-})Q=(QQ^{-}Q)Q^{-}Q=Q\Rightarrow X\in Q\{1\}$

$Now, XQ=(Q^{-}QQ^{-})QQ^{-}P$

$=Q^{-}(QQ^{-}Q)Q^{-}P$

$=\left(Q^{-}QQ^{-}\right)P$

$= XP$

Similarly, QX=PX

Hence $P \geq Q$ with respect to $X \in Q\{1\}$

**Remark :3.2** In general, in the definition of minus ordering $P \geq Q$, P need not be regular. This is illustrated in the following example.

**Example:3.2** Let us consider

$$P=\begin{bmatrix} <1,0> & <1,0> & <0,0> \\ <0,0> & <1,0> & <1,0> \\ <0,0> & <0,0> & <1,0> \end{bmatrix}, Q=\begin{bmatrix} <1,0> & <1,0> & <1,0> \\ <1,0> & <1,0> & <1,0> \\ <1,0> & <1,0> & <1,0> \end{bmatrix}$$

Since Q is idempotent ,Q is regular and Q itself is a g- inverse of Q. Here Q= QP = PQ. Hence $P \geq Q$ which implies $Q = Q^{-} \in Q\{1\}$. If P is not regular ,since there is no $X \in F_3$ such that PXP = P

**Theorem:3.2** Let $P,Q \in (IF)^{-}_{m,n}$. If $P \geq Q$,then $P\{1\} \subseteq Q\{1\}$

**Proof:** $P \geq Q \Rightarrow Q = QQ^{-}P = PQ^{-}Q$

$For, P^{-} \in P\{1\}$

$QP^{-}Q = (QQ^{-}P)P^{-}(PQ^{-}Q)$

$QP^{-}Q = QQ^{-}(PP^{-}P)Q^{-}Q$

$= (QQ^{-}P)Q^{-}Q = QQ^{-}Q = Q$

$Hence, QP^{-}Q = Q \;\; for \;\; each \;\; P^{-} \in P\{1\}$

Therefore, $P\{1\} \subseteq Q\{1\}$

**Theorem:3.3** If $P \geq Q$ and Q is idempotent then Q is a g-inverse of P.

**Proof.** Let P itself is a g-inverse of P then P is regular, P is idempotent . Here P ∈P{1}. Then by above property P{1} ⊆ Q{1}. Hence P is a g-inverse of Q.

**Example.3.3** Let us Consider

$$P=\begin{bmatrix} <1,0> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}, Q=\begin{bmatrix} <1,0> & <1,0> \\ <0.5,0.5> & <0,1> \end{bmatrix}$$

P is not idempotent.

$$Q\{1\} = \left\{ X : X = \begin{bmatrix} <1,0> & <\beta,0> \\ <1,0> & <\alpha,0> \end{bmatrix}, 0.5 \leq \beta \leq 1, and\ 0 \leq \alpha \leq 1 \right\}$$

Here, $P \geq Q$ for

$$Q^{-} = \begin{bmatrix} <0,1> & <0.5,0> \\ <1,0> & <1,0> \end{bmatrix} \; but \; P \notin Q\{1\}$$

**Theorem3.3** For $P,Q \in (IF)^{-}_{m,n}$ then the given conditions are equivalent

(i) $P \geq Q$

(ii) $Q = QP^{-}P = PP^{-}Q = QP^{-}Q \;\; for\, all\, P^{-} \in P\{1\}$

(iii) $R(Q) \subseteq R(P), C(Q) \subseteq C(P) \;\; and\; QP^{-}Q = Q$

**Proof:** (i) $\Rightarrow$(ii): $Q = PQ^{-}P$ (By theorem 3.1)

$= PQ^{-}(PP^{-}P)$

$= (PQ^{-}P)P^{-}P$

$= QP^{-}P$ (By theorem 3.1)

Therefore, $Q = QP^{-}P \;\; for \;\; each \;\; P^{-} \in P\{1\}$

$Similarly, we\, have\, Q = PP^{-}Q \;\; for \;\; each \;\; P^{-} \in P\{1\}$

$Also,\; Q = QP^{-}Q$ (By theorem 3.1)

(ii) $\Rightarrow$(iii): $Q = QP^{-}P = PP^{-}Q = QP^{-}Q \;\; for\, all\, P^{-} \in P\{1\}$

$Q = QP^{-}P \;\; for\, all\, P^{-} \in P\{1\}$

$Q = XPP^{-}P$, Q = XP

$Q = XP \Leftrightarrow R(Q) \subseteq R(P)$

$Q = PP^{-}Q \; for\, all\, P^{-} \in P\{1\}$

$Q = PP^{-}PY$

$Q = PY \Leftrightarrow C(Q) \subseteq C(P),$

(iii) $\Rightarrow$(i):Let $X = P^{-}QP^{-}$

$QXQ = Q(P^{-}QP^{-})Q$

$QXQ = (QP^{-}Q)P^{-}Q$

$= QP^{-}Q = Q \Rightarrow X \in Q\{1\}$

$Now, QX = Q(P^{-}QP^{-})$

$= PP^{-}Q(P^{-}QP^{-})$

$= PP^{-}(QP^{-}Q)P^{-}$

$= PP^{-}QP^{-}$

$= PX$

Similarly, XQ=XP and $QP^{-}Q = Q$

Hence $P \geq Q$ with respect to $X \in Q\{1\}$

**Theorem3.4** For $(IF)^{-}_{m,n}$ the minus ordering $\geq$ is a partial ordering.

**Proof:** (i) $R \geq R$ is obvious .Hence $\geq$ is reflexive.

ii. $R \geq Q \Rightarrow R = QR^{-}Q$

$Q \geq R \Rightarrow Q = QQ^{-}R = RQ^{-}Q$

$R = QR^{-}Q$

$= (QQ^{-}R)R^{-}(RQ^{-}Q)$

$= QQ^{-}(RR^{-}R)Q^{-}Q$

$= QQ^{-}(RQ^{-}Q)$

$= QQ^{-}Q$

$= Q$

Thus , $R \geq Q$ and $Q \geq R \Rightarrow R = Q$ .Hence $\geq$ **is** antisymmentric.

iii. $R \geq Q \Rightarrow R = RQ^{-}A \text{ and } R = RQ^{-}Q = QQ^{-}R$

$Q \geq P \Rightarrow Q = QQ^{-}P = PQ^{-}Q$

Let $X = Q^{-}RQ^{-}.Then\ RXR = R(Q^{-}RQ^{-})R = (RQ^{-}R)Q^{-}R = RQ^{-}R = R$

$Since, R \geq Q \text{ and } Q \geq P$ Applying Theorem 3.2 repeatedly ,we have

$RX = R(Q^{-}RQ^{-})$

$= QQ^{-}R(Q^{-}RQ^{-})$

$= QQ^{-}(RQ^{-}R)Q^{-}$

$= QQ^{-}RQ^{-}$

$= (PQ^{-}Q)Q^{-}RQ^{-}$

$= PQ^{-}(QQ^{-}R)Q^{-}$

$= P\left(Q^{-}RQ^{-}\right)$

$= PX$

Similarly, XR=XP. Since $X \in R\{1\}$ with RX = PX and XR=XP it follows that, $R \geq P$.

**Theorem 3.5** For $P \in (IF)^{-}_{m,n}$ and $Q \in (IF)_{m\times n}$ the given conditions are equivalent

(i) $P \geq Q \Leftrightarrow P^{t} \geq Q^{t}$

(ii) $P \geq Q \Leftrightarrow RPS \geq RQS$ for some invertible matrices R and S

**Proof:** $P \geq Q \Leftrightarrow QQ^{-} = PQ^{-} \text{ and } Q^{-}Q = Q^{-}P$

$\Leftrightarrow \left(QQ^{-}\right)^{t} = \left(PQ^{-}\right)^{t}$

$\Leftrightarrow \left(Q^{-}\right)^{t} Q^{t} = \left(Q^{-}\right)^{t} P^{t}$

$\Leftrightarrow \left(Q^{t}\right)^{-} Q^{t} = \left(Q^{t}\right)^{-} P^{t}$

$QQ^{-} = PQ^{-} \Leftrightarrow \left(Q^{t}\right)^{-} Q^{t} = \left(Q^{t}\right)^{-} P^{t}$

$Similarly, Q^{-}Q = Q^{-}P \Leftrightarrow Q^{t}\left(Q^{t}\right)^{-} = P^{t}\left(Q^{t}\right)^{-}$

Hence, $P \geq Q \Leftrightarrow P^t \geq Q^t$

(ii) $P \geq Q \Leftrightarrow RPS \geq RQS$ for some invertible matrices R and S

$P \geq Q \Leftrightarrow QQ^- = PQ^-$ *and* $Q^-Q = Q^-P$

Since P is regular which implies $RPS$ is also regular and $S^T P^- R^T$ a g-inverse of $RPS$

$(RQS)^-(RQS) = (S^t Q^{-1} R^t) SQR$

$(RQS)^-(RQS) = S^t Q^{-1}(R^t R) QS$

$(RQS)^-(RQ) = S^t (Q^{-1}Q) S$

$(PDQ)^-(PDQ) = S^t (Q^{-1}P) S$

$(PDQ)^-(PDQ) = (S^t Q^{-1} R^t)(RPS)$

$(PDQ)^-(PDQ) = (RQS)^-(RPS)$

$Similarly, (RQS)(RQS)^- = (RPS)(RQS)^-$

$Hence, P \geq Q \Rightarrow (RPS) \geq (RQS)$

$Conversly,\ (RPS) \geq (RQS) \Rightarrow R^t(RPS)S^t \geq R^t(RQS)S^t$

$\Rightarrow P \geq Q$

**Corollary:3.1** For $P,Q \in (IF)^+_{m,n}$ , $P \geq Q$ with respect to $P^+ \Leftrightarrow P^+ \geq Q^+$ with respect to C.

**Theorem 3.6** For $P \in (IF)^-_{m,n}$ and $Q \in (IF)_{m\times n}$ with $P \geq Q$

(i) If $P = P^2, then\ Q = Q^2$

(ii) If $P^2 = 0, then\ Q^2 = 0$

**Proof:** $Q^2 = QQ$

$= (QQ^-P)(PQ^-Q)$

$= QQ^-P^2Q^-Q$

$= (QQ^-P)Q^-Q$

$= QQ^-Q = Q$

$Q^2 = QQ = (QQ^-P)(PQ^-Q) = QQ^-P^2Q^-Q = 0$

**Remark:3.3** In the above Theorem 3.1 if $P \geq Q$ with Q idempotent then P need not be idempotent .Consider $P = \begin{bmatrix} <0,0> & <1,0> \\ <1,0> & <0,0> \end{bmatrix}, Q = \begin{bmatrix} <1,0> & <1,0> \\ <1,0> & <1,0> \end{bmatrix}$ . Here $P \geq Q$ with respect to $Q^- = Q$ .But P is not idempotent.

**Theorem 3.7** For $P,Q \in (IF)^+_{m,n}$ , $P \overset{T}{\geq} Q \Leftrightarrow P \geq Q$ and $PQ^+P = Q$

**Proof:** $P\overset{T}{\geq}Q$ and by remark (3.1) it follows that $P\geq Q$ and $QQ^{+}B=Q\Rightarrow Q=PQ^{+}P$

Conversely: if $P\geq Q$ by Theorem 3.3 $Q=QP^{-}Q$ for all $P^{-}\in P\{1\}$ .Since $P\in(IF)^{+}_{m,n}$

$P^{+}$ exist and $P^{t}=P^{+}$ is a g-inverse of P, hence $Q=QP^{+}Q=QP^{t}Q$

Now, $QQ^{t} = Q(PQ^{+}P)^{t}$

$QQ^{t}=QP^{t}QP^{t}$

$=\left(QP^{t}Q\right)P^{t}$

$QQ^{t}=QP^{t}$

$Thus, QQ^{t}=QP^{t}\Rightarrow QQ^{t}=PQ^{t}$ (Taking transpose on both sides)

Similarly, we have $Q^{t}Q=Q^{t}P$

Hence, $P\overset{T}{\geq}Q$

**Theorem 3.8** For $P,Q\in(IF)^{+}_{m,n}$ the following conditions are equivalent

i. $P\geq Q$ with respect to $Q^{+}\left(P\overset{T}{\geq}Q\right)$

ii. $P\in Q^{+}\{1,3,4\}$

iii. $P^{+}\in Q\{1,3,4\}$

Proof: (i) $\Rightarrow(ii)$ $P\geq Q$ with respect to $Q^{+}\Rightarrow Q^{+}Q=Q^{+}P$ $and$ $QQ^{+}=PQ^{+}$

$Now, Q^{+}=Q^{+}QQ^{+}=Q^{+}PQ^{+}\Rightarrow P\in Q^{+}\{1\}$

$(Q^{+}P)^{t}=\left(Q^{+}Q\right)^{t}=Q^{+}Q=Q^{+}P\Rightarrow P\in Q^{+}\{3\}$

$\left(PQ^{+}\right)^{t}=\left(QQ^{+}\right)^{t}=QQ^{+}=PQ^{+}\Rightarrow P\in Q^{+}\{4\}$

(ii) $\Rightarrow(iii)$ Since $Q^{+}=Q^{t}$ $and$ $P^{+}=P^{t}$, $We\ have$, $P\in Q^{+}\{1,3,4\}\Rightarrow P^{+}\in Q\{1,3,4\}$

(iii) $\Rightarrow(i)$ $P^{+}\in Q\{1,3,4\}\Rightarrow QP^{+}Q=Q,\left(QP^{+}\right)^{t}=QP^{+}=PQ^{+}$ $and$ $\left(P^{+}Q\right)^{t}=P^{+}Q=Q^{+}P$

$Q^{+}Q=Q^{+}Q\left(P^{+}Q\right)=\left(Q^{+}QQ^{+}\right)P=Q^{+}P$

$QQ^{+}=\left(QP^{+}Q\right)Q^{+}=\left(QP^{+}\right)QQ^{+}=PQ^{+}QQ^{+}=PQ^{+}$

Hence $P\geq Q$ with respect to $Q^{+}$

**Theorem 3.9** For $P,Q,R\in(IF)^{+}_{m,n}$, $R\in P\{2\}$ and $Q\geq R$ then $Q\in P\{2\}$.

**Proof:**

$Q\geq R\Rightarrow RR^{-}Q=QR^{-}R=QR^{-}Q=Q$

$QPQ=\left(QR^{-}R\right)P\left(RR^{-}Q\right)$

$=QR^{-}\left(RPR\right)R^{-}Q$

$= QR^{-}\left(RR^{-}Q\right)$

$= QR^{-}Q$

$= Q$

$Hence, Q \in P\{2\}.$

**Conclusion**:

We derive some equivalent conditions for each ordering by using generalized inverses. Additionally, we demonstrate that these orderings are the same for a particular class of IFM. we study the minus ordering for IFM as an analogue of minus ordering for complex matrix studied and as a generalization of T- ordering for IFM introduced . We show that the minus ordering is only a partial ordering in the set of all regular fuzzy matrices. Finally, we characterize the minus ordering on matrix in terms of their generalized inverses.

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