PERISTALTIC FLOW OF A FOURTH GRADE FLUID IN PERMEABLE WALLS CHANNEL WITH SUCTION AND INJECTION

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Abstract: The Peristaltic flow of a fourth grade fluid in permeable walls channel with suction and injection is investigated. The perturbation technique in terms of small Deborah number is employed to determine the expressions for the velocity, the pressure rise and friction force under long wavelength and low Reynolds number assumptions. The effects of different parameters on the pumping characteristics and frictional forces are discussed graphically.

Key Words: Peristaltic transport, Fourth grade fluid, Permeability, Suction, Iinjection .

1.Introduction:

Peristalsis is a well-known mechanism for pumping biological and industrial fluids. This mechanism generally occurs in the gastrointestinal, urinary and reproductive tracts in the living body. Latham [1}. A detailed review on peristalsis was presented by Jaffrin and Shapiro[2] . Fung and Tang [3] investigated longitudinal dispersion of particles in the blood flowing in a pulmonary alveolar. Thereafter quite a good number of analytical/ numerical studies pertaining to the flow of an Newtonian fluids were considered by many researchers in different physical constraints (Mishra et al. [4], Elshahawey et al. [5], Hayat et al. [6], Nadeem and Akbar [7]. Sreenadh and Arunachalam [8] studied the Couette flow between two permeable beds with suction and injection. Mishra and Ramachandra Rao [9] made a detailed analysis on the peristaltic transport with permeable walls. Haroun [10] have studied the non-linear peristaltic flow of a fourth grade fluid in an inclined asymmetric channel Vajravelu et al. [11] studied the peristaltic transport of Casson fluid in contact with Newtonian fluid in a circular tube with permeable wall. Narahari and Sreenadh [12] investigated peristaltic transport of Bingham fluid in contact with Newtonian fluid. Vajravelu et al. [13] studied the influence of heat transfer on the peristaltic transport of Jeffrey fluid in a vertical porous stratum. Bohme and Muller [14] investigated the impact of nonlinear viscoelastic fluid properties on the peristaltic pumping characteristics of a non-Newtonian fluid in a tube. Kavitha et al. [15] discussed the peristaltic pumping of a Jeffrey fluid between porous walls with suction and injection. Hemadri Reddy *et al.* [16] investigated peristaltic flow of a carreau fluid in a porous channel with suction and injection.Nandagopal.et.al. [17] discussed Couette flow of a Bingham fluid in a channel bounded by permeable beds with suction and injection. Recently, Chakradhar K et.al[18] investigates peristaltic motion of a viscous fluid in a porous channel with suction and injection by using Numerical technique.

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In this paper the peristaltic flow of a fourth grade permeable walls channel with suction and injection is investigated. The perturbation technique in terms of small Deborah number is employed to determine the expressions for the velocity, fluid is injected into the channel perpendicular to the lower layer with constant velocity V_o and is sucked bent to the upper permeable layer with the identical velocity V_o, the speed, the pressure rise and friction force under long wavelength and low Reynolds number assumptions. The effects of different parameters on the pressure rise and friction forces are obtained. The results are obtained and discussed.

2.Mathematical Formulation:

Consider the peristaltic flow of an incompressible fourth grade fluid of half width a. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower permeable walls of the channel. The fluid is injected into the channel perpendicular to the lower permeable wall with a constant velocity v_0 and is sucked out of the upper permeable wall with the same velocity v_0 as shown in figure 1. For simplicity, we restrict our discussion to the half width of the channel.

The wall deformation is given by

$$y = H(X,t) = a + b\sin\frac{2\pi}{\lambda}(X - ct) \qquad (1)$$

where b is the amplitude, λ is the wave length and c is the wave speed.

We introduce a wave frame of reference (x, y) moving with the velocity c in which the motion becomes independent of time when the channel length is an integral multiple of the wave length and the pressure difference at the ends of the channel is a constant. The transformation from the fixed frame of reference (X,Y) to the wave frame of reference (x, y)) is given by

$$x = X - ct, y = Y, u = U - c, v = V, p(x) = p(X,t) \qquad (2)$$

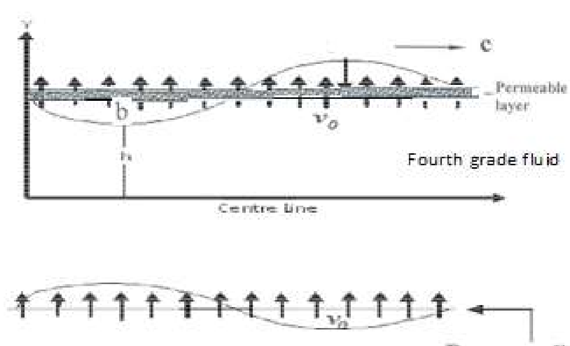

Figure 1: Physical Model

where (u,v) and (U,V) are the velocity components, p and P are the pressures in the wave and fixed frames of reference respectively. The equations governing the flow field, in the wave frame of reference are

In order to write the governing equations and the boundary conditions indimensionless form, the following non-dimensional quantities are introduced.

$$\bar{x}=\frac{x}{\lambda},\ \bar{y}=\frac{y}{a},\ \bar{t}=\frac{ct}{\lambda},\ \bar{u}=\frac{u}{c},\ \bar{v}=\frac{v}{c},\ \bar{\phi}=\frac{b}{a},\mathrm{Re}=\frac{\rho ac}{\mu},\ \bar{p}=\frac{2\pi a^2}{\lambda c\mu}p$$

$$\delta=\frac{2\pi a}{\lambda},\ \bar{h}=\frac{H}{a},\lambda_i=\frac{\alpha_i c}{\mu a}\ \text{(i=1,2)} \tag{3}$$

where Re and δ represents the Reynolds number and wave number respect-ively. In view of dimensionless quantities , Under lubrication approach, the governing equation is

$$\frac{\partial}{\partial y}[\frac{\partial u}{\partial y}(1+2\Gamma(\frac{\partial u}{\partial y})^2)-k\frac{\partial u}{\partial y}=\frac{\partial p}{\partial x} \tag{4}$$

where $k=\mathrm{Re}\,v_0$, $v_0$is suction/injection velocity and Γ is the Deborah number. The corresponding dimensionless boundary conditions in thewave frame of reference are given by

$$\frac{\partial u}{\partial y}=0\ at\ y=0 \tag{5}$$

$$u=-1-\beta\frac{\partial u}{\partial y}\quad \text{at} \quad y=h \tag{6}$$

The volume flow rate q in a wave frame of reference is given by

$$q=\int_0^h u\,dy \tag{7}$$

The instantaneous flux $Q(x,t)$ in a fixed frame is

$$Q(X,t)=\int_0^h U\,dY=q+h \tag{8}$$

The time average flux Q over one period of the peristaltic wave is

$$\bar{Q}=\frac{1}{T}\int_0^T Q\,dt=q+1 \tag{9}$$

3.Perturbation Solution:

The equation (4) is non-linear and its closed form solution is not possible. So,we expand u, p and q in terms of as

$$u = u_0 + \Gamma u_1 + O(\Gamma^2)$$
$$\frac{\partial p}{\partial x} = \frac{\partial p_0}{\partial x} + \Gamma \frac{\partial p_1}{\partial x} + O(\Gamma^2) \quad (10)$$
$$q = q_0 + \Gamma q_1 + O(\Gamma^2)$$

Substituting the equations of (10) in (4) and solving the resulting systems,we get

$$u_0 = -1 + \frac{P_0}{k^2}(e^{ky} - e^{kh} - k\beta e^{kh}) - \frac{P_0}{k}(y - h - \beta) \quad (11)$$

$$u_1 = (\frac{P_1}{k^2} + \frac{3P_0^3}{k^3})(e^{ky} - e^{kh} - k\beta e^{kh}) - \frac{P_1}{k}(y - h - \beta) - \frac{6P_0^3}{k^2}([\frac{e^{3ky}}{6k^2} - \frac{e^{3kh}}{6k^2} - \beta\frac{e^{3kh}}{2k^2}]$$
$$-[\frac{2e^{2ky}}{2k^2} + \frac{e^{2kh}}{k^2} + \beta\frac{e^{2kh}}{k^2}] + [\frac{ye^{ky}}{k} - \frac{he^{kh}}{k} - \beta\frac{e^{kh}}{k} - \beta he^{kh}])$$
(12)

$$\frac{\partial p_0}{\partial x} = \frac{(q_0 + h)}{k_1} \quad (13)$$

$$\frac{\partial p_1}{\partial x} = \frac{q_1}{k_1} - P_0^3\frac{k_2}{k_1} \quad (14)$$

$$q_0 = \int_0^h u_0 dy = P_0 k_1 - h \quad (15)$$

$$q_1 = \int_0^h u_1 dy = P_1 k_1 + P_0^3 k_2 \quad (16)$$

Where

$$P_0 = \frac{\partial P_0}{\partial x} \quad \text{and} \quad P_1 = \frac{\partial P_1}{\partial x} \quad (17)$$

$$k_1 = \frac{e^{kh}}{k^3} - \frac{he^{kh}}{k^2} + \frac{kh^2}{2k^2} - \frac{1}{k^3} - kh\beta(1 + e^{kh}) \quad (18)$$

$$k_2 = \frac{he^{3kh}}{k^4} - \frac{e^{3kh}}{k^5} + \frac{he^{2kh}}{k^4} + \frac{e^{2kh}}{2k^5} + \frac{3e^{kh}}{k^4} - \frac{3}{k^4}$$

$$-\frac{3he^{kh}}{k^3} + \frac{6he^{kh}}{k^4} - \frac{6e^{kh}}{k^5} - \frac{6h^2e^{kh}}{k^3} + \frac{10}{3k^5} +$$

$$\beta[\frac{3he^{3kh}}{k^3}+\frac{12\,he^{2kh}}{k^3}-\frac{6he^{kh}}{k^3}+\frac{6h^2e^{kh}}{k^2}-\frac{3he^{kh}}{k^3}-\frac{3he^{kh}}{k^2}]$$

(19)

$$\frac{dP}{dx}=\frac{(q+h)}{k_1}-\Gamma\frac{(q+h)^3}{k_1^4}k_2 \quad (20)$$

Pressure rise over one wave cycle is

$$\Delta P=\int_0^1\frac{dP}{dx}dx \quad (21)$$

The dimensionless frictional force F across one wave length is

$$F=\int_0^1 h(-\frac{dp}{dx})dx \quad (22)$$

4. **Results and Discussion:**

The variation of pressure difference as a function of $\bar{Q}$ for different values of amplitude ratio is shown in Fig. 2. We observe that the larger the amplitude ratio the greater, the pressure rise against which the pump works. For a given flux $\bar{Q}$, the pressure rise decreases with increasing permeable and suction parameters shown in.Fig.3 and Fig.4, we find that the larger the Deborah number the greater the pressure rise against which the pump works, Fig.5. The frictional force F as a function of Q for different values of amplitude ratio and Deborah number first decreases and then increases. with an increase in amplitude ratio and Deborah number shown in Fig.6 and Fig.9. The frictional force F as a function of $\bar{Q}$ for different values of permeable and suction parameters decreases with an increase permeable and suction parameters shown in Fig.7 and Fig.8.

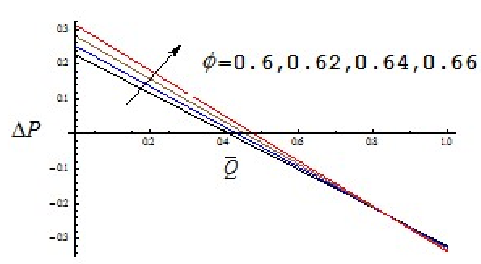

Fig 2.The variation of pressure rise ΔP against time average volume flow rate $\overline{Q}$ for different values of ϕ with fixed β=0.2, k=2, Γ=0.001.

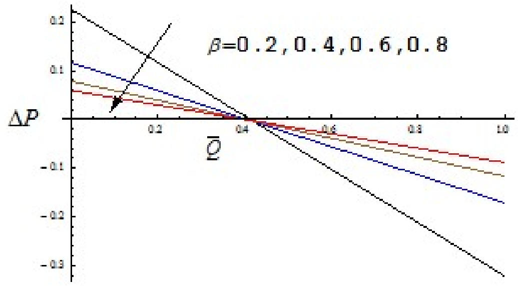

Fig 3.The variation of pressure rise ΔP against time average volume flow rate $\overline{Q}$ for different values of β with fixed ϕ =0.6, k=2, Γ=0.001.

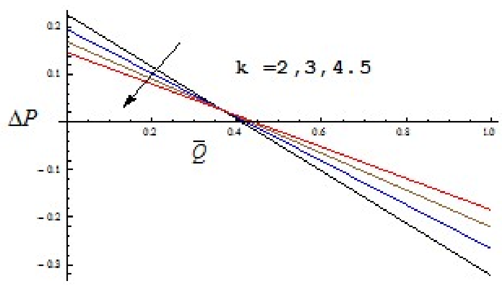

Fig 4. The variation of pressure rise ΔP against time average volume flow rate $\overline{Q}$ for different values of k with fixed ϕ=0.6, , β=0.2, Γ=0.001.

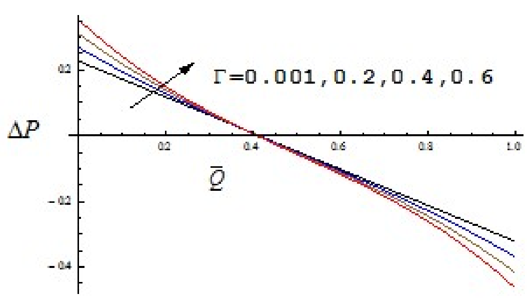

Fig 5.The variation of pressure rise ΔP against time average volume flow rate $\overline{Q}$ for different values of Γ with fixed ϕ=0.6, β=0.2, k=2.

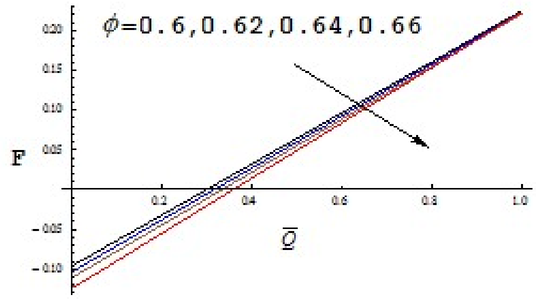

Fig 6 .The variation of frictional force F against time average volume flow rate $\overline{Q}$ for different values of ϕ with fixed β=0.2, k=2, Γ=0.001.

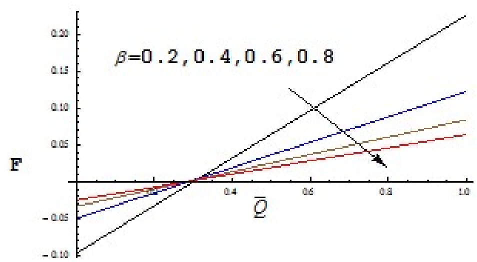

Fig 7 .The variation of frictional force F against time average volume flow rate $\overline{Q}$ for different values of β with fixed ϕ =0.6, k=2, Γ=0.001.

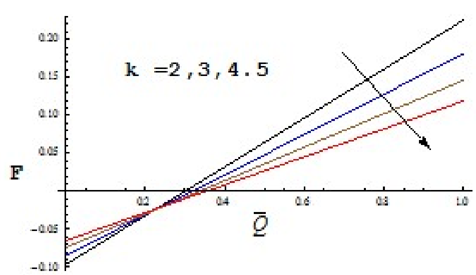

Fig 8 . The variation of frictional force F against time average volume flow rate $\overline{Q}$ for different values of k with fixed ϕ=0.6, β=0.2, Γ=0.001.

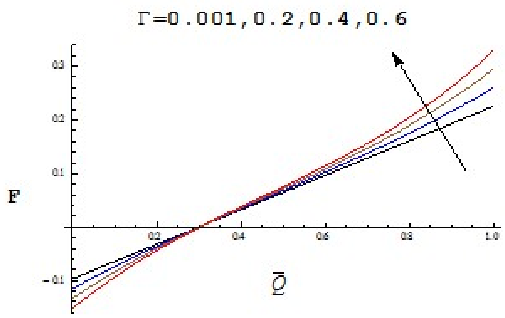

Fig 9 . The variation of frictional force F ΔP against time average volume flow rate $\overline{Q}$ for different values of Γ with fixed ϕ=0.6, β=0.2, k=2.

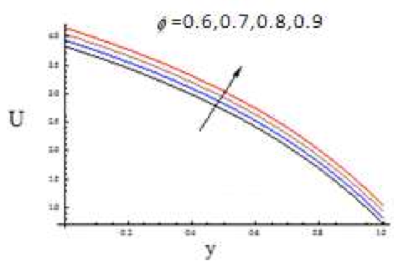

fig 10(a)

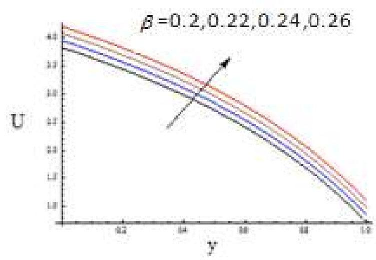

fig 10(b)

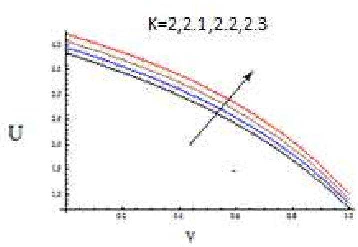

fig 10(c)

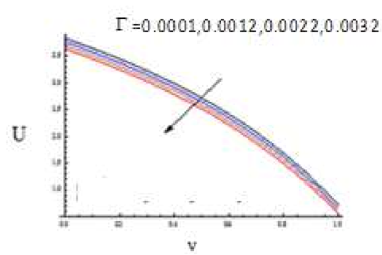

fig 10(d)

Fig.10(a) Illustrates the velocity distribution against y for several values of the amplitude ratios. It is found that the increase in the amplitude ratio increases to the velocity of the flow field.

Fig. 10(b). Illustrates the velocity distribution against y for several values of the slip parameters. It is found that increase in the slip parameter increases to the velocity of the flow field.

Fig. 10(c). It is observed that the increase in the suction parameter increases to the velocity of the flow field .

Fig.10 (d) Illustrates the velocity distribution against y for several values of the Deborah number parameters. It is found that increase in the parameter decreases to the velocity of the flow field.

5. Conclusion:

The Peristaltic transport of a fourth grade fluid in a permeable walls channel with suction and injection has been studied in the present work under the assumption of long wavelength and low Reynolds number approximations. The expressions for velocity field, pressure rise and frictional force are determined.

The results discussed through graphs.

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