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DIOPHANTINE EQUATIONS – A PIVOTAL WAY IN BUILDING ARCHITECTURE AND COMPUTER NETWORK FLOW

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Abstract:

In this paper I use Diophantine equation to find out the solution for measuring the land to make garden and to measure the flow of liquid in pipe line and the traffic in a particular place. We can find out solution for so many things in our day to day life using this Diophantine equation. The application of Diophantine equation is very interesting. In this paper I tried and solve some examples based on this**.**

Keywords:

Diophantine equations,Euclideanalgorithm, Gnome, Bricks, Long division method, Network flow, Inflow, Outflow, City, Streets.

I.Introduction

Number Theory is not only a systematic mathematical study but also a popular diversion.

The linear Diophantine Equation is

$ax_1+bx_2=c$

Where a, b, c are integers and a, b are not both zero.

II.Euclidean Algorithm

Let a, b be two positive integers. Then

$a=q_1b+r_1 \quad 0\leq r_1<b$

$b=q_2r_1+r_2 \quad 0\leq r_2<r1$

$r_1=q_3r_2+r_3 \quad 0\leq r_3<r2$

.

Continue until remainder is zero

$r_{n-2}=q_nr_{n-1}+r_n 0\leq r_n<r_{n-1}$

$r_{n-1}=q_{n+1}r_n+0$

Hence $gcd(a,b)=r_n$

Example 2.1:

Mr and Mrs Lenin is a traditional happy gnome couple. Unfortunately they have been in a condition to shift them into my garden are they are not in a condition to hope up with their rising mortgage repayments. However they have been faced with some trouble in building

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their new sweet home. Gnome by laws state that the total number of bricks used in any construction project must be 599 or planning permission will not be granted. As gnome houses form formally the shape of a triangular prism and a wall(namely the garden fence) is already in the place, only two walls ought to be built. The plot of the land they have required is shown below with all dimensions measured in bricks. The equation is

$$49x+23y=599$$

What is the least number of bricks they have to purchase to equalize with the local council, still no waste of single penny.

Given:$49x+23y=599$

By Euclid's Algorithm, we get

$3=49-2.23$ (1) ⟶

$2=23-7.3$ (2) ⟶

$1=3-1.2$(3) ⟶

Then (3) can be involves as 49 and 23.

$$1=3-23+7.3$$

$$1=49-2.23-23+7.49-14.23$$

$$1=49(8)+23(-17)$$

But the equation of RHS needs 599.So multiply both sides by 599.

$$49(8\times599)+23(-17\times 599)=599$$

$$49(4792)+23(-10183)=599$$

This is the answer, but the bricks cannot be negative. So make y as positive without changing the equation. Then the above equation can be written as

$$49(4792)+23(-10183+n)-23n=599$$

$$49(4792-\frac{23n}{49})+23(-10183+n)=599$$

Taken=49t for some t.

$$49(4792-23t)+23(-10183+49t)=599$$

To find t.Using long division method, then

$$49(4792-(23\times208))+23(-10183+(49\times208))=599$$

$$49(8)+23(9)=599$$

Hence the solution is x=8 and y=9.

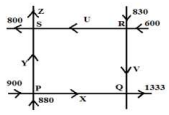
III.Network flow:

In modelling material flow, the directed graph of flow network is used.Flow rates are involved in our life in so many critical positions,some are self evident,such as traffic flow and the flow of oil in a pipeline.

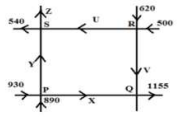
Example 3.1:

The following diagram defines as,In Bangalore city,the flow of traffic in vehicles per hour,over so many one way streets during a peak early afternoon. Determine the general flow patterns for the network.

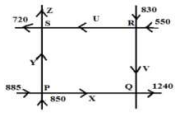
City A:



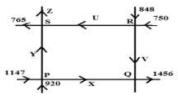
CityB:



City C:



City D:



The above diagram can be represented as the following:

City A:

Intersection points	**Inflow**	**Outflow**
P	880+900	x+y
Q	x+v	1333
R	830+600	u+v
S	y+u	800+z

The Total Inflow=Total Outflow

$$\Rightarrow 880+900+x+v+830+600+y+u=x+y+1333+u+v+800+z$$

$$\Rightarrow z=1077$$

We show the above situation as

$$x+y=1780$$

$$x+v=1333$$

$$u+v=1430$$

$$y+u=1877$$

This is a system of Linear Diophantine equations in five variables.Here x,y,z,u,v represents vehicle.So x,y,z,u,v must be whole numbers.Then the solution is

$$x=1333-v$$
$$y=447+v$$
$$z=1077$$
$$u=1430-v$$
$$v=\text{free variable}$$

The variables cannot be negative in anyway.Because the streets in the problem are one way and next to it,a negative flow in network branch corresponding to flow in opposite direction to the above shown model. This fact paves a way to certain limitations on the possible variables. So, the solution depend upon the choice of v from 0 to 1332.

City B:

Intersection points	Inflow	Outflow
P	890+930	x+y
Q	x+v	1155
R	500+620	u+v
S	y+u	540+z

The Total Inflow=Total Outflow

$$\Rightarrow 890+930+x+v+500+620+y+u=x+y+1155+u+v+540+z$$
$$\Rightarrow z=1245$$

We show the above situation as

$$x+y=1820$$
$$x+v=1155$$
$$u+v=1120$$
$$y+u=1785$$

This is a system of Linear Diophantine equations in five variables. Here x, y, z, u, v represents vehicle. So x, y, z, u, v must be whole numbers. Then the solution is

$$x=1155-v$$
$$y=665+v$$
$$z=1245$$
$$u=1120-v$$
$$v=\text{free variable}$$

The variables cannot be negative in anyway.Because the streets in the problem are one way and next to it, a negative flow in network branch corresponding to flow in opposite direction to the above shown model. This fact paves a way to certain limitations on the possible variables. So, the solution depend upon the choice of v from 0 to 1119.

City C:

Intersection points	Inflow	Outflow

P	850+885	x+y
Q	x+v	1240
R	550+830	u+v
S	y+u	720+z

The Total Inflow=Total Outflow

$$\Rightarrow 850+885+x+v+550+830+y+u=x+y+1240+u+v+720+z$$

$$\Rightarrow z=1155$$

We show the above situation as

$$x+y=1735$$
$$x+v=1240$$
$$u+v=1380$$
$$y+u=1875$$

This is a system of Linear Diophantine equations in five variables. Here x, y, z, u, v represents vehicle. So x, y, z, u, v must be whole numbers. Then the solution is

$$x=1240-v$$
$$y=495+v$$
$$z=1145$$
$$u=1380-v$$
$$v=\text{free variable}$$

The variables cannot be negative in anyway.Because the streets in the problem are one way and next to it, a negative flow in network branch corresponding to flow in opposite direction to the above shown model. This fact paves a way to certain limitations on the possible variables. So, the solution depend upon the choice of v from 0 to 1239.

City D:

Intersection points	Inflow	Outflow
P	920+1147	x+y
Q	x+v	1456
R	750+848	u+v
S	y+u	765+z

The Total Inflow=Total Outflow

$$\Rightarrow 920+1147+x+v+750+848+y+u=x+y+1456+u+v+765+z$$

$$\Rightarrow z=1444$$

We show the above situation as

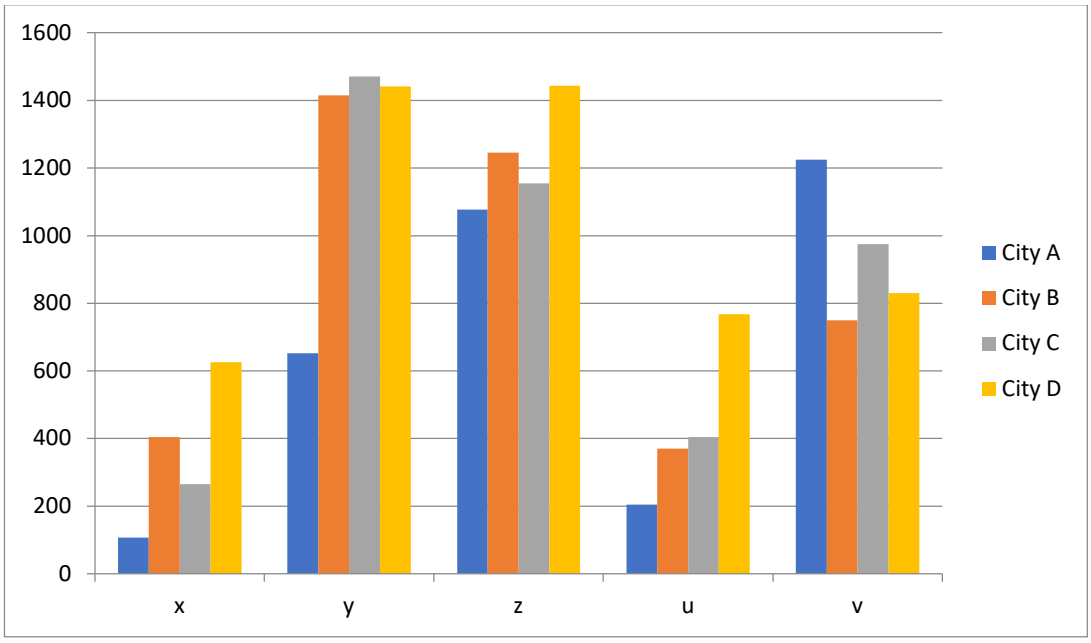
$$x+y=2067$$
$$x+v=1456$$
$$u+v=1598$$
$$y+u=2209$$

This is a system of Linear Diophantine equations in five variables. Here x, y, z, u, v represents vehicle. So x, y, z, u, v must be whole numbers.Then the solution is

$$x=1456-v$$

y=611+v
z=1444
u=1598-v
v=free variable

The variables cannot be negative in anyway.Because the streets in the problem are one way and next to it, a negative flow in network branch corresponding to flow in opposite direction to the above shown model. This fact paves a way to certain limitations on the possible variables.So,the solution depend upon the choice of v from 0 to 1455.



IV.Conclusion

By using the application of Diophantine equation,I found out the integer solution for things that happened in our day to day life. In this paper I proved how Diophantine equation is highly useful and played a vital role in the field of Building Architecture and in Computer Science.

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