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ON (C_1, C_2, C_3) PSEUDO AND (F_1, F_2, F_3) TOTALLY PSEUDO REGULAR HESITANCY FUZZY GRAPHS

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Abstract- In this paper, we examined (c_1, c_2, c_3) pseudo regular Hesitancy Fuzzy Graph and (f_1, f_2, f_3) totally pseudo regular Hesitancy Fuzzy Graph and we talked about the properties of pseudo ordinary, absolutely pseudo normal and ordinary in the fresh charts $G^*(V, E)$.

I. INTRODUCTION

L.A.Zadeh Presented the idea of a fuzzy subset of a set in 1965[1]. From that point forward the hypothesis of fuzzy set ended up being a critical idea of examination in various regions like Measurements, Topology, Engineering, Logic, Pattern Acknowledgment, Robotics, Decision Making, Signal Handling and so forth

A.Rosenfeld [3] present the idea of fuzzy graphs in 1975. A.Nagoor Gani and S.R.Latha presented and examined the irregular fuzzy graph and their properties.

T.Pathinathan and J.Jon Arockiaraj [7] presented another fuzzy graph called the Hesitancy Fuzzy Graphs. In this paper , we characterize the idea of Pseudo regular and totally Pseudo regular Hesitancy Fuzzy Graphs and examined the a portion of its properties.

II. BASIC DEFINISIONS

Definition 2.1

A Hesitancy Fuzzy Graphs is of the structure $G = (V, E)$, where $V = \{v_1, v_2, \cdots v_n\}$ such an extent that $\mu_1 : V \to [0,1]$, $\gamma_1 : V \to [0,1]$ and $\beta_1 : V \to [0,1]$ indicate the level of enrollment, non-participation and aversion of the vertex $v_i \in V$ separately and $\mu_1(v_i) + \gamma_1(v_i) + \beta_1(v_i) = 1$ for every $v_i \in V$, where $\beta_1(v_i) = 1 - [\mu_1(v_i) + \gamma_1(v_i)]$ and $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$, $\gamma_2 : V \times V \to [0,1]$ and $\beta_2 : V \times V \to [0,1]$ are to such an extent that, $\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)]$, $\gamma_2(v_i, v_j) \le \max[\gamma_1(v_i), \gamma_1(v_j)]$,
$\beta_2(v_i, v_j) \le \min[\beta_1(v_i), \beta_1(v_j)]$ and $0 \le \mu_2(v_i, v_j) + \gamma_2(u_i, v_i) + \beta_2(u_i, v_i) \le 1$ for each $(v_i, v_j) \in E$

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Definition 2.2

A HFG $H=(V',E')$ is said to be a Hesitancy Fuzzy Subgraph of $G=(V,E)$ if

(i) $V' \subseteq V$,where $\mu_1' = \mu_1, \gamma_1' = \gamma_1, \beta_1' = \beta_1$ for all $v_i \in V'$, $i = 1,2,\ldots,n$.

(ii) $E' \subseteq E$, where $\mu_2' = \mu_2, \gamma_2' = \gamma_2, \beta_2' = \beta_2$ for all $(v_i, v_j) \in E'$, $i, j = 1,2,\ldots,n$.

Definition 2.3

Hesitancy fuzzy graph $G=(V,E)$ is supposed to be normal if each vertex of G have same degree.

Definition 2.4

Let $G=(V,E)$ be an Hesitancy Fuzzy Graph on $G^*(V,E)$. Then d_2 - degree of a vertex $v \in G$ is defined by $d_2(v) = (d_{2\mu_1}(v), d_{2\gamma_1}(v), d_{2\beta_1}(v))$, where $d_{2\mu_1}(v) = \sum d_{\mu_1}(u)$ $d_{2\gamma_1}(v) = \sum d_{\gamma_1}(u)$ and $d_{2\beta_1}(v) = \sum d_{\beta_1}(u)$, , the vertex u is the adjoining the vertex v.

Definition 2.5

Let $G(V,E)$ be an Hesitancy Fuzzy Graph on $G^*(V,E)$. Then, at that point Pseudo (average) level of a vertex v in Hesitancy Fuzzy Graph G is characterized by $d_p(v) = (d_{p\mu_1}(v), d_{p\gamma_1}(v), d_{p\beta_1}(v))$, where $d_{p\mu_1}(v) = \frac{d_{2\mu_1}(v)}{d_G^*(v)}$, $d_{p\gamma_1}(v) = \frac{d_{2\gamma_1}(v)}{d_G^*(v)}$ and $d_{p\beta_1}(v) = \frac{d_{2\beta_1}(v)}{d_G^*(v)}$ where $d_G^*(v)$ is the quantity of edges occurrence at v.

III. (c_1, c_2, c_3) Pseudo and Totally Pseudo Regular Hesitancy Fuzzy Graphs

Definition 3.1

Let $G=(V,E)$ be an Hesitancy Fuzzy Graph on $G^*(V,E)$. The absolute pseudo degree of a vertex v in Hesitancy Fuzzy Graph G is characterized by $td_p(p) = (td_{p\mu_1}(v), td_{p\gamma_1}(v), td_{p\beta_1}(v))$, where $td_{p\mu_1}(v) = d_{p\mu_1}(v) + \mu_1(v)$, $td_{p\gamma_1}(v) = d_{p\gamma_1}(v) + \gamma_1(v)$ and $td_{p\beta_1}(v) = d_{p\beta_1}(v) + \beta_1(v)$, for all v in G.

Example: 3.2

Consider the following $G=(V,E)$ Hesitancy Fuzzy Graph

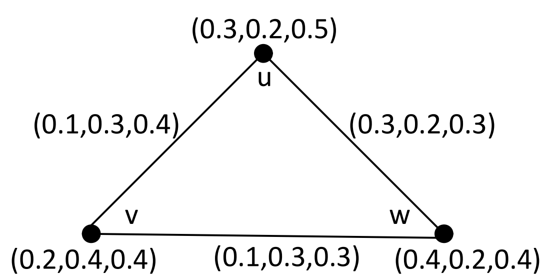

Figure-1 Hesitancy Fuzzy Graph

Pseudo degree of a vertex:

$d_p(u) = (d_{p\mu_1}(u), d_{p\gamma_1}(u), d_{p\beta_1}(u))$,where $d_{p\mu_1}(v) = \frac{d_{2\mu_1}(v)}{n_G^*(v)}$, $d_{p\gamma_1}(v) = \frac{d_{2\gamma_1}(v)}{n_G^*(v)}$ and $d_{p\beta_1}(v) = \frac{d_{2\beta_1}(v)}{n_G^*(v)}$, $d_p(u) = \left(\frac{0.6}{2}, \frac{1.1}{2}, \frac{1.3}{2}\right) = (0.3,0.55,0.65)$, $d_p(v) = \left(\frac{0.8}{2}, \frac{1}{2}, \frac{1.3}{2}\right) = (0.4,0.5,0.65)$ and $d_p(w) = \left(\frac{0.6}{2}, \frac{1.1}{2}, \frac{1.4}{2}\right) = (0.3,0.55,0.7)$.

Definition 3.3

Let $G = (V, E)$ be an Hesitancy Fuzzy Graph on $G^*(V, E)$. In the event that $d_p(v) = (c_1, c_2, c_3)$for all v in V , then G is called $(c_1, c_2, c_3) -$ pseudo regular Hesitancy Fuzzy Graph.

Example 3.4

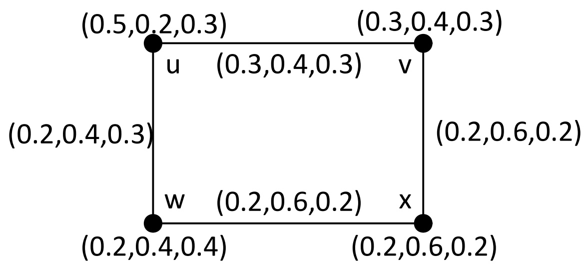

Fig. 2 $(c_1, c_2, c_3) -$ pseudo regular Hesitancy Fuzzy Graph

Pseudo degree of a vertex:

$d_p(u) = (d_{p\mu_1}(u), d_{p\gamma_1}(u), d_{p\beta_1}(u))$, where $d_{p\mu_1}(v) = \frac{d_{2\mu_1}(v)}{n_G^*(v)}$, $d_{p\gamma_1}(v) = \frac{d_{2\gamma_1}(v)}{n_G^*(v)}$ and $d_{p\beta_1}(v) = \frac{d_{2\beta_1}(v)}{n_G^*(v)}$

$$d_p(u) = d_p(v) = d_p(w) = d_p(x) = \left(\frac{0.9}{2}, \frac{2}{2}, \frac{1}{2}\right) = (0.45,1,0.5)$$

Here all the vertices have the similar pseudo degree. Hence G is called $(c_1, c_2, c_3) -$ pseudo regular Hesitancy Fuzzy Graph.

Definition **3.5**

Let $G = (V, E)$be an Hesitancy Fuzzy Graph on $G^*(V, E)$. In the event that $d_p(v) = (f_1, f_2, f_3)$, for all v in V , G is called (f_1, f_2, f_3) - totally pseudo regular Hesitancy Fuzzy Graph.

Example **3.6**

Consider an Hesitancy Fuzzy Graph on $G^*(V, E)$.

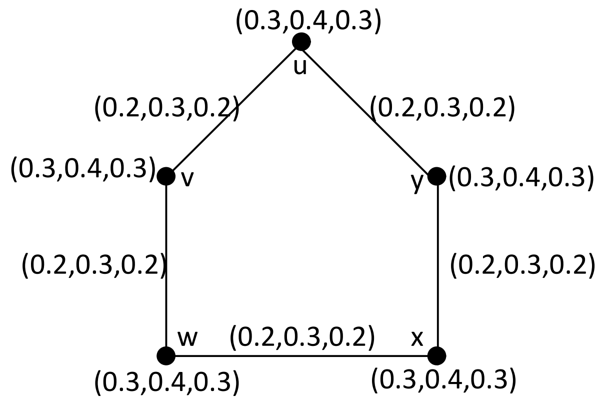

Fig. 3 Pseudo and Totally Pseudo Irregular HFG

Pseudo degree of a vertex

$d_p(u) = d_p(v) = d_p(w) = d_p(x) = d_p(y) = \left(\frac{0.8+0.8}{2}, \frac{1.2+12}{2}, \frac{0.8+0.8}{2}\right) = (\frac{1.6}{2}, \frac{2.4}{2}, \frac{1.6}{2})$ =(0.8,1.2,0.8)

Here all vertices have same pseudo degree. Consequently G is (0.8,1.2,0.8)- Pseudo regular Hesitancy Fuzzy Graph..

Total Pseudo degree of a vertex:

Also $td_p(u) = td_p(v) = td_p(w) = td_p(x) = td_p(y) = (0.8 + 0.3, 1.2 + 0.4, 0.8 + 0.3) = (1.1, 1.6, 1.1)$.

So , all vertices have same total pseudo degree .

Consequently , G is totally (1.1,1.6,1.1)- Pseudo regular Hesitancy Fuzzy Graph.

Remark 3.7

i. A pseudo regular Hesitancy Fuzzy Graph need not be a totally pseudo regular Hesitancy Fuzzy Graph.

ii. A totally pseudo regular Hesitancy Fuzzy Graph need not be a pseudo regular Hesitancy Fuzzy Graph.

Theorem 3.8

Let $G = (M, N)$ be an Hesitancy Fuzzy Graph on $G^*(V, E)$. Then ,at that point $M(v) = (\mu_1(v), \gamma_1(v), \beta_1(v))$ for all $v \in V$ is a steady capacity if and only if coming up next are same.

(i) G is a pseudo regular Hesitancy Fuzzy Graph.

(ii) $G$is a totally pseudo regular Hesitancy Fuzzy Graph.

Proof :

Assume that $M(v) = (\mu_1(v), \gamma_1(v), \beta_1(v))$ is a constant function . Let $M(v) = \left(\mu_1(v), \gamma_1(v), \beta_1(v)\right) = (f_1, f_2, f_3)$, for all $v \in V$. Suppose G is a pseudo regular Hesitancy Fuzzy Graph .Then $d_p(u) = (a_1, a_2, a_3)$, for all $u \in V$. Now, $td_p(u) = d_p(u) + M(u) = (a_1, a_2, a_3) + (\mu_1(v), \gamma_1(v), \beta_1(v)) = (a_1 + f_1, a_2 + f_2, a_3 + f_3)$ for all $u \in V$. Hence , G

is a totally pseudo regular Hesitancy Fuzzy Graph. Thus $(i) \Rightarrow (ii)$ is proved . Suppose G is a totally pseudo regular Hesitancy Fuzzy Graph. Then $td_p(u) = (b_1, b_2, b_3)$ for all $u \in V$
$\Longrightarrow d_p(u) + M(u) = (b_1, b_2, b_3)$ for all u$\in V$, $\Longrightarrow d_p(u) + (f_1, f_2, f_3) = (b_1, b_2, b_3)$ for all $u \in V$.
$\Longrightarrow d_p(u) = (b_1 - f_1, b_2 - f_2, b_3 - f_3)$ for all $u \in V$. Hence , G is a pseudo regular Hesitancy Fuzzy Graph. Thus $(ii) \Rightarrow (i)$ is proved . Hence (i) and (ii) are Equivalent. Conversely , Suppose (i) and (ii) are equivalent . Let G be a pseudo regular Hesitancy Fuzzy Graph and a totally pseudo regular Hesitancy Fuzzy Graph.
Then $d_p(u) = (a_1, a_2, a_3)$ and $td_p(u) = (b_1, b_2, b_3)$ for all $u \in V$.
Now , $td_p(u) = (b_1, b_2, b_3)$, for all $u \in V$. $\Rightarrow d_p(u) + M(u) = (b_1, b_2, b_3)$ for all u$\in V$,
$\Rightarrow (a_1, a_2, a_3) + M(u) = (b_1, b_2, b_3)$ for all $u \in V$
$\Rightarrow M(u) = (b_1 - a_1, b_2 - a_2, b_3 - a_3)$ for all $u \in V$.
Thus , M is a Constant capacity .

Theorem 3.9

Let $G = (M, N)$ be a Hesitancy Fuzzy Graph on $G^*(V, E)$. Assuming G is both pseudo regular and totally pseudo regular Hesitancy Fuzzy Graph then M is a Constant capacity.

Proof : Assume that G is both pseudo regular and totally pseudo regular Hesitancy Fuzzy Graph . Then ,at that point $d_p(u) = (a_1, a_2, a_3)$ and $td_p(u) = (b_1, b_2, b_3)$, for all $u \in V$.
Now , $td_p(u) = (b_1, b_2, b_3)$, $\Rightarrow d_p(u) + M(u) = (b_1, b_2, b_3)$,$\Rightarrow (a_1, a_2, a_3) + M(u) = (b_1, b_2, b_3)$
$\Rightarrow M(u) = (b_1, b_2, b_3) - (a_1, a_2, a_3)$,$\Rightarrow M(u) = (b_1 - a_1, b_2 - a_2, b_3 - a_3)$= Constant .
Subsequently , $M(u)$ is Constant capacity .

Theorem 3.10

Let $G = (M, N)$ be a Hesitancy Fuzzy Graph on $G^*(V, E)$, a cycle of length n . If $N(u, v)$ is a constant capacity , then G is a pseudo regular Hesitancy Fuzzy Graph.

Proof :

Let $G = (M, N)$ be a Hesitancy Fuzzy Graph on $G^*(V, E)$, a cycle of length n. If $N(u, v)$ is a constant capacity , Say, $N(u, v) = \big(\mu_2(u, v), \gamma_2(u, v), \beta_2(u, v)\big) = (k_1, k_2, k_3)$ for all $(u, v) \in E$. Then , at that point $d_p(u) = (2k_1, 2k_2, 2k_3)$, for all $u \in V$. Thus G is a $(2k_1, 2k_2, 2k_3)$ $-$pseudo regular Hesitancy Fuzzy Graph.

Remark 3.11

Converse of the above hypothesis need not be true .

Theorem 3.12

If $G = (M, N)$ is a regular Hesitancy Fuzzy Graph on $G^*(V, E)$, an r-regular graph , Then, at that point $d_a(v) = d_G(v)$, for all $v \in V$.

Proof :

Let G is a (l_1, l_2, l_3) - regular Hesitancy Fuzzy Graph on $G^*(V, E)$, an r-regular graph.

Then, at that point $d_G(v) = (l_1, l_2, l_3)$, for all $v \in G$ and $d_{G^*}(v) = r$, for all $v \in G^*$.

So, $d_2(v) = \sum d_G(v_i)$, where each v_i (for $i = 1,2, \dots, r$) is adjacent with vertex v.

$\Longrightarrow d_2(v) = \sum d_G(v_i) = r(l_1, l_2, l_3)$. Also , $d_p(v) = \frac{d_2(v)}{d_{G^*}(v)}$, $\Rightarrow d_p(v) = \frac{d_2(v)}{r}$,$\Longrightarrow d_p(v) = \frac{r(l_1,l_2,l_3)}{r}$

$\Rightarrow d_p(v) = (l_1, l_2, l_3)$,$\Rightarrow d_p(v) = d_G(v)$.

Theorem 3.13

Let $G = (M, N)$ be a Hesitancy Fuzzy Graph on $G^*(V, E)$, an r-regular graph . Then, at that point G is a pseudo regular Hesitancy Fuzzy Graph if G in case is a regular Hesitancy Fuzzy Graph .

Proof :

Let G be a (l_1, l_2, l_3)- regular Hesitancy Fuzzy Graph on $G^*(V, E)$, an r-regular graph .

$\Rightarrow d_p(v) = d_G(v)$, for all $v \in G$, $\Longrightarrow d_p(v) = (l_1, l_2, l_3)$ for all $v \in G$

$\Rightarrow$ all the vertices have same pseudo degree (l_1, l_2, l_3) .

Consequently G be a (l_1, l_2, l_3) - pseudo regular Hesitancy Fuzzy Graph.

Conclusion:

In this paper, we presented the idea of (c_1, c_2, c_3) pseudo regular Hesitancy Fuzzy Graph and $(\mathrm{f}_1, \mathrm{f}_2, \mathrm{f}_3)$ totally pseudo regular Hesitancy Fuzzy Graph dependent on their d_2 degree and pseudo degree and their properties. Further we talked about the pseudo unpredictable Hesitancy Fuzzy Graphs.

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