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STATISTICAL PROCESS CONTROL USING NEW GENERALIZATION OF EXPONENTIATED MUKHERJEE- ISLAM DISTRIBUTION

S.Abirami[1] and N.Vijayasankar[2]
[1,2]Department of Statistics, Annamalai University, Annamalai Nagar, Tamil Nadu, India
[1]Corresponding Author: abiramistat@gmail.com , [2]Email: vijaystat@gmail.com

Abstract:

Statistical Process Control (SPC) is often used in the area to monitor processes when the quality of interest follows a normal distribution. In practice, it is not always true that the variable of interest follows the normal distribution, but may also follow non-normal distributions. The variable of interest may follow some non-normal distribution such as an exponential distribution or a gamma distribution or any other. The use of control charts designed for a normal distribution may not be workable in this situation and may cause an increase in the proportion of non-conforming products.

In this paper, the techniques of Length Biased distributions have been used to the EMID to the applications of SPC to check the performances of the production process. The main objective of this paper is to introduce a control chart using New Generalization of EMID in order to study the production system and monitor the same.

Keywords: *Statistical Process Control (SPC), Control Limits, Length Biased distributions, Exponentiated Mukherjee- Islam Distribution (EMID)*

Introduction

Statistical process control (SPC) is a method of continuous improvement is commonly utilized in modern manufacturing and service businesses. This method primarily focuses on using in the use of control charts and frequency distributions of development and quality data. The Shewhart control chart is a well-known and well-established control chart. W. A. Shewhart was the first to propose the usage of control charts. Control charts are essential tools for quality assurance. Control charts are used to manage current processes by detecting and fixing problems as they occur, making predictions the estimated range of outcomes from a process, attempting to determine whether a process is stable, investigating patterns of process variation caused by special causes, non-routine events, or common causes built into the process, and determining whether the quality improvement project should aim to prevent specific problems or make fundamental changes to the process. Control charts help to develop higher-quality products. Control charts should be used to take timely action on the process, that is, no action should be performed when an examination of a control chart reveals that the progression is under control. However, if the control chart indicates that the progression has become uncontrollable, suitable action should be done to restore control. As a result, a control chart shows when or for which statement remedial action should be implemented. The two control limits, known as the upper and lower control limits (UCL and LCL), exist used to monitor the progression target mean or variation. These constraints are extremely beneficial in both

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minimizing faulty goods and improving industry revenues. The preservation of quality through control charts results in a positive market reputation for an industry.

When the quality of interest follows a normal distribution, Shewhart control charts are frequently used to monitor operations. In run-through, the variable of interest may not necessarily follow the normal distribution but may also follow non-normal distributions. Several scholars have created several sorts of control charts. Amin and Venkatesan [12] discoursed the recent developments in control charts methods and they also discussed the Bayesian Approach in Control Charts Techniques. When the time between occurrences follows an exponential distribution, Santiago and Smith (2013) propose the t-chart as a control chart. They employed Nelson's variable transformation [5] to approximate normal data from exponentially dispersed data. Reyad et al. (2017) discovered the length biased weighted Erlang distribution and described its features and uses. The length biased Erlang-truncated exponential distribution using life time data was studied by Rather and Subramanian [2]. Control charts intended for a normal distribution may not be applicable in this case, thereby increasing the quantity of nonconforming goods. Furthermore, normal distribution is used when data is gathered in subgroups so that the central limit theorem may be used for creating control charts. Again, in actuality, data collection in groups is not always practicable. As a result, in many actual circumstances, classical distributions provide insufficient correction for real data. If the data is asymmetric, for example, the normal distribution is not a viable choice. As a result, many generators based on one or more factors have been proposed to generate new distributions.

Length Biased distributions have been explored dynamically in commonly occurring real experimental statistical data for model selection and related challenges. The Mukherjee-Islam Distribution was subjected to Exponentiated Distributions methods. A probability distribution can be represented in a variety of ways, with the conditional expectation of lower record values defining the Mukherjee-Islam Distribution. Mukherjee and Islam pioneered Mukherjee-Islam distribution (1983). It is finite range distribution, which is one of the utmost important properties of reliability analysis in recent times Its mathematical procedure is simple and can be controlled easily, which is why, it is desired to use over more complex distributions such as normal, Weibull beta etc. Dar et al. (2018) discussed weighted Mukherjee-Islam distribution and its applications. Rather and Subramanian [1] obtained the EMID and its various statistical properties. Recently, Subramanian and Rather [1], obtained weighted EMID. Several distributions can be used to monitor the production process, and in this paper, the control chart has results using the EMID to monitor the process.

Notation: EMID- Exponentiated Mukherjee-Islam distribution

2. Description of the Distribution

Definition 1

The pdf of EMID which was obtained by Rather and Subramanian (2019), is given by

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$f(x) = \frac{\alpha\beta x^{\alpha\beta-}}{\theta^{\alpha\beta}}$, $0 < x < \theta$, $\alpha, \beta, \theta > 0$ (1)

Definition 2

The cdf of EMID is

$F(x) = \left(\left(\frac{x}{\theta}\right)^{\beta}\right)^{\alpha}$ (2)

LENGTH BIASED EXPONENTIATED MUKHERJEE-ISLAM (LBEMI) DISTRIBUTION

Definition 3

Let X is a real arbitrary variable with probability density function (pdf) $f(x)$ Let $w(x)$ be the real weight function, then the possibility density function of the weighted random variable X_w is given by

$$f_w(x) = \frac{w(x)\,f(x)}{E(w(x))}, \quad x > 0$$

Definition 4

Substitute (1) in above equation, we will get the required pdf of LBEMI distribution as

$f_1(x) = \frac{(\alpha\beta+1)x^{\alpha\beta}}{\theta^{\alpha\beta+}}$, $0 < x < \theta$ (3)

Where $E(x) = \frac{\alpha\beta\theta}{\alpha\beta+1}$ and the corresponding cdf of length-biased EMID is obtained as

$$F_1(x) = \int_0^x f_1(x)\,dx$$

$F_1(x) = \left(\frac{x}{\theta}\right)^{\alpha\beta+1}$ (4)

Performance measures of the LBEMI Distribution

Let X means the random variable of LBEMI distribution with the parameters α, β and θ, then the r-th order moment $E(X^r)$ of LBEMI distribution can be acquired as

$$E(X^r) = \mu_r^1 = \int_0^{\theta} x^r\, f_1(x)\,dx$$

$$E(X^r) = \int_0^{\theta} \frac{(\alpha\beta+1)x^{\alpha\beta+1}}{\theta^{\alpha\beta+1}}\,dx$$

$$E(X^r) = \frac{(\alpha\beta+1)\theta^{\alpha\beta+r+1}}{\theta^{\alpha\beta+1}\ (\alpha\beta+r+1)}$$

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$$E(r) = \frac{(\alpha\beta+1)\theta^r}{(\alpha\beta+r+1)} \quad (5)$$

Put r =1 in (5) we determination the mean of LBEMI Distribution which is known by

$$E(x) = \frac{(\alpha\beta + 1)\theta}{(\alpha\beta + 2)}$$

Put r = 2 in (5) we have

$$E(x^2) = \frac{(\alpha\beta + 1)\theta^2}{(\alpha\beta + 3)}$$

After simplification we get the variance of LBEMI Distribution is

$$V(x) = \frac{(\alpha\beta+1)\theta^2}{(\alpha\beta+3)(\alpha\beta+2)^2}$$

4. Control Limits for the LBEMI Distribution

$$\text{UCL} = \frac{(\alpha\beta+1)\theta}{(\alpha\beta+2)} + 3\ \frac{\theta}{(\alpha\beta+2)} \sqrt{\frac{(\alpha\beta+1)}{(\alpha\beta+3)}}$$

$$\text{CL} = \frac{(\alpha\beta+1)\theta}{(\alpha\beta+2)}$$

$$\text{LCL} = \frac{(\alpha\beta+1)\theta}{(\alpha\beta+2)} - 3\ \frac{\theta}{(\alpha\beta+2)} \sqrt{\frac{(\alpha\beta+1)}{(\alpha\beta+3)}}$$

5. Numerical illustration

A illustration regarding the structure of control limits is measured in order to illustrate the practicability of the proposed model. The control limits of the EMID are acquired by using simulated data set. For parameters α, β and θ being random variables the table 1- 4 is constructed. Therefore, UCL and LCL are reported in table 1- 4.

Table.1: Control limits for EMID for $\theta = 2$

α	β	CL	UCL	LCL
0.1	0.3	1.014778	2.738045	-0.70849
	0.5	1.02439	2.741674	-0.69289
	0.7	1.033816	2.745028	-0.67739
	1	1.047619	2.749572	-0.65433
	3	1.130435	2.767772	-0.5069
0.3	0.3	1.043062	2.748119	-0.662
	0.5	1.069767	2.755957	-0.61642
	0.7	1.095023	2.761883	-0.57184

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	1	1.130435	2.767772	-0.5069
	3	1.310345	2.754446	-0.13376
0.6	0.3	1.082569	2.759141	-0.594
	0.5	1.130435	2.767772	-0.5069
	0.7	1.173554	2.77115	-0.42404
	1	1.230769	2.769231	-0.30769
	3	1.473684	2.679625	0.267743
1	0.3	1.130435	2.767772	-0.5069
	0.5	1.2	2.771169	-0.37117
	0.7	1.259259	2.765558	-0.24704
	1	1.333333	2.747547	-0.08088
	3	1.6	2.579796	0.620204
3	0.3	1.310345	2.754446	-0.13376
	0.5	1.428571	2.706325	0.150818
	0.7	1.512195	2.653136	0.371254
	1	1.6	2.579796	0.620204
	3	1.818182	2.316111	1.320252

Table.2: Control limits for EMID for $\theta = 3$

A	β	CL	UCL	LCL
0.1	0.3	1.522167	4.107068	-1.06273
	0.5	1.536585	4.112511	-1.03934
	0.7	1.550725	4.117541	-1.01609
	1	1.571429	4.124357	-0.9815
	3	1.695652	4.151658	-0.76035
0.3	0.3	1.564593	4.122179	-0.99299
	0.5	1.604651	4.133936	-0.92463
	0.7	1.642534	4.142824	-0.85776
	1	1.695652	4.151658	-0.76035
	3	1.965517	4.13167	-0.20064
0.6	0.3	1.623853	4.138711	-0.891
	0.5	1.695652	4.151658	-0.76035
	0.7	1.760331	4.156725	-0.63606
	1	1.846154	4.153846	-0.46154
	3	2.210526	4.019438	0.401615
1	0.3	1.695652	4.151658	-0.76035
	0.5	1.8	4.156753	-0.55675
	0.7	1.888889	4.148337	-0.37056
	1	2	4.12132	-0.12132
	3	2.4	3.869694	0.930306

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3	0.3	1.965517	4.13167	-0.20064
	0.5	2.142857	4.059487	0.226227
	0.7	2.268293	3.979704	0.556881
	1	2.4	3.869694	0.930306
	3	2.727273	3.474167	1.980378

Table.3: Control limits for EMID for $\theta = 5$

A	β	CL	UCL	LCL
0.1	0.3	2.536946	6.845113	-1.77122
	0.5	2.560976	6.854186	-1.73223
	0.7	2.584541	6.862569	-1.69349
	1	2.619048	6.873929	-1.63583
	3	2.826087	6.91943	-1.26726
0.3	0.3	2.607656	6.870299	-1.65499
	0.5	2.674419	6.889893	-1.54106
	0.7	2.737557	6.904707	-1.42959
	1	2.826087	6.91943	-1.26726
	3	3.275862	6.886116	-0.33439
0.6	0.3	2.706422	6.897852	-1.48501
	0.5	2.826087	6.91943	-1.26726
	0.7	2.933884	6.927875	-1.06011
	1	3.076923	6.923077	-0.76923
	3	3.684211	6.699063	0.669358
1	0.3	2.826087	6.91943	-1.26726
	0.5	3	6.927922	-0.92792
	0.7	3.148148	6.913895	-0.6176
	1	3.333333	6.868867	-0.2022
	3	4	6.44949	1.55051
3	0.3	3.275862	6.886116	-0.33439
	0.5	3.571429	6.765811	0.377046
	0.7	3.780488	6.63284	0.928136
	1	4	6.44949	1.55051
	3	4.545455	5.790279	3.300631

Table.4: Control limits for EMID for $\theta = 7$

A	β	CL	UCL	LCL
0.1	0.3	3.551724	9.583158	-2.47971
	0.5	3.585366	9.59586	-2.42513
	0.7	3.618357	9.607597	-2.37088
	1	3.666667	9.623501	-2.29017

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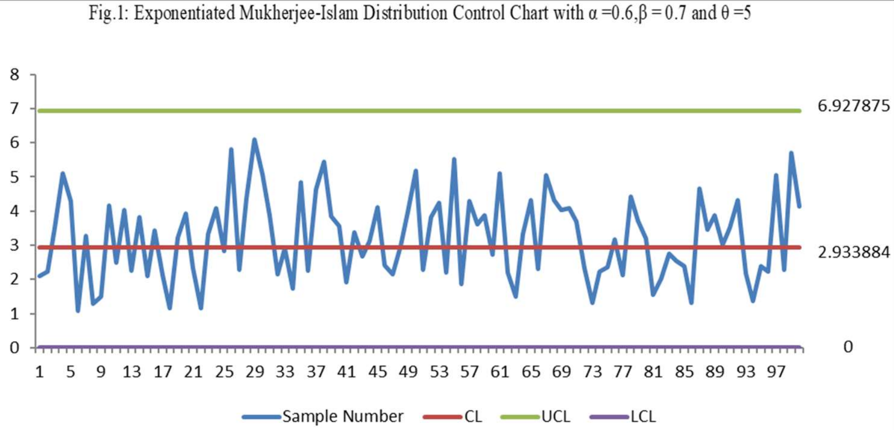
	3	3.956522	9.687202	-1.77416
0.3	0.3	3.650718	9.618418	-2.31698
	0.5	3.744186	9.64585	-2.15748
	0.7	3.832579	9.666589	-2.00143
	1	3.956522	9.687202	-1.77416
	3	4.586207	9.640562	-0.46815
0.6	0.3	3.788991	9.656993	-2.07901
	0.5	3.956522	9.687202	-1.77416
	0.7	4.107438	9.699025	-1.48415
	1	4.307692	9.692308	-1.07692
	3	5.157895	9.378688	0.937101
1	0.3	3.956522	9.687202	-1.77416
	0.5	4.2	9.699091	-1.29909
	0.7	4.407407	9.679453	-0.86464
	1	4.666667	9.616414	-0.28308
	3	5.6	9.029286	2.170714
3	0.3	4.586207	9.640562	-0.46815
	0.5	5	9.472136	0.527864
	0.7	5.292683	9.285976	1.29939
	1	5.6	9.029286	2.170714
	3	6.363636	8.10639	4.620883

It is detected that from table 1- 4, the static significance of the parameter α, the control limits increase at any time the parameter β increases also they increase at any time θ increases. As well as for the increasing of parameter α, with a fixed β the control limits increase at any time the parameter θ is fixed and the more θ increases, the more the control limits increase. The UCL and LCL should be around the center line. Though, in the case the LCL becomes negative, it can be rounded to zero. The EMID Control Chart is shown in Fig. 1 for α =0.6, β =0.7 and θ = 5.

Fig.1: Exponentiated Mukherjee-Islam Distribution Control Chart with α =0.6,β = 0.7 and θ =5



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It is also pointed out that, the explanations of the process control are essential or else the process will be in control. That is, depending on the manufacturing products, the manufacturing engineers would set the parameter values based on the type of data they are dealing with as shown in Table 1–4, the greater the parameter values, the greater the control limits.

6. Conclusion

In this article, the progression control has established the usage of EMID. Whilst the technique is non-normal distribution, a novel procedure is given for sentencing the method whilst production. The manipulation limits are given for the EMID with exceptional values of parameters α, β and θ. Table.1 is built to assist in the choice of the parameters based totally completely surely on the form of documents the manufacturing engineer is confronted with. The control chart is drawn by way of wondering about the parameters $\alpha = 0.6$, $\beta = 0.7$ and $\theta = 5$ for this reason it is recommended to keep θ elevated than the located approach facts.

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