COMPARATIVE ANALYSIS OF REPLACING EQUIPMENT IN UNCERTAIN ENVIRONMENTS WHERE MAINTENANCE COSTS RISE WITH AMBIGUOUS VALUES OVER TIME

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Abstract In an imprecise environment, this article assists organisations or businesses in determining the optimum replacement time and the lowest weighted average cost of machineries. In this article, fuzzy and intuitionistic fuzzy numbers are used to reveal unknown quantities. Practical implementation becomes difficult when numerical values are expressed as unknown quantities, such as intuitionistic fuzzy numbers. Novel approaches to overcoming this computational complexity have been proposed, such as expressing intuitionistic fuzzy numbers into their respective parametric forms involving left fuzziness index, right fuzziness index, and location index of membership and non - membership functions. Arithmetic operations and a centroid-based ranking function are proposed in terms of parametric form. The primary goal of this article is to provide industries with a more precise result so that they can determine the best time to replace machineries or equipment. The original problem's fuzzy and intuitionistic fuzzy nature is preserved, indicating the proposed work carried out using the proposed arithmetic operations and ranking function. This paper discusses a different solution approach for the replacement problem in a fuzzy environment. This alternative method is used to determine the optimal age of the machine, which we used it to compare the two different machines to determine the best machine among that.
Keywords Fuzzy optimization, Intuitionistic Triangular fuzzy number, replacement theory

1. Introduction

Replacement of machinery and facilities is a constant and complicated issue in today's dynamic world as a result of rapid technological growth and globalisation, and it is a shared concern among all business owners. This requires precise knowledge of decision parameters such as maintenance cost, resale value, capital cost, and interest rate. In 1987, M Abdel-Hameed [3] discussed the block replacement policy problem with classical parameters in the traditional replacement model. It replaces devices when they fail and replaces each failed device with a new device every T units of time. Richard M. Feldman et al. [4] presented a minimal repair/replacement problem in 1996 to demonstrate the use of post-optimal analyses during policy implementation. In 2021, Bhattacharyya D investigated a two-sample nonparametric test for comparing mean time to failure functions in age replacement. In 2022, Vijay C. Makwana [11] proposed a new hypothesis and solution for fuzzy equations. According to

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reports, many real-world problems cannot be solved using classical set theory, necessitating its extension. As a result, in the 1960s, the eminent scientist Zadeh [1] proposed Fuzzy Sets as an extension of classical set theory. The fuzzy set theory and the classical set theory take different approaches to ambiguity. As a fuzzy set generalisation, Atanassov [2] proposed and demonstrated intuitionistic fuzzy sets in 1986.

The expression of intuitionistic fuzzy numbers into their corresponding parametric forms using left, right, and location fuzziness indices of membership and non-membership functions is one of the novel ways suggested to overcome this computational difficulty. In terms of parametric form, arithmetic operations and a centroid-based ranking function are suggested. This article's main objective is to give industries a more accurate result so they can choose when it is better to replace machinery or equipment. The fuzzy and intuitionistic quality of the original challenge is kept, proving that the suggested work was completed using the suggested arithmetic operations and ranking function. In this study, a novel way to solving the replacement problem in a fuzzy environment is discussed. By comparing the two distinct machines, we were able to establish which one was the best using this alternate method, which is utilised to estimate the ideal age of the machine.

2.Preliminaries This division's goal is to provide fundamental definitions, annotations, and results that will be used in subsequent calculations.

Definition 2.1.Let a non empty set be X. An Intuitionistic fuzzy set is defined as ,where and denotes the degree of membership and degree of non-membership functions respectively where **Definition 2.1.**Let a non empty set be X. An Intuitionistic fuzzy set $\tilde{A}^{IFS}$ is defined as $\tilde{A}^{IFS} = \left\{\left(x, \mu_{\tilde{A}^{IFS}}(x), \gamma_{\tilde{A}^{IFS}}(x) / x \in X \right)\right\}$,where $\mu_{\tilde{A}^{IFS}} : X \to [0,1]$ and $\gamma_{\tilde{A}^{IFS}} : X \to [0,1]$ denotes the degree of membership and degree of non-membership functions respectively where $x \in X$,

Forevery $x \in X$ **,** $0 \le \mu_{\tilde{A}^{IFS}}(x) + \gamma_{\tilde{A}^{IFS}}(x) \le 1$ **.**

Definition 2.2. A fuzzy number $\tilde{A}^{IFN}$ on R is said to be a triangular intuitionistic fuzzy number if its membership function $\mu_{\tilde{A}^{IFN}} : R \to [0,1]$ and non- membership function $\gamma_{\tilde{A}^{IFN}} : R \to [0,1]$ satisfies the following conditions,

$$\mu_{\tilde{A}^{IFN}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, \text{for } a_1 \le x \le a_2 \\ = 1, \text{ for } x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, \text{for } a_2 \le x \le a_3 \\ = 0, \text{otherwise} \end{cases}$$

$$\gamma_{\tilde{A}^{IFN}}(x) = \begin{cases} \dfrac{x - d_1}{d_2 - d_1}, \text{for } d_1 \le x \le d_2 \\ = 1, \text{ for } x = d_2 \\ \dfrac{d_3 - x}{d_3 - d_2}, \text{for } d_2 \le x \le d_3 \\ = 0, \text{otherwise} \end{cases}$$

and is given by $\tilde{A}^{IFN} = (a_1, a_2, a_3; d_1, d_2, d_3)$ where

$d_1 \le a_1 \le (d_2 = a_2) \le a_3 \le d_3$.

Let $\tilde{A}^{IFN} = (a_1, a_2, a_3; d_1, d_2, d_3)$ be a triangular intuitionistic fuzzy number then the following cases arise.

Case 1:If $a_1 = d_1, a_2 = d_2, a_3 = d_3$, then $\tilde{A}^{IFN}$ represent a triangular fuzzy number.

Case2: If $a_1 = d_1 = a_2 = d_2 = a_3 = d_3 = R$, then $\tilde{A}^{IFN}$ represent a real number R. The parametric form of triangular intuitionistic fuzzy number $\tilde{A}^{IFN}$ is represented as $\tilde{A}^{IFN} = (\alpha_{\mu_a}, a_\mu, \beta_{\mu_a}; \alpha_{\gamma_a}, a_\gamma, \beta_{\gamma_a})$ where a_μ, a_γ are the mid value of membership functions and non-membership functions and $\alpha_{\mu_a}, \alpha_{\gamma_a}$ & $\beta_{\mu_a}, \beta_{\gamma_a}$ represents the left spread and right spread membership and non – membership functions respectively.

2.3 Ranking of triangular intuitionistic fuzzy number:

$R(\tilde{A}^{IFN}) = \sqrt{\frac{1}{2}\left(\left[\tilde{z}_\mu(\tilde{A}) - \tilde{w}_\mu(\tilde{A})\right]^2 + \left[\tilde{z}_\gamma(\tilde{A}) - \tilde{w}_\gamma(\tilde{A})\right]^2\right)}$ **where** $\tilde{z}_\mu(\tilde{A}), \tilde{w}_\mu(\tilde{A}), \tilde{z}_\gamma(\tilde{A}), \tilde{w}_\gamma(\tilde{A})$ **are centroid point of the membership and non-membership functions and it can be define by**

$$\tilde{z}_\mu = \left[\frac{(a_3 + a_1 + a_2)}{3}\right]$$

$$\tilde{z}_\gamma = \left[\frac{(d_1 - (d_2 - d_1) + 2d_3)}{3}\right]$$

,

$$\tilde{w}_\mu = \frac{1}{3}\left[\frac{(a_1 - a_3)}{(a_1 - a_3)}\right] = \frac{1}{3}, \quad \tilde{w}_\gamma = \frac{1}{3}\left[\frac{2(d_3 - d_1)}{(d_3 - d_1)}\right] = \frac{2}{3}$$

For the intuitionistic Fuzzy Number $\tilde{A}^{IFN}$

3. MODEL SPECIFICATIONS:

Machine that need to be replaced because their annual maintenance costs rise over time and the worth of money changes over time, We must compute the fuzzy present value when the money value changes over time.

Present value of a rupee spent n years = $(1+\tilde{i})^{-n} = \tilde{v}^n$

Where $\tilde{v}$ is called fuzzy discount rat e or fuzzy present worth factor.

$\tilde{C}^{IFN}$ be the price of the machine and

$\tilde{R}_1^{IFN}, \tilde{R}_2^{IFN}, \tilde{R}_3^{IFN}, \ldots \tilde{R}_n^{IFN}$ be the fuzzy running cost.

The present value of the expenditure in n years is assuming that the machine has no scrap value and that all payments (cash outflows) are made at the beginning of each year.

$$\tilde{P}_n^{IFN} = \tilde{C}^{IFN} + \tilde{R}_1^{IFN} + \tilde{v}\tilde{R}_2^{IFN} + \tilde{v}^2\tilde{R}_3^{IFN} + \ldots + \tilde{v}^{n-1}\tilde{R}_n^{IFN}$$

The fuzzy present worth of fixed annual payments, each of value $\tilde{x}^{IFN}$, for n years is

$$\tilde{x}^{IFN} + \tilde{v}\tilde{x}^{IFN} + \tilde{v}^2\tilde{x}^{IFN} + \ldots + \tilde{v}^{n-1}\tilde{x}^{IFN} = \frac{1-\tilde{v}^n}{1-n}\tilde{x}^{IFN}$$

$$\tilde{P}_n^{IFN} = \frac{1-\tilde{v}^n}{1-\tilde{v}}\tilde{x}^{IFN} \quad \ldots\ldots(1)$$

"since $1-\tilde{v}$ is a positive constant"

$\tilde{F}_n^{IFN}$ can be minimum if

$$\Delta\tilde{F}_{n-1}^{IFN} < 0 < \Delta\tilde{F}_n^{IFN} \quad \ldots\ldots(.2)$$

$$\Delta\tilde{F}_n^{IFN} = \tilde{F}_{n+1}^{IFN} - \tilde{F}_n^{IFN} \quad \ldots\ldots(.3)$$

$$\tilde{P}_{n+1}^{IFN} = (\tilde{C} + \tilde{R}_1^{IFN} + \tilde{v}\tilde{R}_2^{IFN} + \ldots + \tilde{v}^{n-1}\tilde{R}_n^{IFN}) + \tilde{v}^n\tilde{R}_{n+1}^{IFN} = \tilde{P}_n^{IFN} + \tilde{v}^n\tilde{R}_{n+1}^{IFN} \quad \ldots\ldots(.4)$$

From (4)

$$\Delta\tilde{F}_n^{IFN} = \frac{1}{(1-\tilde{v}^{n+1})(1-\tilde{v}^n)}[(\tilde{v}^n\tilde{R}_{n+1}^{IFN} + \tilde{v}^{n+1}\tilde{P}_n^{IFN} - \tilde{v}^n\{\tilde{P}_n^{IFN} + \tilde{v}^n\tilde{R}_{n+1}^{IFN}\}] \quad \ldots\ldots(5)$$

$$= \frac{\tilde{v}^n(1-\tilde{v})}{(1-\tilde{v}^{n+1})(1-\tilde{v}^n)}\left[\frac{1-\tilde{v}^n}{1-v}\tilde{R}_{n+1}^{IFN} - \tilde{P}_n^{IFN}\right] \quad \ldots\ldots(6)$$

From (2)

$$\frac{1-\tilde{v}^{n-1}}{1-\tilde{v}}\tilde{R}_n^{IFN}-\tilde{P}_{n-1}^{IFN}<0<\frac{1-\tilde{v}^{n}}{1-\tilde{v}}\tilde{R}_{n+1}^{IFN}-\tilde{P}_n^{IFN}$$

……………………………...…………….………(.7)

From (7)

$$\tilde{R}_{n+1}^{IFN}>\frac{(\tilde{C}+\tilde{R}_1^{IFN}+\tilde{v}\tilde{R}_2^{IFN}+.....+\tilde{v}^{n-1}\tilde{R}_n^{IFN})}{1+\tilde{v}+\tilde{v}^2+........+\tilde{v}^{n-1}} \quad \ldots\ldots(8)$$

$$\tilde{R}_{n+1}^{IFN}>\frac{\tilde{C}^{IFN}+\sum_{r=1}^{n}\tilde{R}_r^{IFN}\tilde{v}^{r-1}}{\sum_{r=1}^{n}\tilde{v}^{r-1}} \quad \ldots\ldots(9)$$

The weights $1,\tilde{v},\tilde{v}^2,......\tilde{v}^{n-1}$ are the fuzzy discount factors

Similarly (7) will be

$$\tilde{R}_{n}^{IFN}<\frac{(\tilde{C}+\tilde{R}_1^{IFN}+\tilde{v}\tilde{R}_2^{IFN}+.....+\tilde{v}^{n-2}\tilde{R}_{n-1}^{IFN})}{1+\tilde{v}+\tilde{v}^2+........+\tilde{v}^{n-2}} \quad \ldots\ldots(10)$$

$$\tilde{R}_{n+1}^{IFN}>\frac{\tilde{C}^{IFN}+\sum_{r=1}^{n}\tilde{R}_r^{IFN}\tilde{v}^{r-2}}{\sum_{r=1}^{n}\tilde{v}^{r-2}} \quad \ldots\ldots(11)$$

The machine should be replaced if the next period's fuzzy cost is greater than the weighted average of previous fuzzy costs, according to expressions (8) and (11).

4. Computational descriptions:

4.1 Example:

Two machines (A_1, A_2) are shown to a manufacturer. Whose respective purchase prices are ($35000, $36000, $37000; $34,000, $36,000, $38,000) and ($37000, $38000, $39000; $36000, $38000, $40000). The Running expenses are listed below. Which machine should be purchased if the annual discount rate is 10%?

Table 1: Running Fuzzy Cost of Machine A_1 and Machine A_2

Age Or Year	Running fuzzy cost $\tilde{R}(x)$ of Machine A_1	Running fuzzy cost $\tilde{R}(x)$ of Machine A_2

1	(3500,3600,3700;3400,3600,3800)	(3700,3800,3900; 3600,3800,4000)
2	(3900,4000,4100;3800,4000,4200)	(4000,4100,4200; 3900,4100,4300)
3	(4400,4500,4600;4300,4500,4700)	(4300,4400,4500; 4200,4400,4600)
4	(4900,5000,5100;4800,5000,5200)	(4900,5000,5100; 4800,5000,5200)
5	(5200,5300,5400;5100,5300,5500)	(5100,5200,5300; 5000,5200,5400)
6	(5400,5500,5600;5300,5500,5700)	(5500,5600,5700; 5400,5600,5800)
7	(5600,5700,5800;5500,5700,5900)	(5800,5900,6000; 5700,5900,6100)
8	(6100,6200,6300;6000,6200,6400)	(14600,14700,14800; 14500,14700,14900)
9	(7800,7900,8000;7700,7900,8100)	(14800,14900,15000; 14700,14900,15100)
10	(11000,11100,11200;10900,11100,11300)	(14900,15000,15100; 14800,15000,15200)

Table 2: Discounted Maintenance Cost of Machine A_1

Age Or Year	Discounted Maintenance Cost $(\tilde{R}_n^{IFN}\tilde{v}^{n-1})$	$\sum_{r=1}^{n}(\tilde{R}_n^{IFN}\tilde{v}^{n-1})$
1	(3600,100,100;3600,200,200)	(3600,100,100;3600,200,200)
2	(3636.4,100,100;3636.4,200,200)	(7236.4,100,100;7236.4,200,200)
3	3718.8,100,100;3718.8,200,200)	10955.2,100,100;10955.2,200,200)
4	(3756.5,100,100;3756.5,200,200)	(14711.7,100,100;14711.7,200,200)
5	(3619.9,100,100;3619.9,200,200)	(18331.6,100,100;18331.6,200,200)

6	(3414.95,100,100;3414.95,200,200)	(21746.55,100,100;21746.55,200,200)
7	(3217.65,100,100;3217.65,200,200)	(24964.2,100,100;24964.2,200,200)
8	(3181.84,100,100;3181.84,200,200)	(28146.04,100,100;28146.04,200,200)
9	(3685.35,100,100;3685.35,200,200)	(31831.39,100,100;31831.39,200,200)
10	(4707.51,100,100;4707.51,200,200)	(36538.9,100,100;36538.9,200,200)

Table 3: Cumulative total discounted cost of Machine A_1

Age Or Year	Cumulative total discounted cost $\tilde{C}^{IFN} + (\tilde{R}_n^{IFN} \tilde{v}^{n-1})$	Machine A_1 average fuzzy cost $\frac{\tilde{C}^{IFN} + (\tilde{R}_n^{IFN} \tilde{v}^{n-1})}{\sum_{n=1}^{r} (\tilde{v}^{n-1})}$
1	(39600,1000,1000;39600,2000,2000)	(39600,1000,1000;39600,2000,2000)
2	(43236.4,1000,1000;43236.4,2000,2000)	(22647.5,1000,1000;22647.5,2000,2000)
3	(46955.2,1000,1000;46955.2,2000,2000)	(17165.12,1000,1000;17165.12,2000,2000)
4	(50711.7,1000,1000;50711.7,2000,2000)	(14543.9,1000,1000;14543.9,2000,2000)
5	(54331.6,1000,1000;54331.6,2000,2000)	(13029.7,1000,1000;13029.7,2000,2000)
6	(57746.55,1000,1000;57746.55,2000,2000)	(12053.8,1000,1000;12053.8,2000, 2000)
7	(60964.2,1000,1000;60964.2,2000,2000)	(11384.1,1000,1000;11384.1,2000,2000)
8	(64146.04,1000,1000;64146.04,2000,2000)	(10930.7,1000,1000;10930.7,2000,2000)
9	(67831.39,1000,1000;67831.39,2000,2000)	(10707.5,1000,1000;10707.5,2000,2000)(replace)
10	72538.9,1000,1000;72538.9,2000,2000)	10732.19,1000,1000;10732.1,2000,2000)

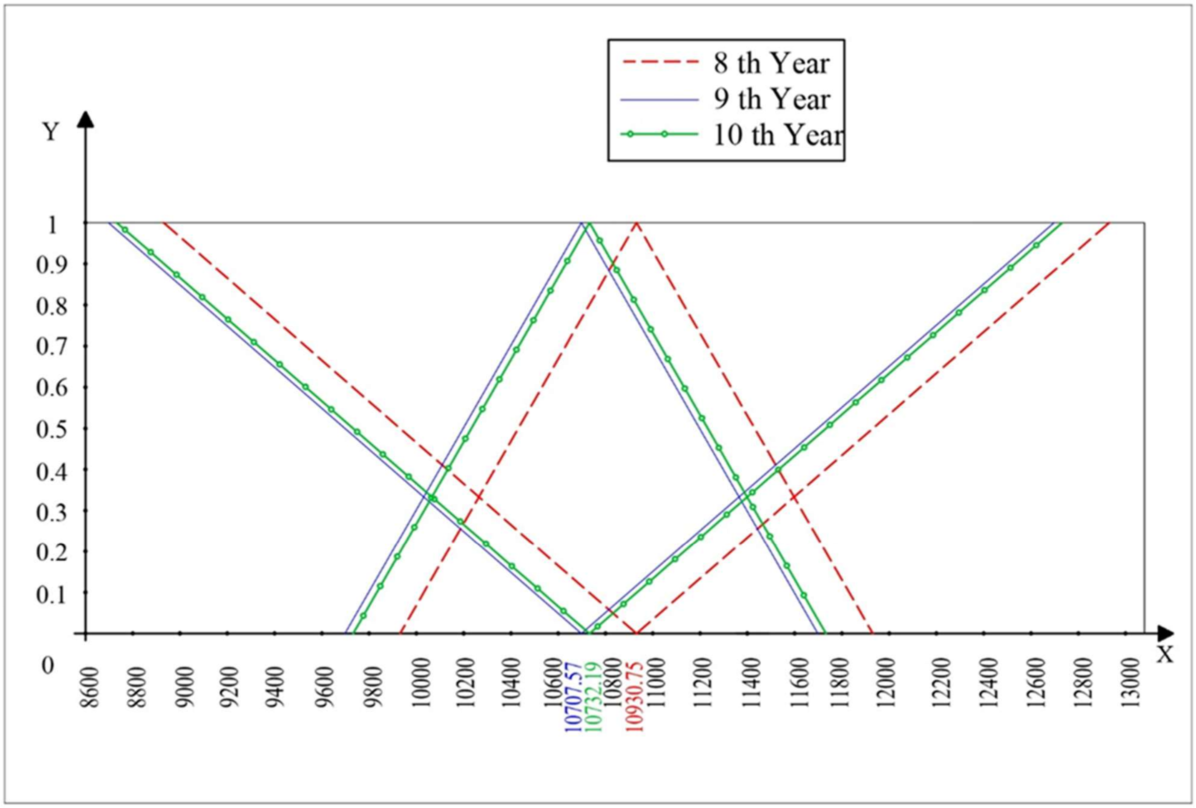

Figure 1: The graph depicts the annual average fuzzy cost of Machine A_1 for the eighth, ninth and tenth years.

Based on the foregoing, we can conclude that Machine A_1 minimum average fuzzy cost is 10707.5, and that it occurred in the ninth year. As a result, according to the proposed replacement policy, machine A_1 must be replaced at the end of the ninth year. Otherwise, it could result in a loss due to rising maintenanc costs. Figure 1 depicts the machine A_1 annual average costs of eighth, ninth, tenth year.

Table 4: Discounted Maintenance Cost of Machine A_2

Age Or Year	Discounted Maintenance Cost $(\tilde{R}_n^{IFN}\tilde{v}^{n-1})$	$\sum_{r=1}^{n}(\tilde{R}_n^{IFN}\tilde{v}^{n-1})$
1	(3800,100,100;3800,200,200)	(3800,100,100;3800,200,200)
2	(3727.31,100,100;3727.31,200,200)	(7527.31,100,100;7527.31,200,200)

3	(3636.16,100,100;3636.16,200,200)	(11163.47,100,100;11163.47,200,200)
4	(3756.5,100,100;3756.5,200,200)	(14919.97,100,100;14919.97,200,200)
5	(3551.6,100,100;3551.6,200,200)	(18471.57,100,100;18471.57,200,200)
6	(3477.04,100,100;3477.04,200,200)	(21948.61,100,100;21948.61,200,200)
7	(3330.55,100,100;3330.55,200,200)	(25279.16,100,100;25279.16,200,200)
8	(7544.04,100,100;7544.04,200,200)	(32823.2,100,100;32823.2,200,200)
9	(6950.85,100,100;6950.85,200,200)	(39774.05,100,100;39774.05,200,200)
10	(6361.5,100,100;6361.5,200,200)	(46135.55,100,100;46135.55,200,200)

Table 5: Cumulative total discounted cost of Machine A_2

Age Or Year	Cumulative total discounted cost $\tilde{C}^{IFN} + (\tilde{R}_n^{IFN}\tilde{v}^{n-1})$	$\frac{\tilde{C}^{IFN} + (\tilde{R}_n^{IFN}\tilde{v}^{n-1})}{\sum_{n=1}^{r}(\tilde{v}^{n-1})}$
1	(41800,100,100;41800,200,200)	(41800,100,100;41800,200,200)
2	(45527.31,100,100;45527.31,200,200)	(23847.5,100,100;23847.5,200,200)
3	(49163.47,100,100;49163.47,200,200)	(17972.3,100,100;17972.3,200,200)
4	(52919.97,100,100;52919.97,200,200)	(15177.2,100,100;15177.2,200,200)
5	(56471.57,100,100;56471.57,200,200)	(13542.9,100,100;13542.9,200,200)
6	(59948.61,100,100;59948.61,200,200)	(12513.54,100,100;12513.5,200,200)
7	(63279.16,100,100;63279.16,200,200)	(11816.4,100,100;11816.4,200,200)(replace)
8	(70823.2,100,100;70823.2,200,200)	(12068.57,100,100;12068.5,200,200)
9	(77774.05,100,100;77774.05,200,200)	(12277.0,100,100;12277.0,200,200)
10	(84135.55,100,100;84135.55,200,200)	(12447.9,100,100;12447.9,200,200)

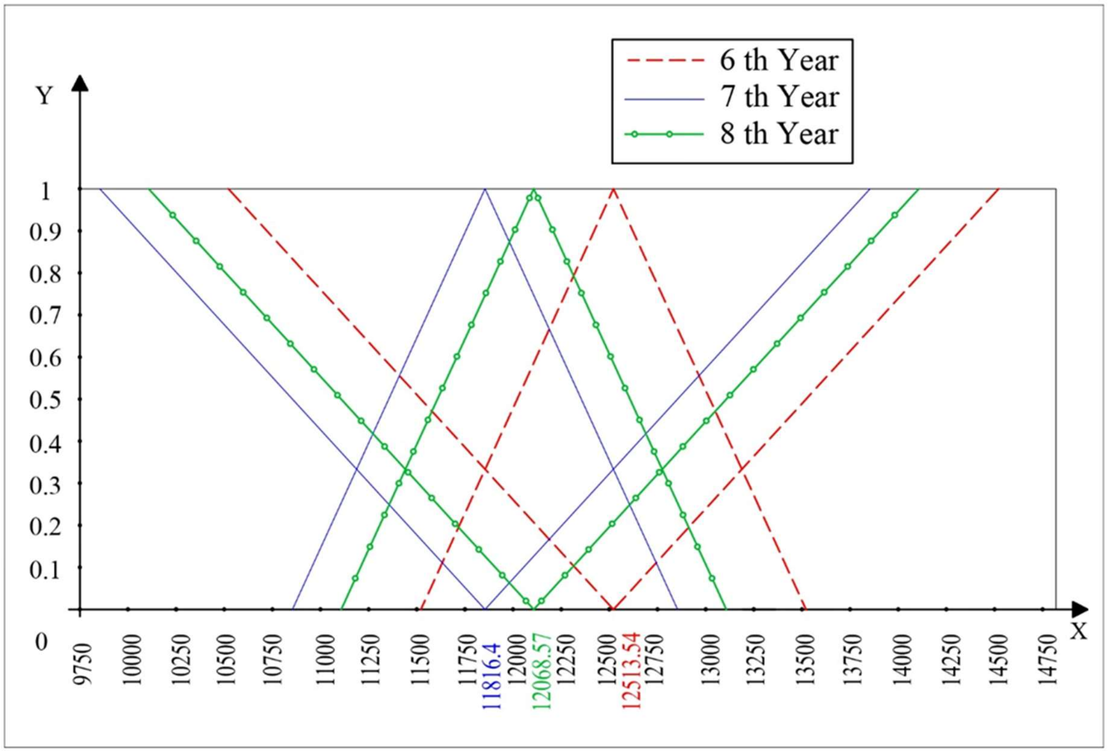

Figure 2: The graph depicts the annual average fuzzy cost of Machine A_2 for the sixth, seventh, eighth years.

We can conclude that Machine A_2 minimum average fuzzy cost is 11816.4, and that it occurred in the seventh year. As a result, according to the proposed replacement policy, machine A_2 must be replaced at the end of the seventh year. Otherwise, it could result in a loss due to rising maintenance costs. Figure 2 depicts the machine A_2 annual average costs of sixth, seventh, eighth year.

5. Conclusion

In this article, fuzzy and intuitionistic fuzzy numbers are used to reveal unknown quantities. Practical implementation becomes difficult when numerical values are expressed as unknown quantities, such as intuitionistic fuzzy numbers. Novel approaches to overcoming this computational complexity have been proposed, such as converting intuitionistic fuzzy numbers into their respective parametric forms using the left fuzziness index, right fuzziness index, and location index of membership and non-membership functions. Arithmetic operations and a centroid-based ranking function are proposed in terms of parametric form. The primary goal of this article is to provide industries with more precise results so that they can determine the best time to replace machineries or equipment. From (example 4.1) In A_1 machine, the annual maintenance fuzzy cost for the eighth, ninth, and tenth years are 10930.7 >10707.510732.1, indicating that the proposed method finds the lowest average fuzzy cost in the ninth year, allowing the machine to be replaced at the end of the ninth year. In A_2 machine, the annual maintenance fuzzy cost for the sixth, seventh, and eighth years are 12513.5>11816.4<12068.5, indicating that the proposed method finds the lowest average fuzzy cost in the seventh year, allowing the machine to be replaced at the end of the seventh year. In comparison to the results of both machines, machine A_1 can work for a longer period of time than machine A_2. As a result, we can conclude that machine A_1 will be superior to machine A_2.

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