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PRODUCT CORDIAL LABELING OF DOUBLE WHEEL RELATED GRAPHS

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ABSTRACT
A graph G is said to be a product cordial graph if there exists a mapping g from $V(G)$ to $\{0,1\}$ such that if each line rt is given the label $g(r).g(t)$, then the cardinality of points with value 0 and the cardinality of points with value 1 differ at most by 1 and the cardinality of lines with value 0 and the cardinality of lines with value 1 differ by at most 1. In this case, g is said to be a product cordial labelling of G. In this article, we find the product cordial labelling of double wheel related graphs and we prove that the graphs such as $DW_n \odot \overline{K_1}, DW_n \odot \overline{K_2}, DW_n \odot \overline{K_3}, DW_n \odot K_2, DW_n@P_3$ all are product cordial graphs.
Keywords and phrases: Cordial labeling, Product cordial labeling, Double wheel graph, Corona product.

1. INTRODUCTION

Simple and finite graphs with p points and q lines consider in this article. For entire survey of graph labeling we refer [2]. Several variations of graph labelling have been developed including prime labeling and product cordial labeling [4,5,6,11]. Many researchers have studied product cordial graphs [7,8,9,10]. In this article, we find the existence of product cordial labeling of double wheel-related graphs such as $DW_n \odot \overline{K_1}, DW_n \odot \overline{K_2}, DW_n \odot \overline{K_3}, DW_n \odot K_2, DW_n@P_3$.

Definition 1.1.

The corona product of two graphs P and Q is define as the graph got by take one copy of P and $|V(P)|$ copies of Q and connecting the i^{th} vertex of P to all point in the i^{th} copy of Q.

Definition 1.2.
A double wheel graph DW_n of dimension n can be composed of $2C_n + K_1$. It consists of 2 cycles of dimension n where points of two cycles are all connecting to a middle point.

Definition 1.3.
$DW_n@P_3$ is the graph got by joining P_3 at every point of the double wheel DW_n.

2. MAIN RESULTS

The product cordial labeling of double wheel-related graphs were investigated in this paper.

Theorem 2.1.
$DW_n \odot \overline{K_1}$ *is a product cordial graph.*

Proof:
Let $G = DW_n \odot \overline{K_1}$

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Let $\{s_0, s_1, \dots, s_n, t_1, t_2, \dots, t_n\}$ denote the points of the double wheel of G where s_0 is apex point, $\{s_1 \dots s_n\}$ are rim points of the first cycle and $\{t_1, t_2, \dots, t_n\}$ are rim points the of second cycle.
Let $\{s_0', s_1', \dots, s_n'\}$ and $\{t_1' t_2' \dots t_n'\}$ be the pendant points attached at $\{s_0 s_1 \dots s_n\}$ and $\{t_1, t_2, \dots, t_n\}$ respectively.
Let $V(G) = \{s_0, s_0', s_1, \dots, s_n, t_1, t_2, \dots, t_n, s_1', \dots, s_n', t_1' t_2' \dots t_n'\}$

$$E(G) = \{s_0 s_\ell \,/\, 1 \le \ell \le n\} \cup \{s_\ell s_{\ell+1} \,/\, 1 \le \ell \le n-1\}$$
$$\cup \{s_\ell s_\ell' \,/\, 0 \le \ell \le n\} \cup \{s_0 t_\ell \,/ 1 \le \ell \le n\}$$
$$\cup \{t_\ell t_{\ell+1} \,/ 1 \le \ell \le n-1\} \cup \{t_\ell t_\ell' \,/\, 1 \le \ell \le n\} \cup \{s_n s_1, t_n t_1\}$$

Give a labeling h from $V(G)$ to $\{0,1\}$by:

$$h(s_0) = 1\,;\ h(s_0') = 0$$

$h(s_\ell) = 1$ for $1 \le \ell \le n$

$h(s_\ell') = 1$ for $1 \le \ell \le n$

$h(t_\ell) = h(t_\ell') = 0$ for $1 \le \ell \le n$

Here, $v_h(0) = 2n + 1$, where $v_h(0)$ is number of points with value 0
$v_h(1) = 2n + 1$, where $v_h(1)$ is number of points with value 1
$e_h(0) = 3n + 1$, where $e_h(0)$ is number of lines with value 0
$e_h(1) = 3n$, where$e_h(1)$ is number of lines with value 1

Thus the absolute difference of $v_h(0)$ and $v_h(1)$is less than or equal to 1 and the absolute difference of $e_h(0)$and $e_h(1)$ is less than or equal to 1.
Hence $DW_n \odot \overline{K_1}$ is a product cordial graph.

Illustration 2.1.1

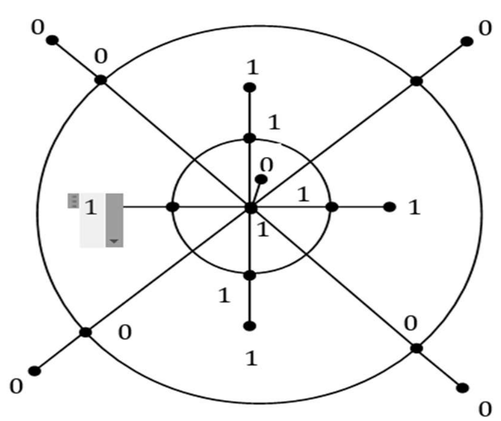

Figure 1: Product cordial labeling of $DW_4 \odot \overline{K_1}$

Theorem 2.2.

$DW_n \odot \overline{K_2}$is a product cordial graph.

Proof:

Let $G = DW_n \odot \overline{K_2}$

Let $\{s_0, s_1, \dots, s_n, t_1, t_2, \dots, t_n\}$ denote the points of double wheel of G where s_0 is apex point, $\{s_1 \dots s_n\}$ are rim points of first cycle and $\{t_1, t_2, \dots, t_n\}$ are rim points of second cycle.

Let $\{s'_0, s'_1, \dots, s'_n, s''_0, s''_1, \dots, s''_n\}$ and $\{t'_1 t'_2 \dots t'_n, t''_1 t''_2 \dots t''_n\}$ be the pendant points attached at $\{s_0 s_1 \dots s_n\}$ and $\{t_1, t_2, \dots, t_n\}$ respectively.

Let $V(G) = \begin{Bmatrix} s_0, s_1, \dots, s_n, t_1, t_2, \dots, t_n, s'_0, s'_1, \dots, s'_n, t'_1 t'_2 \dots t'_n, \\ s''_0, s''_1, \dots, s''_n, t''_1 t''_2 \dots t''_n \end{Bmatrix}$

$$E(G) = \{s_0 s_\ell \,/\, 1 \le \ell \le n\} \cup \{s_\ell s_{\ell+1} \,/\, 1 \le \ell \le n-1\}$$
$$\cup \{s_\ell s'_\ell \,/\, 0 \le \ell \le n\} \cup \{s_\ell s''_\ell \,/\, 0 \le \ell \le n\}$$
$$\cup \{s_0 t_\ell \,/\, 1 \le \ell \le n\} \cup \{t_\ell t_{\ell+1} \,/\, 1 \le \ell \le n-1\}$$
$$\cup \{t_\ell t'_\ell \,/\, 1 \le \ell \le n\} \cup \{t_\ell t''_\ell \,/\, 1 \le \ell \le n\} \cup \{s_n s_1, t_n t_1\}$$

Give a labeling h from $V(G)$ to $\{0,1\}$by:

$h(s_0) = 1;\ h(s''_0) = 1;\ h(s'_0) = 0$

$h(s_\ell) = 1$ for $1 \le \ell \le n$

$h(s'_\ell) = 1$ for $1 \le \ell \le n$

$h(s''_\ell) = 1$ for $1 \le \ell \le n$

$h(t_\ell) = h(t'_\ell) = h(t''_\ell) = 0$ for $1 \le \ell \le n$

Here, $v_h(0) = 3n + 1,\ v_h(1) = 3n + 2$

$e_h(0) = 4n + 1, e_h(1) = 4n + 1$

Thus the absolute difference of $v_h(0)$ and $v_h(1)$is less than or equal to 1 and the absolute difference of $e_h(0)$and $e_h(1)$ is less than or equal to 1.

Hence $DW_n \odot \overline{K_2}$ is a product cordial graph.

Illustration 2.2.1

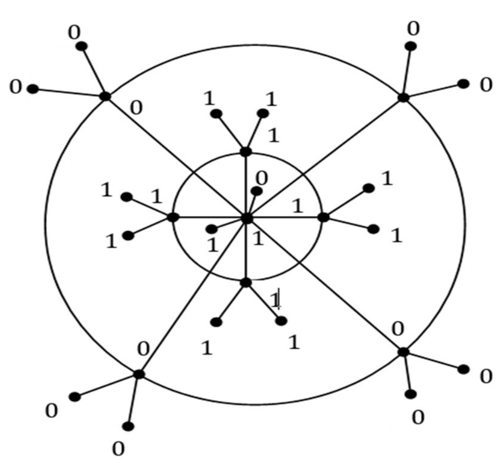

Figure 2: Product cordial labeling of $DW_4 \odot \overline{K_2}$

Theorem 2.3.
$DW_n \odot \overline{K_3}$ *is a product cordial graph.*
Proof.
Let $G = DW_n \odot \overline{K_3}$
Let $\{s_0, s_1, \ldots, s_n, t_1, t_2, \ldots, t_n\}$ denote the points of double wheel of G where s_0 is apex point, $\{s_1 \ldots s_n\}$ are rim points of first cycle and $\{t_1, t_2, \ldots, t_n\}$ are rim points of second cycle.

Let $\{s_0', s_1', \ldots, s_n', s_0'', s_1'', \ldots, s_n'', s_0''', s_1''', \ldots, s_n'''\}$ and $\left\{\begin{matrix} t_1' t_2' \ldots t_n', t_1'' t_2'' \ldots t_n'', \\ t_1''' t_2''' \ldots t_n''' \end{matrix}\right\}$ be the pendant points attached at $\{s_0 s_1 \ldots s_n\}$ and $\{t_1, t_2, \ldots, t_n\}$ respectively.

Let $V(G) = \left\{\begin{matrix} s_0, s_1, \ldots, s_n, t_1, t_2, \ldots, t_n, s_0', s_1', \ldots, s_n', t_1' t_2' \ldots t_n', \\ s_0'', s_1'', \ldots, s_n'', t_1'' t_2'' \ldots t_n'', s_0''', s_1''', \ldots, s_n''', t_1''' t_2''' \ldots t_n''' \end{matrix}\right\}$

$$E(G) = \{s_0 s_\ell \ / \ 1 \le \ell \le n\} \cup \{s_\ell s_{\ell+1} \ / \ 1 \le \ell \le n-1\}$$

$$\cup \{s_\ell s_\ell' \ /0 \le \ell \le n\} \cup \{s_\ell s_\ell'' \ / \ 0 \le \ell \le n\} \cup \{s_\ell s_\ell''' \ / \ 0 \le \ell \le n\}$$

$$\cup \{s_0 t_\ell \ / \ 1 \le \ell \le n\} \cup \{t_\ell t_{\ell+1} \ / \ 1 \le \ell \le n-1\} \cup \{t_\ell t_\ell' \ / \ 1 \le \ell \le n\}$$

$$\cup \{t_\ell t_\ell'' \ / \ 1 \le \ell \le n\} \cup \{t_\ell t_\ell''' \ /1 \le \ell \le n\} \cup \{s_n s_1, t_n t_1\}$$

Give a labeling h from $V(G)$ to $\{0,1\}$ by:

$$h(s_0) = 1; \ h(s_0') = 0; \ h(s_0'') = 0$$

$$h(s_\ell) = h(s_\ell') = h(s_\ell'') = 1 \qquad \text{for } 1 \le \ell \le n$$

$$h(s_\ell''') = 1 \qquad \text{for } 0 \le \ell \le n$$

$$h(t_\ell) = h(t_\ell') = h(t_\ell'') = h(t_\ell''') = 0 \qquad \text{for } 1 \le \ell \le n$$

Here, $v_h(0) = 4n + 2, v_h(1) = 4n + 2, e_h(0) = 5n + 2, e_h(1) = 5n + 1$

Thus the absolute difference of $v_h(0)$ and $v_h(1)$is less than or equal to 1 and the absolute difference of $e_h(0)$and $e_h(1)$ is less than or equal to 1.

Hence $DW_n \odot \overline{K_3}$ is a product cordial graph.

Illustration 2.3.1.

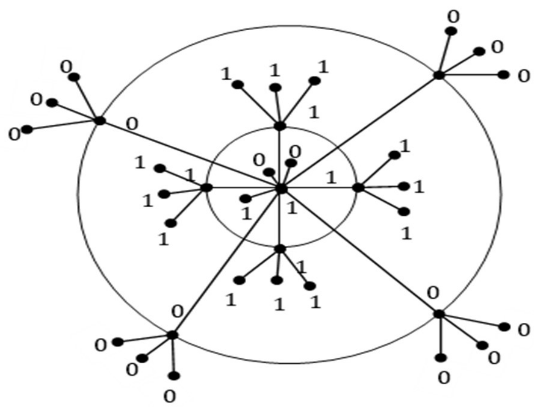

Figure 3: Product cordial labeling of $DW_4 \odot \overline{K_3}$

Theorem 2.4.

$DW_n \odot K_2$ is a product cordial graph.

Proof.

Let $G = DW_n \odot K_2$

Let $\{s_0, s_1, \dots, s_n, t_1, t_2, \dots, t_n\}$ denote the points of double wheel of G where s_0 is apex point, $\{s_1 \dots s_n\}$ are rim points of first cycle and $\{t_1, t_2, \dots, t_n\}$ are rim points of second cycle. Let $\{s_0', s_1', \dots, s_n', s_0'', s_1'', \dots, s_n''\}$ and $\{t_1' t_2' \dots t_n', t_1'' t_2'' \dots t_n''\}$ be the pendant points attached at $\{s_0 s_1 \dots s_n\}$ and $\{t_1, t_2, \dots, t_n\}$ respectively.

Let $V(G) = \begin{Bmatrix} s_0, s_1, \dots, s_n, t_1, t_2, \dots, t_n, s_0', s_1', \dots, s_n', t_1' t_2' \dots t_n', \\ s_0'', s_1'', \dots, s_n'', t_1'' t_2'' \dots t_n'' \end{Bmatrix}$

$$E(G) = \{s_0 s_\ell \,/ 1 \le \ell \le n\} \cup \{s_\ell s_{\ell+1} \,/\, 1 \le \ell \le n-1\}$$
$$\cup \{s_\ell s_\ell' \,/\, 0 \le \ell \le n\} \cup \{s_\ell s_\ell'' \,/\, 0 \le \ell \le n\}$$
$$\cup \{s_\ell' s_\ell'' \,/\, 0 \le \ell \le n\} \cup \{s_0 t_\ell \,/ 1 \le \ell \le n\}$$
$$\cup \{t_\ell t_{\ell+1} \,/\, 1 \le \ell \le n-1\} \cup \{t_\ell t_\ell' \,/\, 1 \le \ell \le n\}$$
$$\cup \{t_\ell t_\ell'' \,/\, 1 \le \ell \le n\} \cup \{t_\ell' t_\ell'' \,/\, 1 \le \ell \le n\} \cup \{s_n s_1, t_n t_1\}$$

Give a labeling h from $V(G)$ to $\{0,1\}$ by:

$$h(s_0) = 1;\ h(s_0'') = 1;\ h(s_0') = 0$$

$h(s_\ell) = h(s_\ell') = h(s_\ell'') = 1$ for $1 \le \ell \le n$

$h(t_\ell) = h(t_\ell') = h(t_\ell'') = 0$ for $1 \le \ell \le n$

Here, $v_h(0) = 3n + 1, v_h(1) = 3n + 2$

$e_h(0) = 5n + 2, e_h(1) = 5n + 1.$

Thus the absolute difference of $v_h(0)$ and $v_h(1)$ is less than or equal to 1 and the absolute difference of $e_h(0)$ and $e_h(1)$ is less than or equal to 1.

Hence $DW_n \odot K_2$ is a product cordial graph.

Illustration 2.4.1.

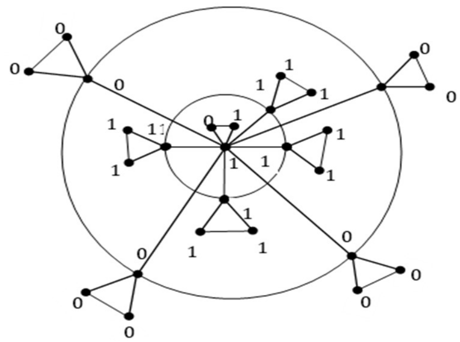

Figure 4: Product cordial labeling of$DW_4 \odot K_2$

Theorem 2.5.

$DW_n@P_3$is a product cordial graph.

Proof.

Let $G = DW_n@P_3$

Let s_0 be the apex point of double wheel in G and let s_0, s_0', s_0'' be the points P_3 attached at s_0.

Let $s_1, s_2, \dots, s_n$ and $t_1, t_2, \dots, t_n$ be the points of two cycles of double wheel. Let $s_\ell, s_\ell', s_\ell''$ be the points of path P_3 attached at s_ℓ for $1 \le \ell \le n$ and let $t_\ell, t_\ell', t_\ell''$ be the points of path P_3 attached at t_ℓ for $1 \le \ell \le n$.

Let $V(G) = \{s_0, s_0', s_0''\} \cup \{s_\ell, s_\ell', s_\ell'', t_\ell, t_\ell', t_\ell''/1 \le \ell \le n\}$

$$E(G) = \{s_0s_0', s_0's_0''\} \cup \{s_0s_\ell, s_0t_\ell\ /1 \le \ell \le n\} \cup \{s_\ell s_\ell'/1 \le \ell \le n\}$$

$$\cup \{s_\ell' s_\ell''/1 \le \ell \le n\} \cup \{t_\ell t_\ell'/1 \le \ell \le n\} \cup \{t_\ell' t_\ell''/1 \le \ell \le n\} \cup$$

$\{s_n s_1, t_n t_1\}$

Give a labeling h from $V(G)$ to $\{0,1\}$by:

$$h(s_0) = h(s_0') = 1;\ \ h(s_0'') = 0$$

$$h(s_\ell') = h(s_\ell'') = h(s_\ell''') = 1 \text{ for } 1 \le \ell \le n$$

$$h(t_\ell') = h(t_\ell'') = h(t_\ell''') = 0 \text{ for } 1 \le \ell \le n$$

Here, $v_h(0) = 3n + 1,\ v_h(1) = 3n + 2,$

$$e_h(0) = 4n + 1, e_h(1) = 4n + 1.$$

Thus the absolute difference of $v_h(0)$ and $v_h(1)$is less than or equal to 1 and the absolute difference of $e_h(0)$and $e_h(1)$ is less than or equal to 1.

Hence $DW_n@P_3$ is a product cordial graph.

Illustration 2.5.1

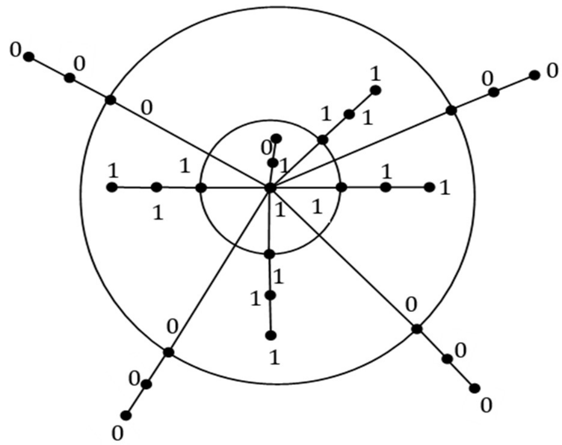

Figure 5: Product cordial labeling of $DW_4@P_3$

3. CONCLUSION

The product cordial labeling of various classes of graphs such as $DW_n \odot \overline{K_1}, DW_n \odot \overline{K_2}, DW_n \odot \overline{K_3}, DW_n \odot K_2, DW_n@P_3$ were investigated. To derive analogous results for some other graph families and other graph labelings in an open research problems.

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