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AUM BLOCK LABELLING FOR KITE, COMB AND DIAMOND SNAKE GRAPHS

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Abstract
Graph labelling, which is a fast-developing branch of Mathematics has a wide range of applications like data structuring, coding theory, machine learning, artificial intelligence, psychology, data mining etc. AUM Block labelling introduced by Uma Maheswari in 2022 is being developed for several graphs, with Python program code to generate block labels. This paper deals with AUM block labelling for Kite, Comb graph and Diamond snake graph. Python program coding to generate the block labels is presented. Apposite illustrations are also encompassed in this paper.
Keywords — AUM Block labelling, Kite graph, Double tailed kite graph, Comb graph, Diamond snake graph.
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Introduction

In graph theory, a graph G is a data structure comprising of a finite set of vertices and a set of edges connecting them. Graph labelling is assigning of labels represented by integers to the vertices [19][1][4], edges [20], faces [21]and blocks [10] [11][13] of a graph. Obtaining a vertex label, edge label, face label or block label for all graphs under certain constraints is an open task for researchers. Popular mathematicians Rosa[1], Prasanna[8], Uma Maheswari et al. (2019a)[15] (2019b)[16] (2019c)[17] (2019d)[18] (2019e)[19] (2022a)[11] (2022b)[12] have given remarkable contributions towards labelling of graphs. Labelled graphs are useful depictions for a varied range of applications in medical [22], astronomy [9], geo mechanics [25], image segmentation[6], data science [3] and communication network [7]. The graph theoretical methods for detecting and describing gestalt clusters [26] is quite interesting and astonishing. AUM block sum labelling for comb and diamond snake graph was studied by Uma Maheswari et al. in 2022[23] wherein the blocks are labelled using the sum of the vertex and edge labels [24]. In this paper, we have used new AUM block labelling induced by the product of vertex and edge labels for Kite graph, Comb graph and Diamond snake graph.

Preliminaries
The definitions and preliminary concepts essential for this paper are given below.

Definition 1[2].: Kite graph

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A (n,t) Kite graph consists of a cycle of length n with a t - edge path (tail) attached to one vertex of a cycle. It has n + t vertices and n + t edges.

Definition 1.: Double tailed Kite graph

We define a double tailed kite graph D(n,t,*l*) which consists of a cycle of length n with two tails one tail being t - edge path and another *l*- edged path attached to one vertex of a cycle. It has n + t vertices and
n + t +*l* edges.

Definition 2[23].: Comb graph

Let P_n be a path graph with n vertices. The comb graph is defined as $P_n \odot K_1$. It has 2n vertices and
2n-1 edges.

Definition 3[23]: Diamond snake graph.

A diamond snake graph D_n is obtained by joining vertices v_i and $v_{i+1}$to new vertices u_i and w_i for
i = 1, 2,,n-1.A diamond snake graph has 3n+1 vertices and 4n edges. n is the number of blocks.

Definition 5[14]: AUM Block labelling

Let G be a graph with p number of vertices, q number of edges and b number of blocks.

p, q, b ≥ 1. The set of vertices, edges and blocks are denoted by V(G)={v_1,v_2,…v_p},

E(G) = {e_1, e_2,…, e_q} and B (G) = {b_1,b_2,…b_n} respectively.

The graph G admits AUM Block labelling if there exists a bijection f: V(G)⟶ $\mathbb{Z}^+$ induced from f by

f *(uv) = f(u)*f(v) and f **(B_k) = B(G) ⟶ $\mathbb{Z}^+$ is defined as follows:

Let B_k be incident with the vertices $v_{k_1,} v_{k_2,},\ldots v_{k_n,}$, 1≤ k_n ≤ p and the edges $e_{k_1,} e_{k_2,},\ldots, e_{k_m,}$, 1≤ k_m ≤ q. then f ** (B_k) = $\sum_{i=1}^{n} f(v_{k_i}) + \sum_{i=1}^{m} f^*(e_{ki})$ and f^{**} (B_k) ≠ f^{**} (B_l) for

1≤ k, *l* ≤ b and k ≠ *l*

Definition 6 [11] AUM Block labelling, block labels induced by the product of vertex and edge labels.

Let graph G have p number of vertices, q number of edges and b number of blocks.

p, q, b $\geq$ 1. The set of vertices, edges and blocks are denoted by $V(G)=\{v_1,v_2,\ldots v_p\}$,

$E(G) = \{e_1, e_2\ldots, e_q\}$ and $B(G) = \{b_1, b_2,\ldots,b_n\}$ respectively.

We say the graph G admits AUM Block labelling, induced by the product of labels, if there exists a bijection $f: V(G)\longrightarrow \mathbb{Z}^+$ induced from f by $f^*(uv) = f(u) . f(v)$ and $f^{**}(B_k) = B(G) \longrightarrow \mathbb{Z}^+$ is defined as follows:

Let B_k be incident with the vertices $v_{k_1,} v_{k_2,}\ldots v_{k_n,}$, $1\leq k_n \leq p$ and the edges $e_{k_1,} e_{k_2,},\ldots, e_{k_m,}$, $1\leq k_m \leq q$. then $f^{**}(B_k) = \prod_{i=1}^{n} f(v_{k_i}) . \prod_{i=1}^{n} f^*(e_{ki}))$ and $f^{**}(B_k) \neq f^{**}(B_l)$ for

$1\leq k, l \leq b$ and $k \neq l$

MAIN RESULTS

In this section, we have proved that Kite, Comb and Diamond snake graphs admit AUM Block labelling induced by the product of vertex and edge labels. Appropriate illustrations are given for better understanding. Python program code is also presented to generate the Block labels.

Theorem 1: A single tailed (n,t) kite graph admits AUM Block labelling induced by the product of vertex and edge labels.

Let G be a (n,t) kite graph with $n \geq 3$ and $t \geq 1$. Let $V(G) = \{v_1,v_2,\ldots v_n\}$, $E(G) = \{e_1,e_2,\ldots,e_n\}$,

$B(G) = \{B_1, B_2,\ldots B_p\}$ signify the set of vertices, edges and blocks.

AUM block labelling for the blocks of G is defined as follows:

Define the function f, $f: V(G)\longrightarrow \{1, 2,\ldots, n\}$ as follows:

$f(v_1) = 1$

$f(v_k) = k \qquad 2 \leq k \leq n$

$f(u_t) = n + t \qquad t \geq 1$

The induced function f* is defined as follows:

$f^*: E(G) \longrightarrow \mathbb{Z}^+$ from f as

$f^*(v_1v_k) = k \qquad k =1, n$

$f^*(v_kv_{k+1}) = k(k+1) \qquad 2 \leq k \leq n-1$

$f^*(u_1v_1) = n+1$

$f^*(u_t u_{t+1}) = (n+t)(n+t+1) \qquad 1 \leq t \leq n, t = n \text{ or } t > n$

We now label the blocks, block labels induced by the product of vertex and edge labels.

Define $f^{**}: B(G) \longrightarrow \mathbb{Z}^+$ by

$f^{**}(B_1) = \{(1.2.3...n\}^3$

$f^{**}(B_k) = (n+1)^2$ for $k = 2$

$f^{**}(B_k) = \{(n + k - 2)(n + k - 1)\}^2 \quad 3 \le k \le n$

$f^{**}(B_k) \ne f^{**}(B_l)$ for $1 \le k, l \le b$ and $k \ne l$

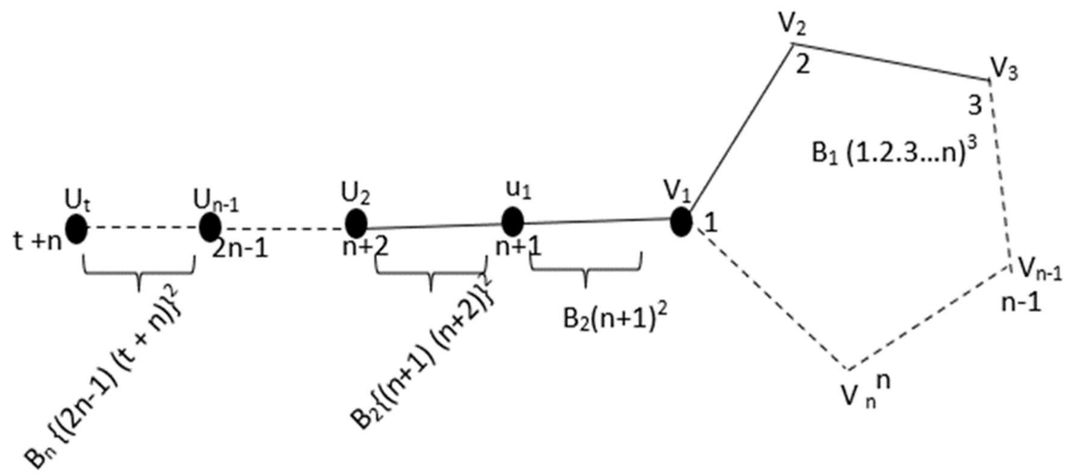

Fig1 AUM Block labelling for (n, t) kite graph

Example 1: A (5,5) kite graph admits AUM Block labelling induced by the product of vertex and edge labels.

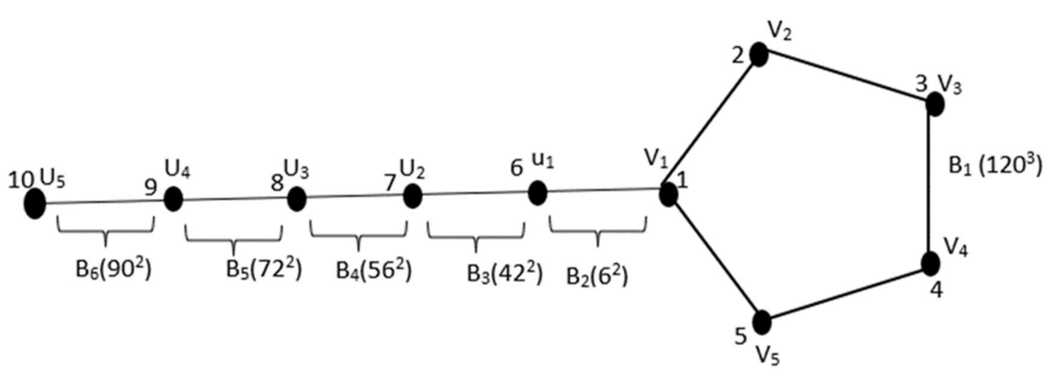

Fig 2 AUM Block labelling for (5,5) Kite graph

Thus, we have proved that Kite graph (5,5) admits AUM Block labelling.

Example 2: A (5,7) kite graph admits AUM Block labelling induced by the product of vertex and edge labels.

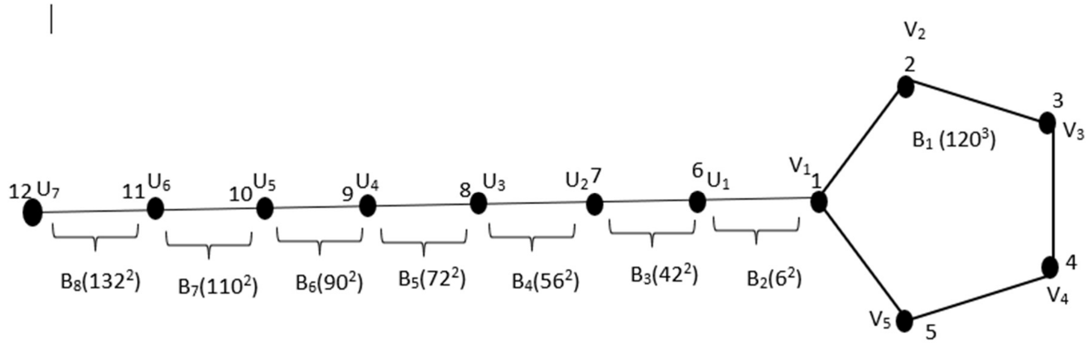

Fig 3 AUM Block labelling for (5,7) Kite graph

Thus, we have proved that (5,7) Kite graph admits AUM Block labelling induced by product of vertex and edge labels.

Theorem 2: A double tailed D (n,t,l) kite graph admits AUM Block labelling induced by the product of vertex and edge labels.

Let G be a D(n, t, l) a double kite graph with $n \geq 3$ and $t \geq 1$and $l > 1$. Let V(G) = $\{v_1, v_2, \ldots v_n\}$, E(G) = $\{e_1, e_2, \ldots, e_n\}$,

B(G) = $\{B_1, B_2, \ldots B_p\}$ signify the set of vertices, edges and blocks.

AUM block labelling for the blocks of G is defined as follows:

Define the function f, f: V(G)$\longrightarrow$ {1, 2,…, n} as follows:

$f(v_1) = 1$

$f(v_k) = k \qquad 2 \leq k \leq n$

$f(u_t) = n + t \qquad t \geq 1$

$f(w_l) = n+t+l \qquad l, t \geq 1$

The induced function f* is defined as follows:

f*: E(G) $\longrightarrow \mathbb{Z}^+$ from f as

$f^*(v_1v_k) = k \qquad 1 \leq k \leq n$

$f^*(v_kv_{k+1}) = k(k+1) \qquad 2 \leq k \leq n-1$

$f^*(u_1v_1) = n+1$

$f^*(u_t u_{t+1}) = (t + n)(t + n + 1) \qquad n \geq 3$

$f^*(u_1w_1) = n+t+1 \qquad t \geq 1$

$f^*(w_kw_{k+1}) = (t + n + k)(t + n+k+1) \qquad 1 \leq k \leq l$

We now label the blocks, block labels induced by the product of vertex and edge labels.

Define f**: B(G) $\longrightarrow \mathbb{Z}^+$ by

$f^{**}(B_1) = \{(1.2.3...n\}^3$

$f^{**}(B_k) = (k+1)^2$ for k =2

$f^{**}(B_k) = \{(n+ k -2)\ (n+k-1)\}^2$ $3 \leq k \leq t+1$

$f^{**}\ (B_k') = \{n+t+1)\}^2$ for k = 2

$f^{**}\ (B_k') = \{(n+t+k-2)\ (n+t+k-1)\}^2$ $3 \leq k \leq l+1$

$f^{**}\ (B_k) \neq f^{**}\ (B_l)$ for $1 \leq k, l \leq b$ and $k \neq l$ & $f^{**}\ (B_k') \neq f^{**}\ (B_l')$ for $k \neq l$

Example 3 : (t=l)

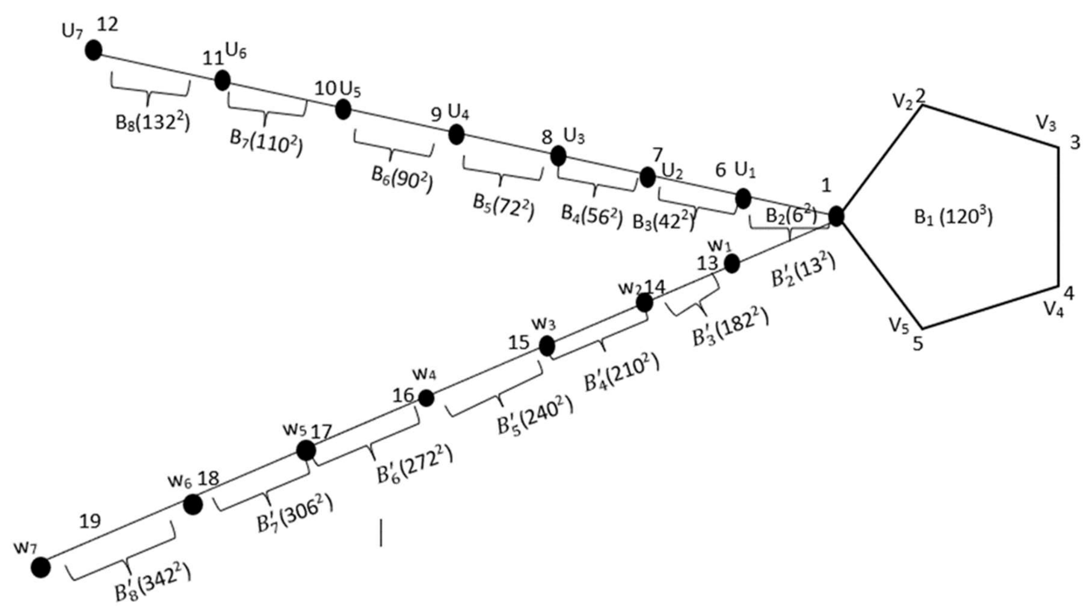

Fig 4 AUM Block labelling for D(5,7,7) Kite graph

Thus, we have proved that D(5,7,7) Double kite graph admits AUM Block labelling induced by product of vertex and edge labels.

Example 4 (t ≠ l)

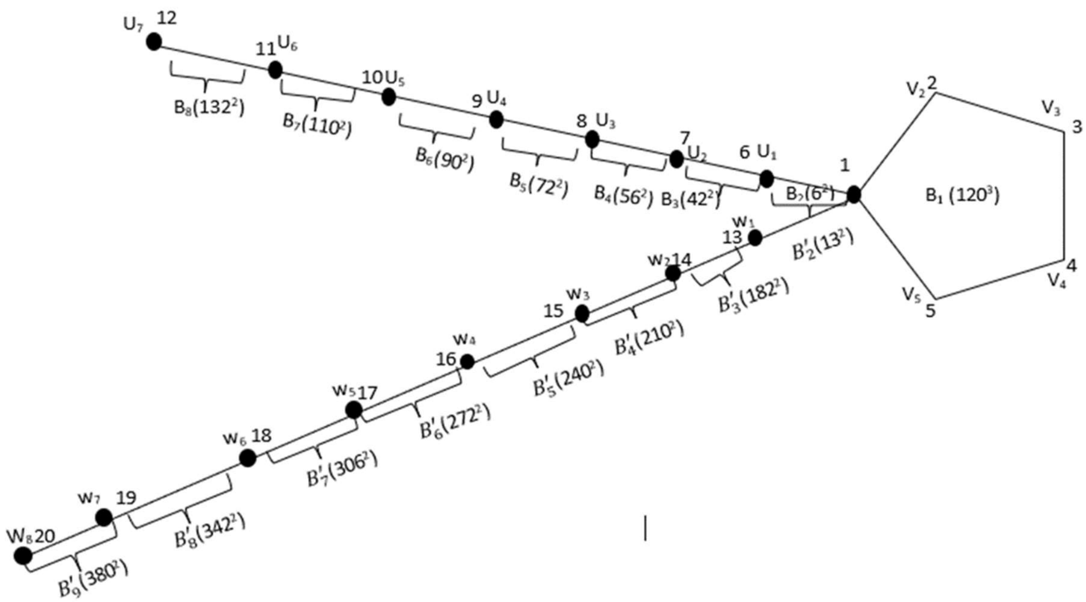

Fig 5 AUM Block labelling for D (5,7,8) Kite graph

Thus, we have proved that D (5,7,8) Double kite graph admits AUM Block labelling induced by product of vertex and edge labels.

Theorem 3: A Comb graph admits AUM Block labelling induced by the product of vertex and edge labels.

Let G be a Comb graph.

AUM block labelling for the blocks of G is defined as follows:

Define the function f: V(G)⟶ {1, 2,…,n}by

$f(v_k) = 2k - 1 \qquad 1 \leq k \leq n.$

$f(u_k) = 2k \qquad 1 \leq k \leq n.$

The induced function f* is defined as follows:

f*: E(G) ⟶ $\mathbb{Z}^+$ from f as

$f^*(u_k u_{k+1}) = 4k(k+1) \qquad 1 \leq k \leq n$

$f^*(v_k u_k) = 2k(2k-1) \qquad 1 \leq k \leq n$

We now label the blocks, block labels induced by the product of vertex and edge labels.

$f^{**}(B_{2k-1}) = \{4k^2 - 2k\}^2$

$f^{**}(B_{2k}) = \{4k^2 + 4k\}^2$

$f^{**}(B_k) \neq f^{**}(B_l)$ for $1 \leq k, l \leq b$ and $k \neq l$

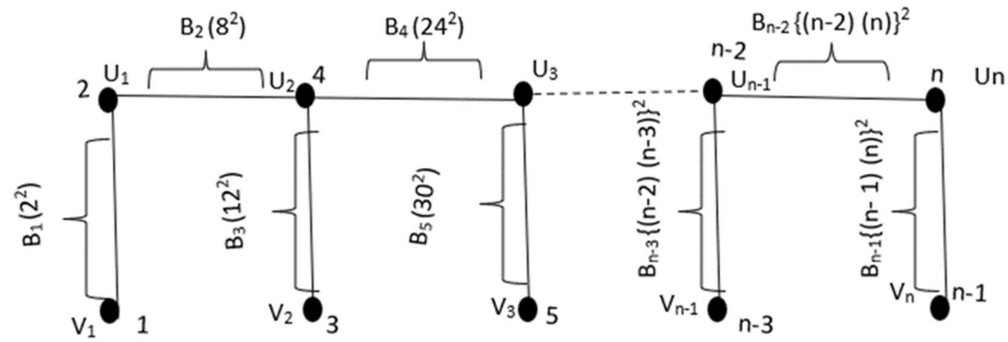

Fig 6 AUM Block labelling for comb graph Pn ⊙ K1

Example 5

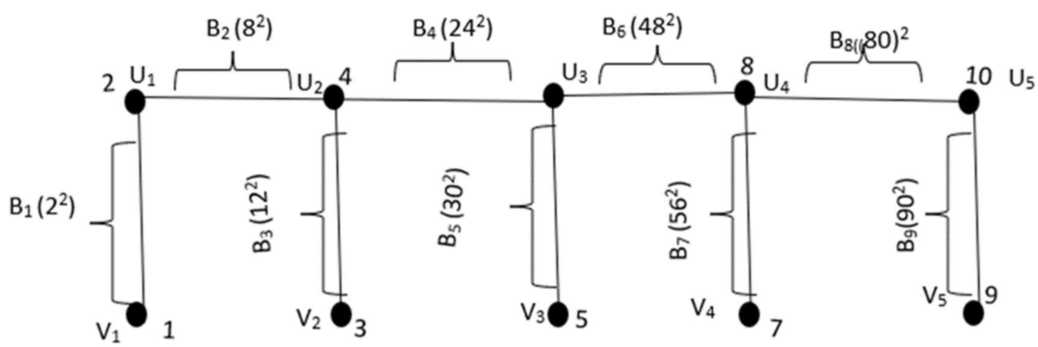

Fig 7 AUM Block labelling for Comb graph P5 ⊙ K1

Thus, we have proved that comb graph admits AUM Block labelling induced by product of vertex and edge labels.

We have generated the block labels of the Comb graph P5 ⊙ K1 using Python program code. The following is the Python program coding link.

Block Labelling for comb graph $B_{2k-1}=(4k^2-2k)^2$

```python
sbs=['₀','₁','₂','₃','₄','₅','₆','₇','₈','₉']

def subs(n):
  n=str(n)
  for i in n:
    print(sbs[int(i)],end='')

k=int(input("Enter value for k   : "))
p1=(2*k)-1
p2=4*k*k
p3=2*k
p4=p2-p3
bk=pow(p4,2)
print("B",end='')
subs(p1)
print("= [",p2,"-",p3,"]" ,"²","=",bk)
```

```
Enter value for k   : 5
B₉= [ 100 - 10 ] ² = 8100
```

Block Labelling for comb graph $B_{2k-1}=(4k^2-2k)^2$

```python
sbs=['₀','₁','₂','₃','₄','₅','₆','₇','₈','₉']

def subs(n):
  n=str(n)
  for i in n:
    print(sbs[int(i)],end='')

k=int(input("Enter value for k   : "))
p1=(2*k)-1
p2=4*k*k
p3=2*k
p4=p2-p3
bk=pow(p4,2)
print("B",end='')
subs(p1)
print("= [",p2,"-",p3,"]" ,"²","=",bk)
```

```
Enter value for k   : 15
B₂₉= [ 900 - 30 ] ² = 756900
```

Theorem 3: A Diamond snake graph Dn admits AUM Block labelling induced by the product of vertex and edge labels.
Let G be a Diamond snake graph.
AUM block labelling for the blocks of G is defined as follows:
Define the function f: V(G)⟶ {1, 2,…,n}by
f(vk) = k $\quad\quad$ $1 \leq k \leq n$.

The induced function f* is defined as follows:

f*: E(G) ⟶ $\mathbb{Z}^+$ from f as

$f^*(v_{3k-2}v_{3k-1}) = 9k^2 - 9k+2$ $\quad\quad$ $1 \leq k \leq n$.

$f^*(v_{3k-1}v_{3k+1}) = 9k^2 - 1$ $\quad\quad$ $1 \leq k \leq n$.

$f^*(v_{3k-2}v_{3k}) = 9k^2 - 6k$ $\quad\quad$ $1 \leq k \leq n$.

$f^*(v_{3k}v_{3k+1}) = 9k^2 + 3k$ $\quad\quad$ $1 \leq k \leq n$.

We now label the blocks, block labels induced by the product of vertex and edge labels**.**

$f^{**}(B_k) = \{(3k-2)\,(3k-1)\,(3k)\,(3k+1)\}^3$

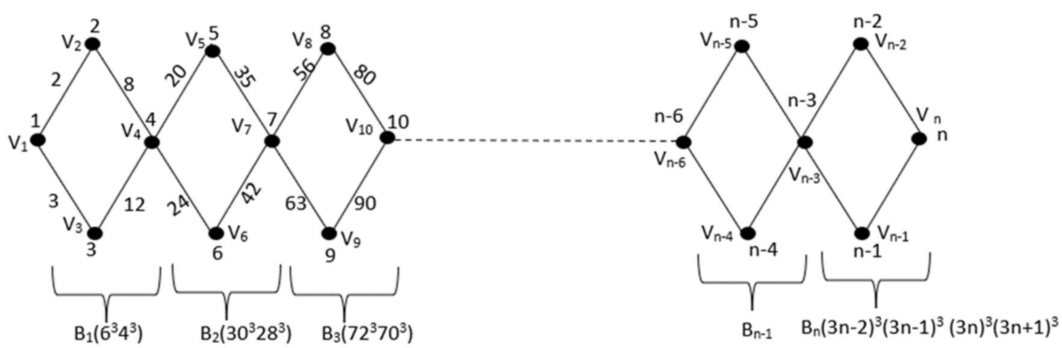

Fig 8 AUM Block labelling for Diamond snake graph Dn

Example 6: A Diamond snake graph D4 admits AUM Block labelling induced by the product of vertex and edge labels.

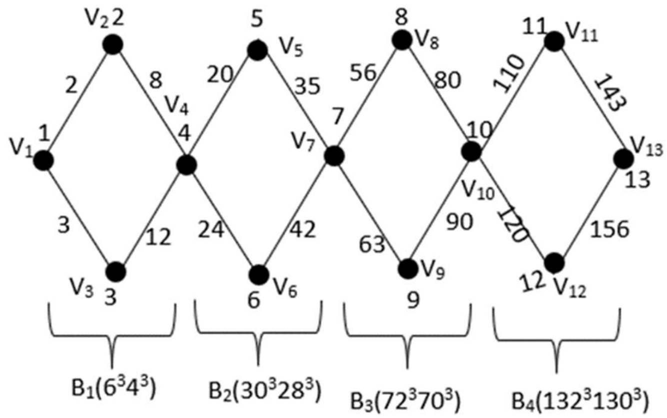

Fig 9 AUM Block labelling for Diamond snake graph D4

Thus, we have proved that Diamond snake graph D4 admits AUM Block labelling induced by product of vertex and edge labels.

Conclusion:

AUM Block labelling for Single tailed kite graph, Double tailed kite graph, Comb graph and Diamond snake graph is given in this paper. Also, Python program code to generate the block labels induced by the product of vertex labels and edge labels for Comb graph P5 $\odot$ K1 is included. In future, this AUM Block labelling can be used in graph-based methods to solve real situations in varied fields like communication network, psychology, psycho-therapy, cryptography, software programming, networking etc.

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