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HERONIAN MEAN LABELING OF SOME NEW GRAPHS

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Abstract : A graph $G = (V, E)$ with p vertices and q edges is said to be a heronian mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1,2, \ldots, q+1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lfloor \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rfloor$ or $\left\lfloor \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rfloor$ then the edge labels are distinct. In this case, f is called heronian mean labeling of G. In this paper, we proved we prove that some new graphs such as Path union of two cycles C_r, $k-$ Path union of two cycles C_r, Path union of two crowns C_r^* and $k-$ Path union of two crowns C_r^* all are heronian mean graphs.

Keywords: Graph, Heronian mean graph, Path, Cycle, Crown.

Mathematics subject classification: 05C78.

I. INTRODUCTION

By a graph, we mean finite undirected graphs without loops or multiple edges. The vertex set is denoted by $V(G)$ and the edge set is denoted by $E(G)$. The cycle of length r is denoted by C_r, the crown of length r is denoted by C_r^* and the path of length k is denoted by P_k. For all other standard terminology and notations , we follow Harary [2]. A detailed survey of graph labeling, we refer to Gallain [1]. The concept of mean labeling has been introduced by S. Somasundaram .et.al [4]. Meena. S and Mani. R [3] investigated Root square mean labeling for some cycle related graphs. Harmonic mean labeling of graph introduced by S. Somasundaram and S.S. Sandhya in [5,6]. The Heronian mean labeling of graphs was introudced by S.S. Sandhya. et.al [7,8,9] . In this paper, we investigate the heronian mean labeling of some new graphs. The following definitions are useful for the present investigation.

Definition 1.1:

A graph $G = (V, E)$ with p vertices and q edges is said to be a **heronian mean graph** if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1,2, \ldots, q+1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lfloor \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rfloor$ or $\left\lfloor \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rfloor$ then the edge labels are distinct. In this case, f is called **heronian mean labeling** of G.

Definition 1.2:

A walk in which $u_1 u_2 \ldots u_r$ are distinct is called a **path**. A path on r vertices is denoted by P_r.

Definition 1.3:

A closed path is called a **cycle**. A cycle on r vertices is denoted by C_r.

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Definition 1.4:

The **union of two graphs** $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and the edge set $= E_1 \cup E_2$.

Definition 1.5:

Let $G_1, G_2, \dots, G_r, m \geq 2$ be r copies of a fixed graph G. The graph G obtained by adding an edge between G_i and G_{i+1} for $i = 1,2, \dots, r-1$ is called a **path union of G.**

Definition 1.6:

The $\boldsymbol{k-}$ **path union of two cycles** $\boldsymbol{C_r}$ is the graph obtained by joining two vertices from two copies of C_r by a path P_k of length $k-1$.

II. MAIN RESULTS

In this paper, we investigate the heronian mean labeling of some new graphs.

Theorem: 2.1

Path union of two cycles C_r is a heronian mean graphs.

Proof:

Let $c_1, c_2, \dots, c_r$ and $d_1, d_2, \dots, d_r$ be the vertices of two cycles C_r in G.

Let $V(G) = \{c_1, c_2, \dots, c_r, d_1, d_2, \dots, d_r\}$

$$E(G) = \{c_j c_{j+1} \,/\, 1 \leq j \leq r-1\} \cup \{d_j d_{j+1} \,/\, 1 \leq j \leq r-1\} \cup \{c_r c_1, d_r d_1, c_r d_1\}.$$

Which are denoted as Figure 1

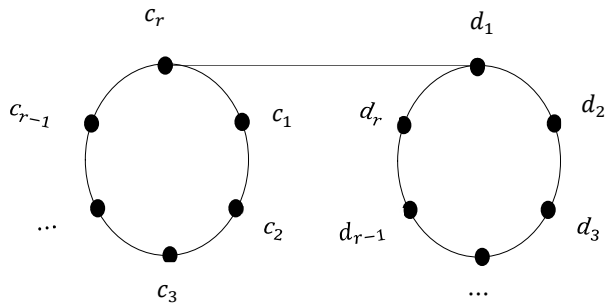

Figure 1: heronian mean labeling of path union of two cycles $\boldsymbol{C_r}$

Define a function $f: V(G) \to \{1,2, \dots, 2r+1\}$ by

$$f(c_j) = j \qquad \text{for } 1 \leq j \leq r$$

$$f(b_r) = r + j + 1 \qquad \text{for } 1 \leq j \leq r$$

Then the edge lables are distinct.

Hence f is a heronian mean labeling of G.

Illustraction:

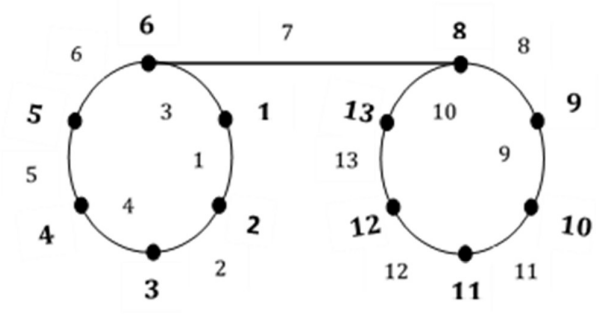

Figure 2: heronian mean labeling of path union of two cycles C_6

Theorem: 2.2

$k-$ Path union of two cycles C_r is a heronian mean graphs.

Proof:

Let $c_1, c_2, \dots, c_r$ and $d_1, d_2, \dots, d_r$ be the vertices of two cycles C_r in G.

Let $a_r = h_1, h_2, \dots, h_k = d_1$ be the vertices of path P_k.

Let $V(G) = \begin{Bmatrix} c_1, c_2, \dots, c_r, d_1, d_2, \dots, d_r, \\ h_1, h_2, \dots, h_k \end{Bmatrix}$

$$E(G) = \{c_j c_{j+1} \,/\, 1 \le j \le r-1\} \cup \{d_j d_{j+1} \,/\, 1 \le j \le r-1\}$$
$$\cup \{h_j h_{j+1} / 1 \le j \le k-1\} \cup \{c_r c_1, d_r d_1\}$$

Which are denoted as Figure 3

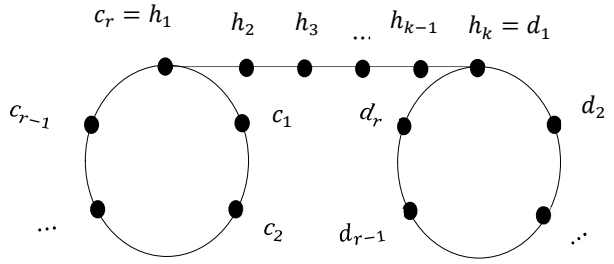

Figure 3: heronian mean labeling of $k-$ path union of two cycles C_r

Define a function $f: V(G) \to \{1,2, \dots, 2r+k-1\}$ by

$$f(c_j) = j \qquad \text{for } 1 \le j \le r$$
$$f(h_j) = r + j - 1 \qquad \text{for } 2 \le j \le k-1$$
$$f(d_j) = r + k + j - 1 \qquad \text{for } 1 \le j \le r$$

Then the edge lables are distinct.

Hence f is a heronian mean labeling of G.

Illustraction:

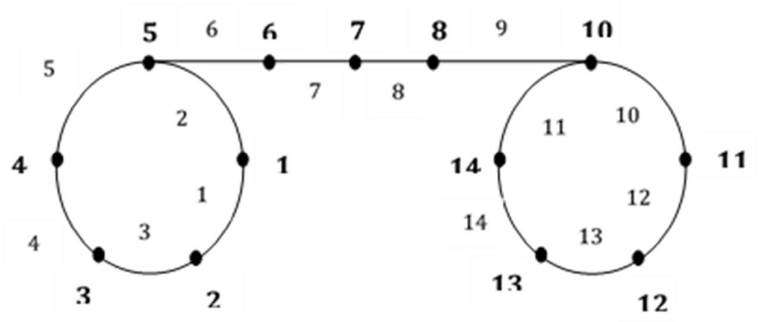

Figure 4: heronian mean labeling of $k-$ path union of two cycles C_5

Theorem: 2.3

Path union of two crowns C_r^* is a heronian mean graphs.

Proof:

Let $c_1, c_2, \dots, c_r$ and $d_1, d_2, \dots, d_r$ be the vertices of two cycles C_r in G.

Let $c_1', c_2', \dots, c_r'$ be the pendant vertices attached at $c_1, c_2, \dots, c_r$ respectively and $d_1', d_2', \dots, d_r'$ be the pendant vertices attached at $d_1, d_2, \dots, d_r$ respectively.

Let $V(G) = \{c_1, c_2, \dots, c_r, d_1, d_2, \dots, d_r, c_1', c_2', \dots, c_r', d_1', d_2', \dots, d_r'\}$

$$E(G) = \{c_j c_{j+1} \,/\, 1 \le j \le r-1\} \cup \{d_j d_{j+1} \,/\, 1 \le j \le r-1\}$$
$$\cup \{c_j c_j' \,/\, 1 \le j \le r\} \cup \{d_j d_j' \,/\, 1 \le j \le r\}$$
$$\cup \{c_r c_1, d_r d_1, c_r d_1\}.$$

Which are denoted as Figure 5

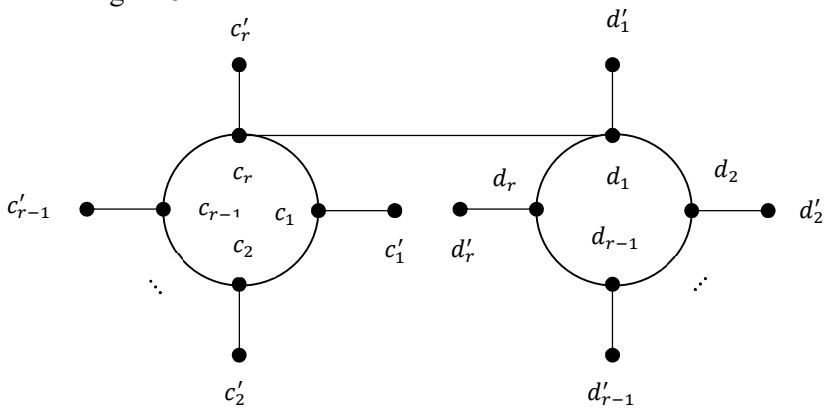

Figure 5: heronian mean labeling of path union of two crowns C_r^*

Define a function $f: V(G) \to \{1,2, \dots, 4r+1\}$ by

$$f(c_j) = 2j \qquad \text{for } 1 \le j \le r$$
$$f(c_j') = 2j - 1 \qquad \text{for } 1 \le j \le r$$
$$f(d_j) = 2r + 2j + 1 \qquad \text{for } 1 \le j \le r$$
$$f(d_j') = 2r + 2j \qquad \text{for } 1 \le j \le r$$

Then the edge lables are distinct.

Hence f is a heronian mean labeling of G.

Illustraction:

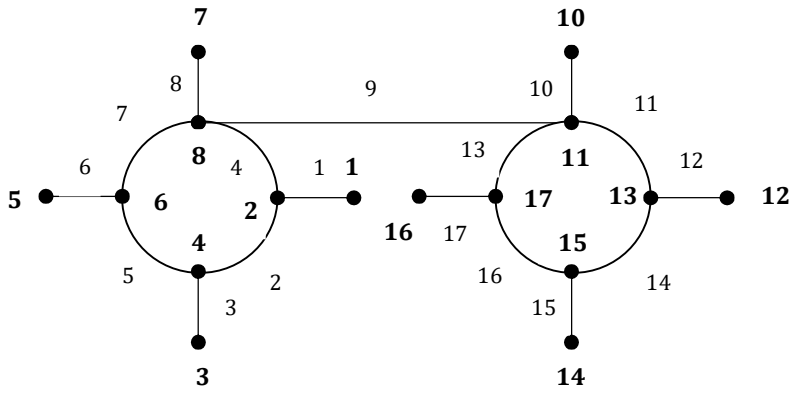

Figure 6: heronian mean labeling of path union of two crowns C_4^*

Theorem: 2.4

$k-$ Path union of two crowns C_r^* is a heronian mean graphs.

Proof:

Let $c_1, c_2, \dots, c_r$ and $d_1, d_2, \dots, d_r$ be the vertices of two cycles C_r in G.

Let $c_1', c_2', \dots, c_r'$ be the pendant vertices attached at $c_1, c_2, \dots, c_r$ respectively and $d_1', d_2', \dots, d_r'$ be the pendant vertices attached at $d_1, d_2, \dots, d_r$ respectively.

Let $a_r = h_1, h_2, \dots, h_k = d_1$ be the vertices of path P_k.

Let $V(G) = \begin{Bmatrix} c_1, c_2, \dots, c_r, d_1, d_2, \dots, d_r, h_1, h_2, \dots, h_k, \\ c_1', c_2', \dots, c_r', d_1', d_2', \dots, d_r' \end{Bmatrix}$

$$E(G) = \{c_j c_{j+1} \,/\, 1 \le j \le r-1\} \cup \{d_j d_{j+1} \,/\, 1 \le j \le r-1\}$$
$$\cup \{h_j h_{j+1} / 1 \le j \le k-1\} \cup \{c_j c_j' \,/\, 1 \le j \le r\}$$
$$\cup \{d_j d_j' \,/\, 1 \le j \le r\} \cup \{c_r c_1, d_r d_1\}$$

Which are denoted as Figure 7

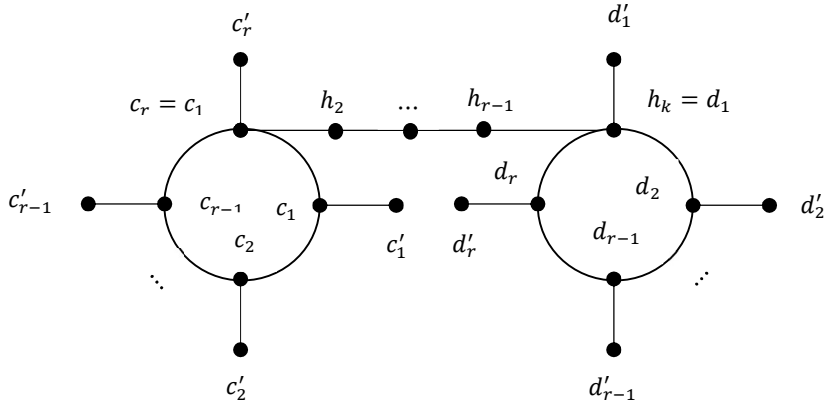

Figure 7: heronian mean labeling of $k-$path union of two crowns C_r^*

Define a function $f: V(G) \to \{1,2, \dots, 4r+k-1\}$ by

$$f(c_j) = 2j \qquad \text{for } 1 \le j \le r$$
$$f(c_j') = 2j-1 \qquad \text{for } 1 \le j \le r$$

$$f(h_j) = 2r + j - 1 \qquad \text{for } 2 \le j \le k-1$$
$$f(d_j) = 2r + k + 2j - 1 \quad \text{for } 1 \le j \le r$$
$$f(d'_j) = 2r + k + 2j - 2 \quad \text{for } 1 \le j \le r$$

Then the edge lables are distinct.

Hence f is a heronian mean labeling of G.

Illustraction:

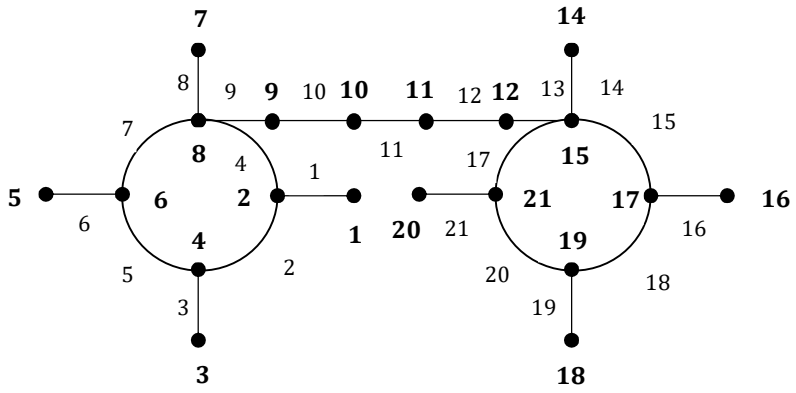

Figure 8: heronian mean labeling of k −path union of two crowns C_4^*

III. CONCLUSION

The variety of applications for labeled graphs make their study extremely important. All graphs are not heronian mean graphs. It is very interesting to investigate graphs which admit heronian mean labeling. In this paper, we proved that Path union some cycles and crowns graphs are heronian mean graph. The resulting conclusions are presented in sufficient illustrations for easier comprehension. Similar results for a variety of various graphs can be investigated.

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