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A NEW CONCEPT OF MICRO-SEMI-OPEN SETS

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Abstract:

Topologists are known to have developed a variety of topologies. Fine topology, supra topology, ideal topology, fuzzy topology, nano topology and micro topology were initiated in 2019, also in micro topology some different types of sets were introduced, like micro (semi-open, pre-open, α-open, β- open, ... etc.) sets and their properties with operators have been given in numerous articles. In this work, a new powerful form of micro-semi-open sets called micro-S_p-open sets are proposed, some of its features are verified as well as their relationships to other micro-open sets, and the idea of micro-S_p-continuity and comparison with others is introduced micro continuity concepts.

Key words: Micro semi-open set, Micro S_p-continuous, Micro S_p-open sets, Micro topological spaces.

Introduction:

As is well known, topology is an essential factor of mathematics, and the concept of topology is growing by the day. Many different topologies have been developed thus far. A non-negative real-valued quasi-pseudo-metric on a set fulfills only the triangle inequality. It's a quasi-metric if the distance between two unique points isn't zero. A conjugate quasi-pseudo-metric q(q(x,y)=p (x,y) is determined by a quasi-pseudo-metric p. Kelly proposed Bitopological spaces in 1963 after studying the effect of employing the topologies provided by these two distance functions simultaneously.

Chandrasekar S. developed micro-topology based on nano-topology in 2019, although Lellis Thivagar first proposed the concept of nano-topology in 20131,2. Lower approximations produce subsets with specific objects that will obviously be part of an interesting subset, while upper approximations produce subsets with unknown objects that may become part of an interesting subset3. Micro semi-open and micro pre-open sets have also been established by Chandrasekar S. during 2019, and micro continuity, micro semi-continuity, and micro pre-continuity were introduced1. Mashhour AS, Abd-El-Monsef ME and El-Deeb SN defined the pre-open set in 1982, Norman Levin provided the semi-open set in 1963, and Njasted presented the α -open set 4,5,6 in 1965 In 2020 Ibrahim H. Z. introduced micro β-open sets 7, and in 2018 Chandrasekar S. and Swathi G. they were studied on micro α-open sets and micro α- continuity in micro topological spaces 8, 9. In 2021 Jassim R. H., Rasheed R. O. and Faris H. I. introduced the concept of θ- micro-open sets and θ-micro continuity 10. In this paper, I describe micro S_p-open, a new type of micro semi-open set, and investigate several of its features and characterizations, as well as present micro S_p-continuity and compare it to other types of micro continuity

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Preliminaries: This section is to gives some results that used in the next sections.

Definitions 1: If V is a non-empty finite set of objects called the universe and Γ is an equivalence relation on V named as the indiscernibility relation. Then the pair (V,Γ) is said to be the approximation space. And let X⊆V, then 3:

(i) The lower approximation of X with respect to Γ is denoted by $L_{\Gamma}(X)$ and defined by $L_{\Gamma}(X) = \bigcup_{x\in V}\{\Gamma(x): \Gamma(x) \subseteq X\}$, where $\Gamma(x)$ denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is denoted by $U_{\Gamma}(X)$ and it is $U_{\Gamma}(X) = \bigcup_{x\in V}\{\Gamma(x): \Gamma(x) \cap X \neq \phi\}$.

(iii) The boundary region of X with respect to R is denoted by $B_{\Gamma}(X)$ and it is $B_{\Gamma}(X) = U_{\Gamma}(X) - L_{\Gamma}(X)$.

Definitions 2: Let V be the universe, Γ be an equivalence relation on V and $\tau\Gamma(X) = \{V, \phi, L_{\Gamma}(X), U_{\Gamma}(X), B_{\Gamma}(X)\}$, where $X \subseteq V$. If $\tau\Gamma(X)$ satisfies the following axioms [2]:

(i) $V, \phi \in \tau\Gamma(X)$.

(ii) If $\{A_i: i \in I\}$ be a family of elements of $\tau\Gamma(X)$, then $\bigcup_{i\in I} A_i$ is in $\tau\Gamma(X)$.

(iii)If A_1 and A_2 are elements of $\tau\Gamma(X)$, then $A_1 \cap A_2$ is in $\tau\Gamma(X)$.

Then $(V, \tau\Gamma(X))$ is called the nano topological space, where $\tau\Gamma(X)$ is a topology on V termed the nano topology with regard to X. The components of $\tau\Gamma(X)$ are known as nano open sets, and $[\tau\Gamma(X)]^c$ is the dual nano topology.

Definition 3: If $(V, \tau\Gamma(X))$ is a nano topological space with respect to X and if $A \subseteq V$; then the nano interior of A is denoted by $Nint(A)$ and defined by $Nint(A) = \cup \{G \subseteq V: G$ is nano open and $G \subseteq A\}$. The nano closure of A is denoted by $Ncl(A)$ and defined by $Ncl(A) = \cap \{F \subseteq V: F$ is nano closed and $A \subseteq F\}$.[2]

Definition 4: If $(V, \tau\Gamma(X))$ is a nano topological space with regard to X, where $X \subseteq V$, then $\mu\Gamma(X) = \{N \cup (N' \cap \mu): N, N' \in \tau\Gamma(X)$ and $\mu \notin \tau\Gamma(X)\}$is called the micro topology on V with respect to X. The triple $(V, \tau\Gamma(X), \mu\Gamma(X))$ is referred to as micro topological space and the members of $\mu\Gamma(X)$ are known as micro-open sets, while the complement of micro-open set is considered to as micro-closed set [1].

Definition 5: The micro closure of a set K of a micro topological space $(V, \tau\Gamma(X), \mu\Gamma(X))$, denoted by $mic - cl(K)$, and defined by $mic - cl(K) = \cap \{H: H$ is micro-closed set in V and $K \subseteq H\}$. The micro interior of K is denoted by $mic - int(K)$ and defined as $mic - int(K) = \cup \{H: H$ is micro-open set and $H \subseteq K\}$[1].

Definition 6: The following are hold, for any sets L and H in $(V, \tau\Gamma(X), \mu\Gamma(X))$[1]:

1. L is in $[\mu\Gamma(X)]^c$ if and only if $L = mic - cl(L)$.
2. L is belong to $\mu\Gamma(X)$ if and only if $L = mic - int(L)$.

3. $L \subseteq H \Rightarrow mic - int(L) \subseteq mic - int(H)$ and $mic - cl(L) \subseteq mic - cl(H)$.
4. $mic - cl(mic - cl(L)) = mic - cl(L)$ and $mic - int(mic - int(L)) = mic - int(L)$.
5. $mic - cl(L \cup H) \supseteq mic - cl(H) \cup mic - cl(H)$.
6. $mic - cl(L \cap H) \subseteq mic - cl(L) \cap mic - cl(H)$.
7. $mic - int(L \cup H) \supseteq mic - int(L) \cup mic - int(H)$.
8. $mic - int(L \cap H) \subseteq mic - int(L) \cap mic - int(H)$.
9. $mic - cl(L^C) = [mic - int(L)]^C$.
10. $mic - int(L^C) = [mic - cl(L)]^C$.

Definition 7: Let K be in $(V, \tau\Gamma(X), \mu\Gamma(X))$, Then:
1. K is in $MicSO(V, X)$ if $K \subseteq mic - cl(mic - int(K))$[1].
2. K is belong to $MicPO(V, X)$ if $K \subseteq min - int(mic - cl(K))$[1].
3. K is θ- micro-open set if for each $x \in K$, there exists a micro-open set G such that $x \in G \subseteq mic - cl(G) \subseteq K$[10].

Proposition 8: Every micro-open set implies micro semi-open and implies micro pre-open set in any micro topological space $(V, \tau\Gamma(X), \mu\Gamma(X))$.

Definition 9: Let $(V, \tau\Gamma(X))$ be a nano space and $B \subseteq V$. Then B is said to be nano S_p-open if for each $x \in B \in NSO(V, X)$, where $NSO(V, X)$ is the family of nano semi-open sets, there exists a nano pre-closed set F such that $x \in F \subseteq B$.[2]

Definition 10: A function $f: (V, \tau\Gamma(X), \mu\Gamma(X)) \rightarrow (V', \tau'\Gamma(Y), \mu'\Gamma'(Y))$, is micro semi-continuous (micro pre-continuous) if the inverse image of every micro-open set in V' is micro semi-open (micro pre-open) set in V[1].

Micro S_p-open sets

Definition 11: A micro semi-open set K of a micro topological space $(V, \tau\Gamma(X), \mu\Gamma(X))$ called micro S_p-open set if for each $x \in K$, a micro pre-closed set F exists such that $x \in F \subseteq A$. The set of all micro S_p-open sets of V is denoted by $MicS_pO(V, X)$ and its complement called micro S_p-closed, and the family of micro S_p-closed sets of V is denoted by $MicS_pC(V, X)$.

Example 12: Consider $V = \{p, q, r, s, t\}$, $V/\Gamma = \{\{p\}, \{q, r, s\}, \{t\}\}$ and $X = \{p, q\} \subseteq V$. Then $\tau\Gamma(X) = \{V, \phi, \{p\}, \{q, r, s\}, \{p, q, r, s\}\}$ is a nano topology on V with respect to X. Now if $\mu = \{t\}$, then $\mu\Gamma(X) = \{V, \phi, \{p\}, \{t\}, \{p, t\}, \{q, r, s, t\}, \{q, r, s\}, \{p, q, r, s\}\}$ is a micro topology on V. Then here $\{p, t\}$ is a micro S_p-open sets

Proposition 13: A micro semi-open subset K of $(V, \tau\Gamma(X), \mu\Gamma(X))$ is micro S_p-open set if and only if K is the union of micro pre-closed subsets of V.

Proof: Let K be micro S_p-open set. Then by **(Definition 11)** K is micro semi-open set and for each $x \in K$, there exists a micro pre-closed F_x such that $x \in F_x \subseteq K$. Then $K = \bigcup_{x \in K} \{x\} \subseteq \bigcup_{x \in K} F_x \subseteq K$ this implies that $K = \bigcup_{x \in K} F_x$. Thus K is the union of micro pre-closed subsets of K.

Conversely: Let the hypothesis be true and let K be a micro semi-open subset of V. Now let $a \in K$ be any point, then since $K = \bigcup_{x \in K} F_x$, where F_x is micro pre-closed for each $x \in K$. And then there exists micro pre-closed F_a such that $a \in F_a \subseteq \bigcup_{x \in K} F_x = K$. Thus $K \in MicS_pO(V, X)$.

It is clear that from the **(Definition 11)** every micro S_p-open set is micro semi-open set. But the converse is not true in general as an example:

Example 14: Let $V = \{1, 2, 3, 4\}$, $V \backslash \Gamma = \{\{1,2\}, \{3\}, \{4\}\}$ and $X = \{2,3,4\}$. Then $\tau\Gamma(X) = \{V, \phi, \{3\}, \{4\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}, \{3, 4\}\}$ and if $\mu = \{2,4\}$, then $\mu\Gamma(X) = \{V, \phi, \{2\}, \{3\}, \{4\}, \{2,3\}, \{2, 4\}, \{3, 4\}, \{1,2\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}\}$. Here $MicSO(V, X) = \mu_R(X)$ and $MicS_pO(V, X) = \{V, \phi, \{3\}, \{4\}, \{1,2\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}\}$. Now the set $\{2\}$is micro semi-open set but not micro S_p-open set.

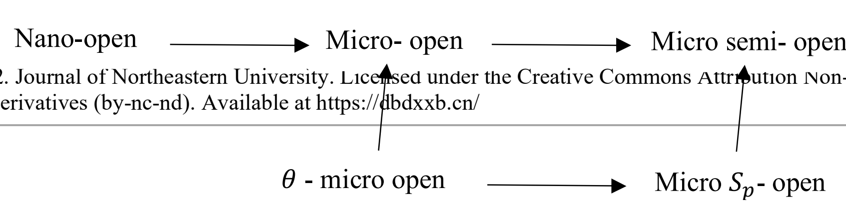
Proposition 15: Every θ-micro open set is micro S_p-open set.

Proof: Let H be a θ-micro open set in micro topological space $(V, \tau\Gamma(X), \mu\Gamma(X))$. Then **(Definition 7 (3))** for each $x \in H$, there exists a micro open set G_x such that $x \in G_x \subseteq mic - cl(G_x) \subseteq H$, this implies that $\bigcup_{x \in H} \{x\} \subseteq \bigcup_{x \in H} G_x \subseteq \bigcup_{x \in H} mic - cl(G_x) \subseteq H$, and then $H = \bigcup_{x \in H} G_x = \bigcup_{x \in H} mic - cl(G_x)$. Now $\bigcup_{x \in H} G_x$ is micro-open set, then by **(Proposition 8 (1))** A is micro semi-open set, and since for each $x \in H$, $mic - cl(G_x)$ is micro closed set, then **(Proposition 8 (2))** $mic - cl(G_x)$ is micro pre-closed and thus H is micro S_p-open set.

The implementation of the above sentence may not be true in general, as illustrate in the following example: Let $V = \{1, 2, 3, 4\}$, $V \backslash \Gamma = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{2, 4\}$, then $\tau\Gamma(X) = \{V, \phi, \{2,4\}\}$, if $\mu = \{1\}$, then $\mu\Gamma(X) = \{V, \phi, \{1\}, \{2, 4\}, \{1, 2, 4\}\}$ is a micro topology on V with respect to X. Now here $\{2, 4\}$ is micro S_p-open set but not θ- micro open set.

Remark 16: The concept of micro-open and micro S_p-open sets are independent as shown in the following example: Let $V = \{1, 2, 3, 4\}$, $V \backslash \Gamma = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{2,4\}$. Then $\tau\Gamma(X) = \{V, \phi, \{ 2, 4\}\}$ and if $\mu = \{1\}$, then the micro topology on V with respect to X is $\mu\Gamma(X) = \{V, \phi, \{1\}, \{2, 4\}, \{1,2,4\}\}$. And then $MicS_pO(V, X) = \{V, \phi, \{1, 3\}, \{2,4\}, \{1, 2, 4\}, \{2,3,4\}\}$. Then $\{1\}$ is in $\mu\Gamma(X)$ but not in $MicS_pO(V, X)$ and $\{1, 3\} \in MicS_pO(V, X)$ but not in $\mu\Gamma(X)$.

From **(Proposition 15)** and **(Remark 16),** the following diagram have:



Lemma 17: The union of a family of micro semi-open sets is micro semi-open.

Proof: Let $\{A_i : i \in \Delta\}$ be a family of micro semi-open sets in micro topological space $(V, \tau\Gamma(X), \mu\Gamma(X))$. Then for each $i \in \Delta$, $A_i \subseteq mic - cl(mic - int(A_i))$. And now $\bigcup_{i\in\Delta} A_i \subseteq \bigcup_{i\in\Delta}[mic - cl(mic - int(A_i))]$

$$\subseteq mic - cl(\bigcup_{i\in\Delta} mic - int(A_i))$$
$$\subseteq min - cl(mic - int(\bigcup_{i\in\Delta} A_i))$$

This implies that $\bigcup_{i\in\Delta} A_i$ is micro semi-open set in V.

Proposition 18: The union of any family of micro S_p-open sets is micro S_p-open.

Proof: Let $\{A_i : i \in \Delta\}$ be a family of micro S_p-open sets in micro topological space $(V, \tau\Gamma(X), \mu\Gamma(X))$. Then for each $i \in \Delta$, A_i is micro semi-open set and by **(Lemma 17)** $\bigcup_{i\in\Delta} A_i$ is micro semi-open set. Now let $x \in \bigcup_{i\in\Delta} A_i$ be any point, then there exists $j \in \Delta$ such that $x \in A_j$, and then there exists a micro pre-closed set F_j such that $x \in F_j \subseteq A_j \subseteq \bigcup_{i\in\Delta} A_i$, this implies that $\bigcup_{i\in\Delta} A_i$ belong to $MicS_pO(V, X)$.

Corollary 19: Arbitrary intersection of micro S_p-closed set is micro S_p-closed.

If $H, K \in MicS_pO(V, X)$, then their intersection need not be micro S_p-open as exists in example of **(Remark 16)** $A = \{1, 3\}$ and $B = \{1, 2, 4\}$ are micro S_p-open sets but $A \cap B = \{1\}$ which is not micro S_p-open set.

Remark 20: The concepts of nano S_p-open set and micro S_p-open set are independent of each other as an example below:

Example: Let $V = \{a, b, c\}$, $V \backslash \Gamma = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$. Then the nano topology on V with respect to X is $\tau\Gamma(X) = \{V, \phi, \{a\}\}$ and if $\mu = \{c\}$, then $\mu\Gamma(X) = \{V, \phi, \{a\}, \{c\}, \{a, c\}\}$. Then here $\{a\}$ is nano S_p-open set but not micro S_p-open and $\{c\}$ is micro S_p-open set but not nano S_p-open set, while every nano open set is micro-open set.

Definition 21: Let $(V, \tau\Gamma(X), \mu\Gamma(X))$ be a micro space. Then $Mic - S_pint(H) = \cup \{G \in MicS_pO(V, X) : G \subseteq H\}$ is the micro S_p interior of H and $Mic - S_pcl(A) = \cap \{F \in MicS_pC(V, X) : H \subseteq F\}$ is the micro S_p-closure of H, so it is the smallest micro S_p-closed set containing H.

Without the proof the following theorem is on micro S_p-interior and micro S_p-closure.

Theorem 22: Let $(V, \tau\Gamma(X), \mu\Gamma(X))$ be a micro space. Then for any subsets A and B of V, have:

1. A is micro S_p-open set if and only if $Mic - S_pint(A) = A$.
2. A is micro S_p-closed set if and only if $Mic - S_pcl(A) = A$.

3. If $A \subseteq B$, then $Mic - S_p int(A) \subseteq Mic - S_p int(B)$ and $Mic - S_p cl(A) \subseteq Mic - S_p cl(B)$.
4. $Mic - S_p int(A) \cup Mic - S_p int(B) \subseteq Mic - S_p int(A \cup B)$.
5. $Mic - S_p int(A \cap B) \subseteq Mic - S_p int(A) \cap Mic - S_p int(B)$.
6. $Mic - S_p cl(A) \cup Mic - S_p cl(B) \subseteq Mic - S_p icl(A \cup B)$.
7. $Mic - S_p cl(A \cap B) \subseteq Mic - S_p cl(B) \cap Mic - S_p cl(A \cup B)$.
8. $Mic - S_p int(A^c) = \left[Mic - S_p cl(A)\right]^c$.
9. $Mic - S_p cl(A^c) = \left[Mic - S_p int(A)\right]^c$.
10. $V \backslash Mic - S_p cl(V \backslash A) = Mic - S_p int(A)$.
11. $V \backslash Mic - S_p int(V \backslash A) = Mic - S_p cl(A)$.

Definition 23: A micro topological space $(V, \tau\Gamma(X), \mu\Gamma(X))$ is called micro locally indiscrete if every micro-open set is micro closed.

Theorem 24: If a micro space $(V, \tau\Gamma(X), \mu\Gamma(X))$ is micro locally indiscrete, then $MicSO(V, X) = MicS_pO(V, X)$.
Proof: It is obvious micro S_p-open implies micro semi-open.
To show every micro semi-open set is micro S_p-open set, so let H be a micro semi-open set in V, then $H \subseteq mic - cl(mic - int(H))$, but V is micro locally indiscrete so $mic - int(H)$ is micro closed, implies that $mic - cl\big(mic - int(H)\big) = mic - int(H)$ and then H is in $MicPC(V, X)$. Thus H is micro S_p-open set.

Lemma 25: If a space V is micro locally indiscrete, then every micro semi-open set is micro pre-open.
Proof: Let H be a micro semi-open set in a micro locally indiscrete space $(V, \tau\Gamma(X), \mu\Gamma(X))$. Then $H \subseteq mic - cl(mic - int(H))$, and since V is micro locally indiscrete implies that $mic - cl\big(mic - int(H)\big) = mic - int(H)$, then H is micro-open set since $H \subseteq mic - cl\big(mic - int(H)\big) = mic - int(H)$, so H is in$MicPO(V, X)$.

Corollary 26: If a space $(V, \tau\Gamma(X), \mu\Gamma(X))$ is micro locally indiscrete, then every micro S_p-open set is micro pre-open.

Lemma 27: A subset H of a micro space $(V, \tau\Gamma(X), \mu\Gamma(X))$ is micro S_p-open set if and only if for each $x \in H$, there exists a micro S_p-open set K such that $x \in K \subseteq H$.

Micro S_p-continuous function

Definition 28: A function $f: V \to V'$, where $(V, \tau\Gamma(X), \mu\Gamma(X))$ and $(V', \tau'\Gamma'(Y), \mu'\Gamma'(Y))$ are micro topological spaces, is called micro S_p-continuous function at a point $x \in V$ if for each micro-open set H in V' containing $f(x)$, there exists a micro S_p-open set K in V containing x such that $f(K) \subseteq H$.

Example 29: Let $V = \{a,b,c\}$, $V\backslash\Gamma = \{\{a\},\{b,c\}\}$ and $X = \{a\}$, then $\tau\Gamma(X) = \{V,\phi,\{a\}\}$ and if $\mu = \{c\}$, then $\mu\Gamma(X) = \{V,\phi,\{a\},\{c\},\{a,c\}\}$ is a micro topology on V with respect to X, and consider if $V' = \{1,2,3\}$, $V'\backslash\Gamma' = \{\{1,2\},\{3\}\}$ and $Y = \{2,3\}$, then $\tau'\Gamma'(Y) = \{V',\phi,\{3\},\{1,2\}\}$ and if $\mu' = \{2\}$, then $\mu'\Gamma'(Y) = \{V',\phi,\{2\},\{3\},\{2,3\},\{1,2\}\}$ is a micro topology on V' with respect to Y and $MicS_pO(V',Y) = \{V',\phi,\{3\},\{1,2\},\{1,3\}\}$.

Now define a function $g: V \to V'$ as: $g(x) = \begin{cases} c & if\ x = 1,2 \\ a & if\ x = 3 \end{cases}$, then g is micro S_p-continuous function.

Theorem 30: A function $f: V \to V'$ is micro S_p-continuous if and only if for every micro-open set H in V', $f^{-1}(H)$ is micro S_p-open set in V.

Proof: Let f be micro S_p-continuous and H be any micro-open set in V'. Then if $f^{-1}(H) = \phi$, then it is clear $f^{-1}(H) \in MicS_pO(V,X)$, but if $f^{-1}(H) \neq \phi$, then there is $h \in f^{-1}(H)$ implies that $f(h) \in H$ and since f is micro S_p-continuous, so there exists a micro S_p-open set $h \in K$ in V such that $f(K) \subseteq H$ implies that $K \subseteq f^{-1}(H)$, and then by **(Lemma 27)** $f^{-1}(H)$ is micro S_p-open set in V.

Conversely: Let the hypothesis be hold and let H be any micro-open set in V' containing $f(h)$, where $h \in V$. Then $h \in f^{-1}(H)$ implies by hypothesis $f^{-1}(H)$ is micro S_p-open in V containing h, now take $K = f^{-1}(H)$ and $f(K) \subseteq H$. Thus f is micro S_p-continuous

Theorem 31: For a function $f: V \to V'$, where $(V,\tau\Gamma(X),\mu\Gamma(X))$ and $(V',\tau'\Gamma'(Y),\mu'\Gamma'(Y))$ are micro spaces, the following are equivalent:

1. f is micro S_p-continuous.
2. For each micro-open set H in V', $f^{-1}(H)$ is micro S_p-open set in V.
3. $f^{-1}(F)$ is micro S_p-closed set in V, for each micro closed set F in V'.
4. For each subset K of V, $f\left(Mic - S_pcl(K)\right) \subseteq Mic - S_pcl(f(K))$.
5. $Mic - S_pcl(f^{-1}(H)) \subseteq f^{-1}(Mic - S_pcl(H))$, for each $H \subseteq V'$.
6. $f^{-1}\left(Mic - S_pint(H)\right) \subseteq Mic - S_p(f^{-1}(H))$, for each $H \subseteq V'$.

Theorem 32: Every micro S_p-continouous function is micro semi-continuous.

Proof: Follows from **(Theorem 30)** and the fact that every micro S_p-open set is micro semi-open.

The converse of the above theorem may not be universal as an example:

From **(Example 29)** if define $f: V' \to V$ as $f(x) = \begin{cases} b \; if \; x = 1 \\ a \; if \; x = 2 \\ c \; if \; x = 3 \end{cases}$, then f is micro semi-continuous but not micro S_p-continuous, since $\{a\} \in \mu\Gamma(X)$ which is micro-open set in V, but $f^{-1}(\{a\}) = \{2\} \notin MicS_pO(V', Y)$.

Corollary 33: Let $f: V \to V'$ be a function, where $(V, \tau\Gamma(X), \mu\Gamma(X))$ and $(V', \tau'\Gamma'(Y), \mu'\Gamma'(Y))$ are micro spaces and V be micro locally indiscrete. then:

1. If f is micro semi-continuous, then f is micro S_p-continuous.
2. If f is micro S_p-continuous, then f is micro pre-continuous.

Proof: Follows from **(Lemma 25)** and **(Corollary 26)**.

Conclusion:

In this article, I introduce micro S_p-opens set in micro topological space which are stronger than micro semi-open sets, and discuss some of their properties and describe relationship to some other micro-open sets. Also, I introduce micro S_p-continuity by using micro S_p-open set.

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