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COMMON FIXED POINTS OF GERAGHTY GENERALIZED RATIONAL TYPE WEAK CONTRACTION MAPS WITH ALTERING DISTANCE FUNCTIONS VIA GRAPH STRUCTURES

Kusuma Tummala 1,∗ A. Sree Rama Murthy 2 V. Ravindranath 3 and P. Harikrishna 4

1∗ Corresponding author:Kusuma Tummala, Department of Mathemat- ics, VNR Vignana Jyothi Institute of Engineering,Hyderabad INDIA. e-mail: kusumatummala9@gmail.com

2A. Sree Rama Murthy, Department of Mathematics, Ideal Institute of Technol- ogy, Kakinada, Andhra Pradesh, India e-mail:sr akella2002@yachoo.co.in 2

3 V. Ravindranath, Department of Mathematics, JNTUK College of Engineering, JNTU, Kakinada-521003, Andhra Pradesh, India. e-mail: nath vr@yahoo.com 3

4P. Harikrishna, Department of Mathematics, N S Raju Institute of Technology, Visakhapatnam , 531 173, INDIA.e-mail: phk.2003@gmail.com 4

Abstract. In this paper we prove the existence of common fixed points of β ψ weak generalized rational contraction mappings with two metrics spaces endowed with a directed graph. We provided examples in support of our results.

Keywords: Fixed Point; directed graph; Geraghty contraction; metric space.

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1. INTRODUCTION AND PRELIMINARIES

Banach contraction principle is one of the most fundamental results in fixed point theory; by extending the contractive condition and the ambi- ent space, there are several extensions and generalizations. Jachmski [17] extended the structure of orders is replaced by the structure of Graphs on metric spaces in extended fixed point theory. The intersection of theories of fixed point findings with single and multi valued mappings is known as fixed point theory and graph theory. Many researchers [2, 3, 6, 8, 9]studied fixed point results on various spaces endowed with graphs. Fixed point results ex- tended using Gerghty [15, 4]contractions with specific properties. Recently [7] proved the existence of fixed point theorems of auxiliary functions frac- tional differential equations with applications.

Note that metric fixed point and graph theory have common application environments. In the multivalued case, the authors in [3] proved a fixed point theorem for Mizoguchi–Takahashi-type contractions on a metric space endowed with a graph. For further results in this direction, we refer to [4–11]. Recently, in [12], the authors introduced a new concept of contrac- tions called F-Khan contractions and proved a related fixed point theorem. The investigation of iterative plans for different classes of contractive and nonexpansive mappings is a focal point in measurement fixed point hypothe- sis. It began with crafted by Banach who demonstrated an old style hypoth- esis, known as the Banach constriction guideline, for the presence of a one of a kind fixed point for a withdrawal. The significance of this outcome is that it likewise gives

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the intermingling of an iterative plan to the one of a kind fixed point. A few creators have likewise given results managing the pres- ence and estimate of fixed marks of specific classes of non expansive-type multi functions. Suzuki laid out some strategies which broaden the no- table withdrawal techniques for mappings and multi functions. It is realized that consolidating a few branches is a regular movement in various areas of science particularly in math. Normally, it is prominent in fixed point hypothesis. Throughout recent many years, there have been a great deal of movement in fixed point hypothesis furthermore, one more branches in arithmetic such differential conditions, calculation and mathematical geog- raphy. In 2005, Echenique gave a short and useful evidence of an expansion of Tarski's proper point hypothesis which is significant in the hypothesis of games. In 2006, Espinola and Kirk provided useful results on combining fixed point theory and graph theory [6]. In 2008 and 2009, Jachymski con- tinued this idea by using different view (see [8] and [7]). Then, Beg, Butt and Radojevic obtained some results in 2010 (see [2]) in the same direction. In this paper, we present some iterative scheme results for G-contractive and G-nonexpansive maps on graphs.

2. DEFINITIONS

Definition 1. [17] Let (X, d) be a metric space and Δ denote the di- agonal of the Cartesian product XxX. The metric space (X, d) is said to be endowed with a directed graph or digraph G = (V (G), E(G)) if G is a directed graph such that the vertex set V (G) contains all the elements of X and the edge set E(G) contain Δ while excluding parallel edges.

Definition 2 [9] Suppose that (X, d) is a metric space with endowed with a graph , and S, T : X $\rightarrow$ X are functions. Let X(S, T) := {u ∈ X : (Su, Tu) ∈ E(G)}
C(S, T) := {u ∈ X : Su = Tu}
C(S, T) is the set of all coincidence points of S and T , and Cm(S, T) := {u ∈
X : Su = Tu = u}, Cm(S, T) is the set of all common fixed points of S and T .
Lemma 1 [17] let (X, d) be a metric space with endowed with a directed graph G = (V (G), E(G)), and let S, T : X X be functions. If C(S, T) = φ, then X(S, T) = φ.
Definition 3 [9] Suppose that (X, d) is a metric space endowed with a digraph G = (V (G), E(G)), and let
(1) A mapping T : X → X is said to be G- continuous at x in X, whenever, for a sequence {xn} in X such that (xn, xn+1) ∈ X for each n ∈ N , we have that if xn x X, then Txn Tx.
More over , T is called G- Continuous whenever it is G- continuous at every element x in X.
(2) The set E(G) is said to transitive property, for all x, y, z X, if (x, z), (z, y) E(G), then (x, y) E(G).
(3) The triple (X, d, G) is said to have the property A whenever, for any sequence {xn} in X such that xn → x ∈ X and (xn, x(n + 1) ∈ E(G) for all n ∈ N , it is true that (xn, x) ∈ E(G) for all n ∈ N .
Definition 4 [18] Let (X, d) be a metric space , and let S, T : X → X be
functions. Then S is said to be d- compatible whenever lim
n→∞

d(STxn, TSxn) =

0 for all sequence xn in X with lim
n→∞

Txn = lim
n→∞

Sxn

Definition 5 [9] Let (X, d) and Y, dJ) be metric spaces , and let S : X → Y and T : X → X be functions. Then T is said to be S- Cauchy on X , for any sequence {xn} in X with {Txn} being Cauchy in (X, d), the sequence
{Sxn} is Cauchy in Y, dJ.
Let φ : [0,) : [0,) be a function such that : φ is an increasing function; φ is a continuous function ; φ(t) = 0 if and only if t = 0.

Consider the class A(X), where X is a metric with metric d. Let h :

XxX [0, 1] , if lim
n→∞

h(tn, sn) = 1 then lim
n→∞

d(tn, sn) = 0 where the se-

quences tn , tn in X.
The existence of common fixed point theorems for auxiliary functions with two metrics endowed with a digraph was defined and proved by Ben Won- gasaiji, Phakdi Charoensawan, Teeranush Suebcharoen and Watchareepan Atiponrat in 2021.[7]

Definition 5 [7] Let (X, d) be a metric space endowed with a directed Graph G = (V (G), E(G)), and let S, T : X X be functions. The pair S, T is an h φ -contraction with respect to d whenever the following conditions hold:
(1) With regard to G, S is T -edge preserving;
(2) there are two functions, h in A(X), and φ in Φ, such that for any x, y
in X with (Tx, Ty) in E(G),

φ(d(Tx, Ty)) h(Sx, Sy)φ(R(Sx, Sy)),

where R : XxX [0,) is a function such that, for any x, y X,
R(Sx, Sy) = max{ , d(Sx, Sy), d(Sx, Tx), d(Sy, Ty), }

Theorem 2.1. [7] Let (X, dJ) be a complete metric space endowed with a directed graph G = (V (G), E(G)), let d be another metric on X, and let S, T : X X be functions. Suppose that (S, T) is an h φ - contraction with respect to d, Further, assume that the following conditions satisfied:

(i) S : (X, dJ) (X, dJ) is a continuous function such that S(X) is dJ- closed;
(ii) T (X) S(X);
(iii) E(G) is transitive;
(iv) If d < dJ , T : (X, d) → (X, dJ) is G- Cauchy on X;
(v) T : (X, d) → (X, dJ) is G- continuous , and T and S are dJ -compatible.

X(T, S) /= φ if and only if C(T, S) /= φ.

In 1973, Geraghty [15]enhanced the Banach Contraction Principle by sub- stituting a function with particular features for the contraction constant.

We denote S is the class of functions

β : R+ → [0, 1) satisfying, for a sequence{tn} in R+ such that β(tn) →

1 =⇒ tn → 0

Definition 5 [15] Let M be a metric space with metric d. A self map T : M M is said to be Geraghty contraction if there exist β S such that d(Tx, Ty) β(d(x, y))d(x, y) for all x, y in X.

In 1884, M.S. Khan, M. Swaleh, S. Sessa, introduced the class of altering distance functions[21] , which we denote Ψ = {ψ : R+ → R+ such that (i) ψ is non decreasing, (ii) ψ is continuous (iii) ψ(t) = 0 =⇒ t = 0}

In this section, we use a Graph Structure to prove the presence Common fixed points of Geraghty generalized rational type weak contraction maps with altering distance functions

3. MAIN RESULTS

Definition 2.1 Let M be a metric space with metric d endowed with a digraph G = (V (G), E(G)), and let f, g : X X be functions. The pair A, B is is said to be β ψ weak rational contraction with respect to d if

(1) with regard to the graph G , A is B-edge preserving;
(2) there exists two functions β ∈ S and ψ ∈ Ψ such that, for all x, y ∈ X

with (Bx, By) ∈ E(G), we have

ψ(d(Ax, Ay)) ≤ β(ψ(K(Bx, By))ψ(K(Bx, By)) + L.N (Bx, By), for L ≥ 0 where K : MxM → [0, ∞) is a function such that, for any x, y ∈ X, K(Bx, By) = max{ d(Bx,Ax)d(Ay,By) , d(By,Ay)[1+d(Bx,Ax)] , d(Ax,Bx)[1+d(Ay,By)] ,
d(By,Ax)[1+d(Ax,By)] , d(Bx, By), d(Bx,Ay)+d(By,Ax) }
N (Bx, By) = min{ d(Bx,Ax)d(Ay,By) , d(By,Ax)[1+d(Ax,By)] , d(Bx, By), d(By, Ay)}

Theorem 3.1. Let M be a complete metric space with metric dJ endowed with a digraph G = (V (G), E(G)), let d be another metric on M, and let A, B : M M be functions. Suppose that (A, B) is an β ψ weak rational contraction with respect to d, Further, assume that the following conditions satisfied:

(i) B : (M, dJ) (M, dJ) is a continuous function such that A(X) is dJ- closed;
(ii) A(M) B(M);
(iii) E(G)is transitive;
(iv) If d < dJ , A : (M, d) (M, dJ) is G- Cauchy on X;
(v) A : (M, d) (M, dJ) is G- continuous , and A and B are dJ -compatible. then A and B have a coincident points.

Proof. Let t0 ∈ M be such that (Bt0, Bt1) ∈ E(G). Since A(M) ⊂ B(M) and A(t0) ∈ M .
Choose t1 ∈ M such that A(t0) = B(t1)
we can construct a sequence {tn} in M such that B(tn) = A(t(n − 1)) for each n ∈ N .
If B(tn+1) = B(tn) for some n N i.e., A(tn) = B(tn), tn is a coincident
point of A and B.
Hence, without loss of generality, we may assume that Btn /= Btn+1 for each

n ≥ N . Since (Bt0, At0) = (Bx0, Ax1) ∈ E(G), and the function A e-edge preserving with respect to G, so that (At0, At1) = (Bt1, Bt2) ∈ E(G).
By proceeding in this way, we have (Btn−1, Bxtn) E(G) for each n N .
Since (A, B) is β ψ- rational contraction with respect to d, for each n 0, we have
ψd(Btn, Btn+1) = ψd(Atn−1, Atn) β(ψ(K(Btn−1, Btn)))(ψ(K(Btn−1, Btn))+LN (Btn−1, Btn)
(3.1)

K(Btn−1, Btn) = max{ d(Btn−1,Atn−1)d(Atn,Btn) , d(Btn,Atn)[1+d(Btn−1,Atn−1)] ,

d(Atn−1,Btn−1)[1+d(Atn,Btn)] ,
1+d(Btn−1,Btn)

d(Btn−1,Btn)

1+d(Btn−1,Btn)

d(Btn,Atn−1)[1+d(Atn−1,Btn)] , d(Btn−1, Btn), d(Btn−1,Atn)+d(Btn,Atn−1) }
1+d(Btn−1,Btn) 2
K(Btn−1, Btn) = max{ d(Btn−1,Btn)d(Btn+1,Btn) , d(Btn,Btn+1)[1+d(Btn−1,Btn)] ,

d(Btn,btn−1)[1+d(Btn+1,Btn)] ,
1+d(Btn−1,Btn)

d(Btn−1,Btn)

1+d(Btn−1,Btn)

Btn), d(Btn−1,Btn+1)+d(Btn,Btn) }
K(Bt , Bt) = max d(Bt , Bt), d(Bt , Bt), d(Btn,Btn−1)[1+d(Btn+1,Btn)] ,
1+d(Btn−1,Btn)
0, d(Btn−1, Btn), d(Btn−1,Btn+1) }
≤ max{d(Btn+1, Btn), d(Btn,Btn−1)[1+d(Btn+1,Btn)] , d(Btn−1, Btn), d(Btn−1,Btn+1) }
1+d(Btn−1,Btn) 2
≤ max{d(Btn+1, Btn), d(Btn−1, Btn)}
N (Btn−1, Btn) = min{ d(Btn−1,Atn−1)d(Atn,Btn) , d(Btn,Btn−1)[1+d(Btn−1,Btn)] , d(Btn−1, Btn), d(Btn, At
d(Btn−1,Btn) 1+d(Btn−1,Btn)
N (Btn−1, Btn) = min{ d(Btn−1,Btn)d(Btn+1,Btn) , d(Btn,Btn)[1+d(Btn,Btn)] , d(Btn−1, gxBtn), d(Btn, Btn
d(Btn−1,Btn) 1+d(Btn−1,Btn)
= min d(Bt , Bt), 0[1+0] , d(Bt , Bt), d(Bt , Bt) = 0
1+d(Btn−1,Btn)
If max{d(Btn+1, Btn), d(Btn−1, Btn)} = d(Btn+1, Btn)
From (2.1), we have ψ(d(Btn, Btn+1)) = ψ(d(Atn−1, Atn)) β(ψ(M (Btn−1, Btn)))(ψ(d(Btn+1, Btn))
LN.0
Since βS, We get ψ(d(Btn, Btn+1)) < ψ(d(Btn+1, Btn)), which is a con- tradiction.

Hence max{d(Btn+1, Btn), d(Btn−1, Btn)} = d(Btn−1, Btn)
Therefore from (2.1), we have
ψ(d(Btn, Btn+1)) = ψ(d(Atn−1, Atn)) < ψd(Btn−1, Btn) (3.2)

So, that the sequence ψ(d(Btn, Btn+1)) is strictly decreasing sequence of

positive real numbers and so lim
n→∞

ψ(d(Btn, Btn+1)) = r ≥ 0

We now show thatr = 0. Suppose that r > 0 then from from (2.1) we have
ψ(d(Btn, Btn+1)) = ψd(Atn−1, Atn) ≤ β(ψ(K(Btn−1, Btn)))(ψ(K(Btn−1, Btn))+ LN (Btn−1, Btn)
≤ β(ψ(K(Btn−1, Btn)))(ψ(d(Btn−1, BTn)) + L.0
ψ(d(Btn, Btn+1)) ≤ β(ψ(M (Btn−1, Btn)))(ψ(d(Btn−1, Btn)) ψ(d(Btn−1,Btn) ≤ β(ψ(M (Btn−1, Btn))) < 1 for each n ≥ 1 Now on letting n → ∞, we get

1 = lim

ψ(d(Btn,Btn+1))

β(ψ(K(Btn−1, Btn))) ≤ 1

n→∞ ψ(d(Btn−1,Btn)

n→∞

so that β(ψ(K(Btn−1, Btn)) → 1 as n → ∞.

From the property of β S, we get lim
n→∞

ψ(K(Btn−1, Btn)) = 0

so that lim
n
i.e., r = 0

ψ(d(Btn−1, Btn)) = 0

Now, we show that the sequence {Btn} is a Cauchy sequence. Suppose that
{Btn} is not a Cauchy sequence. Then there exist an ϵ > 0 such that, for all k ∈ N ,there are sequences of positive integers {m(k)} and {n(k)} with m(k) > n(k) > k and d(Btm(k), Btn(k)) ≥ ϵ and d(Btm(k) − 1, Btn(k)) < ϵ ϵ ≤ d(Btn(k), Btm(k))
≤ d(Btn(k), Btm(k) − 1) + d(Btm(k) − 1, Btm(k))
< ϵ + d(Btm(k) − 1, Btm(k))

Taking as k and using lim
n→∞

d(Btn, Bt(n + 1)) = 0,

we get lim
n→∞

d(Btm(k), Bt(n(k))) = ϵ > 0.

Since E(G) has the transitivity property , we get that (gxm(k), gxn(k)) ∈ E)G) for every k ∈ N , we get
ψ(d(Btn(k) + 1, Btm(k) + 1)) = ψ(d(Atn(k), Atm(k)))
≤ β(K(Btn(k), Btm(k)))ψ(K(Btn(k), BTm(k))) + L.N (Btn(k), Btm(k))
Now K(Btn(k), Btm(k))) = max{ d(Btn(k),Atn(k))d(Atm(k),Btm(k)) , d(Btm(k),Btm(k))[1+d(Btn(k),Atn(k))] ,

d(Atn(k),Btn(k))[1+d(Atm(k),Btm(k))] 1+d(Btn(k),Btm(k))

d(Btn(k),Btm(k))

1+d(Btn(k),Btm(k))

d(Btm(k),Atn(k))[1+d(Atn(k),Btm(k))] , d(Btn(k), Btm(k)), d(Btn(k),Atm(k))+d(Btm(k),Atn(k)) }

K(Bt

(k), Bt

(k))) = max{ d(Btn(k),Btn(k)+1)d(Btm(k)+1,Btm(k)) , d(Btm(k),Btm(k)+1)[1+d(Btn(k),Btn(k)+1)] ,

d(Btn(k)+1,Btn(k))[1+d(Btm(k)+1,Btm(k))] 1+d(Btn(k),Btm(k))
d(Btm(k),Btn(k)+1)[1+d(Btn(k)+1,Btm(k))] 1+d(Btn(k),Btm(k))

n(k), Btm

(k)), d(Btn(k),Btm(k)+1)+d(Btm(k),Btn(k)+1) }

On letting k and using lim
k→∞

d(Btn, Btn+1)) = 0

lim

K(Bt

(k), Bt

(k))) = lim

d(Btm(k),Btn(k)+1)[1+d(Btn(k)+1,Btm(k))] , d(Bt

(k), Bt

(k)), d(Btn(k),B

n m
k→∞

k→∞

1+d(Btn(k),Btm(k)) n m

lim
k
lim
k→∞

K(Btn(k), Btm(k))) = ϵ
N (Btn(k), Btm(k))) = 0

1 = lim

ψ(d(Btn(k),Btn(k)+1))

β(ψ(M (Bt

, Bt

(k)))) ≤ 1

k→∞ ψ(d(Btn(k)−1,Btn(k))

n→∞

n(k)−1 n

so that β(ψ(M (Btn(k)−1, Btn(k))) → 1 as k → ∞.

From the property of β S, we get lim
k→∞

ψ(M (Btn(k)−1, Btn(k))) = 0

so that lim
k→∞

ψ(d(Btn(k)−1, Btn(k))) = 0, which is a contradiction.

Hence the sequence {gxn} is a Cauchy sequence in the metric space (X, d).
Now , we have to prove that the sequence Btn is a Cauchy sequence in the metric space (M, dJ).
When d dJ the proof is trivial. So that we consider d § dJ.
Let ϵ > 0. Since the sequence {Bt(n)} is a Cauchy in the metric space (M, d) and the function A is B- Cauchy on M . We have A(M) ⊂ B(M), we can obtain that {At(n)} is a Cauchy in the metric space (M, dJ). So, there

a number n0 N such that dJ(Btm+1, Btn+1) = dJ(Atm, Atn) < ϵ for all
m, n N0 and hence Btn is Cauchy's sequence in (X, dJ).
B(M) is a d' -closed subset of (M, dJ), and which is complete , then there

exist z = gt ∈ B(M) such that lim

Btn = lim

Atn = z.

J J n→∞

n→∞

We have A : (M, d) (M, d) is a G- continuous function such that A and
B are dJ - compatible

lim

n→∞

d(BAtn, ABtn) = 0/.
J J J

Consider d(Bu, Au) d (Bu, BAtn) + d (BAtn, ABtn) + d (ABtn, Au)
taking limit as n , we get that dJ(Az, Bz) = 0, and from continuity of
B and that A is G- continuous.
Therefore Az = Bz. Which implies that z is a coincidence point of A and
B.
Q
Theorem 3.2. In addition to the hypothesis of the above theorem 2.1 A and
B are weakly compatible the A and B have a common fixed point.

Proof. From the proof of theorem 2.1 Btn is non decreasing sequence and converges to Bz and
Bz = Az.
Since A and B are weakly compatible, we have ABz = BZz. Now Az = Bz = u(say)
Also , Au = ABz = BAz = Bu
If z = u then u is a common fixed point of A and B.
if z u, i.e., d(z, u) > 0
ψ(d(Bz, Bu)) = ψ(d(Az, Au)) ≤ β(ψ(K(Bz, Bu)))ψ(K(Bz, Bu))+L.N (Bz, Bu)
K(Bz, Bu) = max{ d(Bz,Az)d(Au,gBu) , d(Bu,Au)[1+d(Bz,Az)] , d(Az,Bz)[1+d(Au,Bu)] ,
d(Bu,Az)[1+d(Az,Bu)] , d(Bz, Bu), d(Bz,Au)+d(Bu,Az) }

K(Bz, Bu) = max{ d(u,u)d(Au,Bu) , d(Bu,Bu)[1+d(u,u)] , d(u,u)[1+d(Bu,Bu)] ,
d(Bu,u)[1+d(u,Bu)] , d(u, Bu), d(u,Bu)+d(Bu,u) }

K(Bz, Bu) = max{0, 0, 0, d(Bu, u), d(u, Bu), d(u,Bu)+d(Bu,u) }
K(Bz, Bu) = d(Bu, u) = d(Bz, Bu)
N (Bz, Bu) = min{ d(Bz,Bz)d(Au,Bu) , d(Bu,Az)[1+]d(Az,Bu) , d(Bz, Bu), d(Bu, Au)}
N (Bz, Bu) = min{ d(u,u)d(Bu,Bu) , d(Bu,u)[1+]d(u,Bu) , d(Bz, Bu), d(Bu, Bu) =

d(u,Bu)
0}

1+d(u,Bu)

ψ(d(Bz, Bu)) = ψ(d(Az, Au)) ≤ β(ψ(M (Bz, Bu)))ψ(d(Bz, Bu)) + L.0
1 = ψ(d(Bz,Bu)) ≤ β(ψ(M (Bz, Bu))) < 1.
Hence Bz = Bu
Bu = Au = u, there fore u is a common fixed point of A and B.

Q

Theorem 3.3. In addition to the hypothesis of the above theorem 2.1 For any x, y ∈ C(A, B), such that Bx /= By, we have (Bx, By) ∈ E(G). If A and B are dJ- compatible and X(A, gB) /= φ, then Cm(A, B) /= φ.

Proof. From the theorem (2.1), there exists a coincidence point x X,
i.e., gx = fx.
Suppose that there exists another coincidence point y X such that gy =
fy,
Assume that Bx /= By.
From the assumption (Bx, By) ∈ E(G)
ψ(d(Ax, Ay)) ≤ β(ψ(K(Bx, By))ψ(K(Bx, gBy)) + L.N (Bx, By), for L ≥ 0 where K : MxM → [0, ∞) is a function such that, for any x, y ∈ X, K(Bx, By) = max{ d(Bx,Ax)d(Ay,By) , d(By,Ay)[1+d(Bx,Ax)] , d(Ax,Bx)[1+d(Ay,By)] ,
d(By,Ax)[1+d(Ax,By)] , d(Bx, By), d(Bx,Ay)+d(By,Ax) }
K(Bx, By) = max{ d(Bx,Ax)d(By,By) , d(By,By)[1+d(Bx,Bx)] , d(Bx,Bx)[1+d(By,By)] , d(By,Bx)[1+d(Bx,By)] , d(Bx, By), d(Bx,By)+d(By,Bx) } = d(Bx, By)
1+d(Bx,By) 2
N (Bx, By) = 0
therefore ψ(d(Ax, Ay)) β(ψ(M (Bx, By))ψ(M (Bx, By))+L.N (Bx, By) < ψ(d(Ax, Ay))
which is a contradiction.
Therefor Bx = By.
Lett0 = t, and define the sequence {tn} by Btn = Atn−1 for each n ∈ N . Since t is a coincidence point of A and B , we have Btn = At for each n N .
Now let w = Bt
therefore Bw = BBt = BAt
By the definition of the sequence {tn} , Btn = At = Atn−1 for each n ∈ N .

lim
n→∞

Atn = lim
n→∞

Btn = At.
J J

i.e., BAt = Abt.

n→∞

There fore , we have Bw = BAt = ABt = Aw
and hence w is another coincidence point of A and B. Now Aw = Bw = Bt = w
Therefore w is a common fixed point of A and B. i.e., Cm(A, B) /= φ. Q

4. COROLLARIES AND EXAMPLES

When ψ(t) is identity map, we have the following.

Definition 3.1 Let (M, d) be a metric space endowed with a directed Graph G = (V (G), E(G)), and let A, B : M M be functions. The pair A, B is is said to be β rational contraction with respect to d if

(1) A is B -edge preserving with respect to G;
(2) there exists two functions β S and ψ Ψ such that, for all x, y X
with (Bx, By) E(G), we have
(d(Ax, Ay)) β(K(Bx, By)(K(Bx, By) + L.N (Bx, By), for L 0
where M : XxX [0,) is a function such that, for any x, y X,
d(Bx,Ax)d(Ay,By) d(By,Ay)[1+d(Bx,Ax)] d(Ax,Bx)[1+d(Ay,By)]

d(By,Ax)[1+d(Ax,By)] , d(Bx, By), d(Bx,Ay)+d(By,Ax) }
N (Bx, By) = min{ d(Bx,Ax)d(A,By) , d(By,Ax)[1+d(Ax,By) , d(Bx, By), d(By, Ay)}

Corollary 4.1. Let (M, dJ) be a complete metric space endowed with a di- rected graph G = (V (G), E(G)), let d be another metric on M, and let A, B : M M be functions. Suppose that (A, B) is an β - weak rational contraction with respect to d, Further, assume that the following conditions satisfied:

(i) M : (M, dJ) (M, dJ) is a continuous function such that S(X) is dJ- closed;
(ii) A(X) B(X);
(iii) E(G) is transitive ;
(iv) If d § dJ , a : (M, d) (M, dJ) is G- Cauchy on M;
(v) f : (M, d) (M, dJ) is G- continuous , and A and B are dJ -compatible.

then A and A have a coincident points.
When L = 0 , we have the following.
Definition 3.2 Let (M, d) be a metric space endowed with a directed Graph G = (V (G), E(G)), and let A, B : M M be functions. The pair A, B is is said to be β ψ - rational contraction with respect to d if

(1) A is B -edge preserving with respect to G;
(2) there exists two functions β ∈ S and ψ ∈ Ψ such that, for all x, y ∈ X
with (Bx, By) ∈ E(G), we have
ψ(d(Ax, Ay)) ≤ β(ψ(K(Bx, By))ψ(K(Bx, By))+,

where K : MxM → [0, ∞) is a function such that, for any x, y ∈ M ,
K(Bx, By) = max{ d(Bx,Ax)d(f Ay,By) , d(By,Ay)[1+d(Bx,Ax)] , d(Ax,Bx)[1+d(Ay,gBy)] , d(By,Ax)[1+d(Ax,By)] , d(Bx, By), d(Bx,Ay)+d(By,Ax) }

Corollary 4.2. Let (X, dJ) be a complete metric space endowed with a di- rected graph G = (V (G), E(G)), let d be another metric on X, and let A, B : X X be functions. Suppose that (f, g) is an β ψ - rational contraction with respect to d, Further, assume that the following conditions satisfied:

(i) B : (M, dJ) (M, dJ) is a continuous function such that A(X) is dJ- closed;
(ii) A(X) B(X);
(iii) E(G) is transitive ;
(iv) If d § dJ , A : (M, d) (M, dJ) is G- Cauchy on m;
(v) A : (M, d) (M, dJ) is G- continuous , and A and A are dJ -compatible.

then A and B have a coincident points.

Now we present an example in support of Theorem 2.2.

Example 4.3. Let M : [0, 1] and define the metrics d, dJ : M → M defined by d(x, y) = |x − y|, dJ(x, y) = K|x − y| ∀ x, y ∈ M , where k > 1. Clearly
Now we define E(G) = {(x, y)/x = y or x, y ∈ [0, 1] with x ≤ y} Define
the mappings A : M → M and B : M → M by A(x) = x and B(x) = x
∀x ∈ M .

Define ψ(t) = t , for all t, and define β : [0, ∞) → [0, 1) bt β(t) = 1+t , t ≥ 0

2

Let (Bx, By) ∈ E(G), if x = y then (Ax, Ay) ∈ E(G)

1+2t

If (Bx, By) ∈ E(G) with Bx ≤ By, then we get Bx = x ≤ By = y
Ax = x ≤ Ay ≤ y and Ax, Ay ∈ [0, 1] that implies Ax, Ay ∈ E(G)
Let x, y be arbitrary in M and (Bx, By) E(G)
If Bx = By then we get x = y, inequality in Theorem 2.1 holds. Suppose Bx = x, By = y in [0,1].
ψ(d(Ax, Ay)) = ψ(d(x , y))) = ψ(| x − y |)) = | x−y |
d(x, x)d(y ,y) d(y, y)[1+d(x, x)] d(x ,x)[1+d(y ,y)]

K(Bx, By) = max{ 2 2 , 2 2 , 2 2 ,

d(y, x)[1+d(x ,y)]

d(x,y)

d(x, y)+d(y, x)

1+d(x,y)

1+d(x,y)

2 2
1+d(x,y)

2 2 2 }

x . y

y .[1+ x] x [1+ y] |y− |[1+| −y|]

|x− |+|y− |

K(Bx, By) = max{ ‖ 2 2

, 2 2 , 2 2 , 2 2 , |x−y|,

2 2 2 }

x−y‖
K(Bx, By) = |x − y|

1+|x−y| 1+|x−y| 1+|x−y|

d(x, x)d(y ,y) d(y, x)[1+d(x ,y)] y

N (Bx, By) = min{ 2 2 , 2 2 , d(x, y), d(y, 2)} x . y

|y− |[1+| −y|] y

2 2
N (Bx, By) = min{ | 2 2 , , |x − y|, 2 }

N (Bx, By) =

x−y|
| 2y−x |[1+| x−2y |] 1+|x−y|

1+|x−y|

ψ(d(Ax, Ay)) ≤ β(ψ(K(Bx, By))ψ(K(Bx, By)) + L.N (Bx, By)

| x−y | ≤ β(ψ(|x − y|)ψ(|x − y|)) + L.

| 2y−x |[1+| x−2y |] 1+|x−y|

| x−y | ≤ ((

1+ |x−y|
2)
1+2 x−y|

|x−y|) + L.

| 2y−x |[1+| x−2y |]

1+|x−y|

holds for some L ≥ 0

Conclusion We proved the existence of common fixed points of β ψ weak generalized rational contraction mappings with two metrics spaces en- dowed with a directed graph. We used a Graph Structure to prove the presence Common fixed points of Geraghty generalized rational type weak contraction maps with altering distance functions. We derived some corol- laries from main results and provided examples in support of our results.

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