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m-g INVERSES OF m-REGULAR NUETROSOPHIC FUZZY MATRICES

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Abstract:

The goal of this paper is to present about the idea of m-g inverses related with a m-regular Neutrosophic fuzzy matrices as a speculation of g - inverses of a m regular fuzzy matrix. Further, that's what we demonstrated if any two Nuetrosophic Fuzzy Matrices(NSFMs) are said to have a $\{3^m\}$ and $\{4^m\}$ inverses then its enrolment, non-participation and indeterminancy fuzzy matrices are likewise going to have a $\{3^m\}$ and $\{4^m\}$ inverses as well as the other way around.

Keywords: Fuzzy Matrix, Neutrosophic Fuzzy Matrix, g-inverse, m-g inverse.

1. Introduction:

Scholastics in financial aspects, social science, clinical science, modern, climate science and numerous other various fields concur with the ambiguous, uncertain and rarely inadequate with regards to data of displaying estimated information. Therefore, fuzzy set hypothesis was presented by L. A. Zadeh [11]. The issues concerning different sorts of waveings can't tackled by the traditional matrix hypothesis. That sort of issues are settled by utilizing fuzzy matrix [5, 6]. Fuzzy matrices manage just enrolment values. These frame works can't bargain non enrolment values. Intuitionistic fuzzy matrix (IFMs) presented first time by Khan, Shyamal and Pal [6].

Kim and Roush have fostered a hypothesis for fuzzy matrix’s practically equivalent to that for Boolean Matrices by expanding the max.min procedure on fuzzy polynomial math Ғ =[0,1], for components a,b ε Ғ, a+b = max{a,b} and a.b = min{a,b}[3].

Then, the intuitionistic fuzzy sets was created by M. A. Atanassov [1, 2]. Assessment of non-enrolment values is additionally not continually workable for the indistinguishable explanation as in the event of participation esteems thus, there exists an indeterministic part whereupon dithering perseveres. Therefore, Smarandache et al. [4, 9, 10] has presented the idea of Neutrosophic Set (NS) which is a speculation of customary sets, fuzzy set, intuitionistic fuzzy set and so forth. Poongodi have presented the new portrayal on Neutrosophic f fuzzy matrix and concentrated on regular neutrosophic fuzzy matrix in [7]. Princy et, al.[8] has presented the idea of m-ordering on neutrosophic fuzzy matrices as a speculation of normal neutrosophic fuzzy matrices.

In this paper, we concentrate on the idea of m-g inverses related with a m-regular Neutrosophic fuzzy Matrices as a speculation of g - inverses of a regular fuzzy matrix.

2. Preliminaries:

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In this segment, a few essential definitions and results required are given.

Definition 2.1

A matrix C ∈ F_n is known as to be regular that there exist a matrix U ∈ F_n, to such an extent that CUC=C. Then, at that point, U is called g-reverse of C. Let C{1} = {U/CUC = C}. F_n signifies the arrangement of all fuzzy matrices of order nxn.

Definition 2.2

A matrix C∈ F_n is known as right m – regular, assuming there exist a matrix U ∈ F_n , to such an extent that $C^mUC=C^m$, for some sure whole number m. U is named as right m - g inverse of C. Let $C_r\{1^m\}$ = {U/$C^mUC = C^m$ }.

Definition 2.3

A matrix C∈ F_n is known as left m – regular, assuming there exist a matrix V ∈ F_n , to such an extent that $CVC^m=C^m$, for some sure whole number m. V is named as left m - g inverse of C. Let C = {V/ $CVC^m=C^m$ }.

Definition 2.4

A matrix C ∈ F_n is said to have a{1, 2, 3,4}inverse if there exists a matrix U ∈ F_n such that

(i) CUC=C
(ii) UCU=C
(iii) $(CU)^T$=CU
(iv) $(UC)^T$=UC

Then U is called {1 , 2, 3,4}inverse of A and it is denoted by A^+. The Moore-Penrose inverse of A is denoted by A^+.

Definition 2.4

An neutrosophic fuzzy matrix (NSFM) C of order m × n is defined as C= $[X_{ab}, <c_{(ab)T}, c_{(ab)F}, c_{(ab)I}>]_{mxn}$, where $c_{(ab)T}, c_{(ab)F}$ $c_{(ab)I}$ are called enrolment worm (T), the non-participation (F) worm and the indeterminancy worm (I) of U_{ab} in C, which sustaining the condition $0 \le (c_{(ab)T} + c_{(ab)F} + c_{(ab)I}) \le 3$. For simplicity, we write C = $[c_{ab}]_{mxn}$ where $c_{ab} = < c_{(ab)T}, c_{(ab)F}, c_{(ab)I} >$. Let N_n symbolizes the arrangement of all nxn NSFM.

Let C and D be any two NSFMs. The accompanying activities are characterized for any two-component c_{ab} ∈ C and d_{ab} ∈ D, where $c_{ab} = [c_{(ab)T}, c_{(ab)F}, c_{(ab)I}]$ and $d_{ab} = [d_{(ab)T}, d_{(ab)F}, d_{(ab)I}]$ are in [0,1] with the end goal that $0 \le (c_{(ab)T} + c_{(ab)F} + c_{(ab)I}) \le 3$ and $0 \le (d_{(ab)T} + d_{(ab)F} + d_{(ab)I}) \ge 3$, then

$$c_{ab} + d_{ab} = [\max\{c_{(ab)T}, d_{(ab)T}\}, \max\{c_{(ab)F}, d_{(ab)F}\}, \min\{c_{(ab)I}, d_{(ab)I}\}]$$

$$c_{ab} . d_{ab} = [\min\{c_{(ab)T}, d_{(ab)T}\}, \min\{c_{(ab)F}, d_{(ab)F}\}, \max\{c_{(ab)I}, d_{(ab)I}\}]$$

Here we shall track the elementary operations on NSFM.

For C = $(c_{ab}) = [c_{(ab)T}, c_{(ab)F}, c_{(ab)I}]$ and D = $(d_{ab}) = [d_{(ab)T}, d_{(ab)F}, d_{(ab)I}]$ of order mxn, their total indicated as C + D is characterized as,

$$C + D = (c_{ab} + d_{ab}) = [(c_{(ab)T} + d_{(ab)T}), (c_{(ab)F} + d_{(ab)F}), (c_{(ab)I} + d_{(ab)I})] \quad \text{….(2.1)}$$

For $C = (c_{ab})_{mxn}$ and $D = (d_{ab})_{nxp}$ their product indicated as CD is characterized as,

$CD = (e_{ab}) = \sum_{k=1}^{n} c_{bk} . d_{kb}$

$= \left(\sum_{k=1}^{n} (c_{(ak)T}. d_{(kb)T}), \sum_{k=1}^{n} (c_{(ak)F} . d_{(kb)F}), \sum_{k=1}^{n} c_{(ak)I} . d_{(kb)I} \right)$ ....(2.2)

Lemma 2.5

For C, D ∈ N_{mn}

(i) If the row space of D contained in the row space of C then which is equivalent to D = UC for some U ∈ N_m
i.e. R(D) ⊆ R(C) ⇔ D = UC for some U ∈ N_m

(ii) If the column space of D contained in the column space of C then which is equivalent to D = CV for some V ∈ N_n
i.e. C(D) ⊆ C(D) ⇔ D = CV for some V ∈ N_n

Lemma 2.6

For C ∈ N_{mn} and D ∈ N_{nm}, the following hold.

(i) The row space of CD, which is contained in the row space of C, i.e. R(CD) ⊆ R(C)

(ii) The column space of CD, which is contained in the column space of D , i.e. C(CD) ⊆ C(D)

Lemma: 2.7

For $C=[C_T, C_F, C_I] \in N_{mn}$ and $D=[D_T, D_F, D_I] \in N_{nm}$, the following hold.

(i) $C^T = [C_T{}^T, C_F{}^T, C_I{}^T]$

(ii) $CD = [C_TD_T, C_FD_F, C_ID_I]$

Theorem:2.8

For C, D ∈ N_n , with R(C) =R(D) and R (C^m)= R (D^m) then a is right m -regular Neutrosophic Fuzzy Matrix ⇔ D is right m - regular Neutrosophic Fuzzy Matrix.

Theorem:2.9

For C, D ∈ N_n , with C(C) ⊆ C (D) and C (C^m)= C (D^m) then a is left m -regular Neutrosophic Fuzzy Matrix ⇔ D is left m - regular Neutrosophic Fuzzy Matrix.

3. m-g **INVERSES OF** m- **REGULAR NUETROSOPHIC FUZZY MATRICES**

In this section, we introduce m-g inverses associated of m-regular Neutrosophic fuzzy Matrices as a generalisation of g - inverses of a regular fuzzy matrix. Further, we prove that if any two Nuetrosophic Fuzzy Matrices(NSFMs) are said to have a $\{3^m\}$ and $\{4^m\}$ inverses then its membership, non-membership and indeterminancy fuzzy matrices are also going to have a $\{3^m\}$ and $\{4^m\}$ inverses and vice versa.

Definition 3.1. A matrix $C=[C_T, C_F, C_I] \in N_n$ is said to have a $\{3^m\}$inverse if there exists a matrix $U \in N_n$ such that $(C^mU)^T = C^mU$, for some positive integer m. U is called the $\{3^m\}$ inverse of C. Let $C\{3^m\} = \{U/(C^mU)^T=C^mU\}$.

Definition 3.2. A matrix $A=[C_T, C_F, C_I] \in N_n$ is said to have a $\{4^m\}$ inverse if there exists a matrix $U \in N_n$ such that $(UC^m)^T=UC^m$, for some positive integer m. U is called the $\{4^m\}$ inverse of C. Let $C\{4^m\}=\{U/(UC^m)^T=UC^m\}$.

Remark 3.3. In particular for m = 1 and $C_T = C_F = C_I$, Definitions (3.1) and (3.2) reduces to set of {3} and {4} g-inverses respectively of a Fuzzy Matrix.

Theorem3.4

Let $C=[C_T, C_F, C_I] \in N_n$. Then C has a $\{3^m\}$ inverse $\Leftrightarrow$ C_T, C_F and C_I have $\{3^m\}$ inverses.

Proof:

Let $C=[C_T, C_F, C_I] \in N_n$

Since C has a $\{3^m\}$ inverse, Then there exists $U \in N_n$ such that $(C^mU)^T=C^mU$

Let $U=[U_T, U_F, U_I] \in N_n$. Then by lemma (2.7)(ii),

$$(C^mU)^T = C^mU \Leftrightarrow [C_T^m U_T, C_F^m U_F, C_I^m U_I]^T = [C_T^m U_T, C_F^m U_F, C_I^m U_I]$$
$$\Leftrightarrow [[C_T^m U_T]^T, [C_F^m U_F]^T, [C_I^m U_I]^T] = [C_T^m U_T, C_F^m U_F, C_I^m U_I]$$
$$\Leftrightarrow [C_T^m U_T]^T = C_T^m U_T, \quad [C_F^m U_F]^T = C_F^m U_F, \quad [C_I^m U_I]^T = [C_I^m U_I]$$

Hence C has a $\{3^m\}$ inverse $\Leftrightarrow$ C_T, C_F and C_I have $\{3^m\}$ inverses.

Theorem3.5

Let $C=[C_T, C_F, C_I] \in N_n$. Then C has a $\{4^m\}$ inverse $\Leftrightarrow$ C_T, C_F and C_I have $\{4^m\}$ inverses.

Proof:

Let $C=[C_T, C_F, C_I] \in N_n$

Since C has a $\{4^m\}$ inverse, then there exists $U \in N_n$ such that $(UC^m)^T = UC^m$

Let $U=[U_T, U_F, U_I] \in N_n$. Then by lemma(2.7)(ii),

$$(UC^m)^T = U C^m \Leftrightarrow [U_TC_T^m, U_FC_F^m, U_IC_I^m]^T = [U_TC_T^m, U_FC_F^m, U_IC_I^m]$$
$$\Leftrightarrow [[U_T C_T^m]^T, [U_FC_F^m]^T, [U_I C_I^m]^T] = [U_TC_T^m, U_FC_F^m, U_IC_I^m]$$
$$\Leftrightarrow [U_T C_T^m]^T = U_T C_T^m, \quad [U_FC_F^m]^T = U_FC_F^m, \quad [U_IC_I^m]^T = [U_I C_I^m]$$

Hence C has a $\{4^m\}$ inverse $\Leftrightarrow$ C_T, C_F and C_I have $\{4^m\}$ inverses.

Theorem 3.6

Let $C \in N_n$ and m be a positive integer,

(i) If $U \in C\{1_r^m\}$ with $R(U)=R(C^mU)$ then, $C \in U\{1_l^m\}$.

(ii) If $U \in C\{1_l^m\}$ with $C(U)=C(UC^m)$ then, $C \in U\{1_r^m\}$.

Proof:

(i) Since $U \in C\{1_r^m\}$ by Definition (2.2), $C^mUC = C^m$.

Since $R(U)=R(C^mU)$ by lemma (2.5), $U= VC^mU$, for some $V \in N_n$.

$$\begin{aligned} UCU^m &= V(C^mUC)U^m \\ &= VC^mU^m \\ &= VC^mUU^{m-1} \\ &= UU^{m-1} \\ &= U^m \end{aligned}$$

Hence $C \in U\{1_l^m\}$.

(ii) Proof is similar to (i) and hence omitted.

Theorem 3.7

For $C \in N_n$ and for any $G \in N_n$, if $C^mU = C^mG$, where U is a $\{1_r^m, 3^m\}$ inverse of C then G is a $\{1_r^m, 3^m\}$ inverse of C.

Proof:

Since U is a $\{1_r^m, 3^m\}$ inverse of C, by Definitions (2.2) and (3.1),

$$C^mUC = C^m \text{ and } (C^mU)^T = C^mU.$$

Post multiplying by C on both sides of $C^mU = C^mg$, we get $C^mgC = C^mUC = C^m$

Since $C^mU = C^mG \Rightarrow (C^mG)^T = (C^mU)^T = C^mU = C^mG$.

Hence G is a $\{1_r^m, 3^m\}$ inverse of C.

Theorem 3.8

For $C \in N_n$ and for any $G \in N_n$, if $UC^m = GC^m$, where U is a $\{1_\ell^m, 4^m\}$ inverse of C then G is a $\{1_\ell^m, 4^m\}$ inverse of C.

Proof:

Proof is similar to that of Theorem (3.7) and hence omitted.

Theorem 3.9:

For $a \in N_n$, U is a $\{1_r^m, 3^m\}$ inverse of C and G is a $\{1_\ell^m, 3\}$ inverse of C then $C^mU = C^mG$.

Proof:

Since U is a $\{1_r^m, 3^m\}$ inverse of C, by Definitions (2.2) and (3.1),

$$C^mUC = C^m \text{ and } (C^mU)^T = C^mU.$$

Since G is a $\{1_\ell^m, 3\}$ inverse of C, by Definition (2.3) and (3.2),

$$CGC^m = C^m \text{ and } (CG)^T = CG$$

$$\begin{aligned} C^mG &= (C^mUC)G = (C^mU)(CG) \\ &= (C^mU)^T (CG)^T \\ &= U^T(C^T)^mG^TC^T \\ &= U^T(CGC^m)^T \\ &= U^T(C^m)^T \\ &= (C^mU)^T \\ &= C^mU. \end{aligned}$$

Hence the theorem.

Theorem 3.10:

For $C \in N_n$, U is a $\{1_\ell^m, 4^m\}$ inverse of C and G is a $\{1_r^m, 4\}$ inverse of C, then $UC^m = GC^m$.

Proof:

This can be proved in the same manner as that of Theorem (3.9) and hence omitted.

In general, for an Neutrosophic Fuzzy Matrix C, there is no relation between m-regularity of C, C^TC and CC^T. Here, the relation shall be discussed under certain conditions on their row spaces.

Theorem 3.11: For $C \in N_n$, with $R(C) = R(C^TC)$ and $R(C^m) = R((C^TC)^m)$ then, C is right m-regular $\Leftrightarrow$ C^TC is right m-regular.

Proof: This follows from Theorem(2.8), by replacing D by CC^T.

Theorem 3.12: For $C \in N_n$, with $C(C) = C(CC^T)$ and $C(C^m) = C((CC^T)^m)$ then, C is left m-regular $\Leftrightarrow$ CC^T is left m-regular.

Proof: This follows from Theorem(2.9), by replacing D by CC^T.

Theorem 3.13: For $C \in N_n$, if C^TC is a right m-regular NSFM and $R(C^m) \subseteq R((C^TC)^m)$ then C has a $\{1_r^m, 3^m\}$ inverse. In particular for m=1, $Y = (C^TC)^-C^T$ is a $\{1, 3\}$ inverse of C.

Proof: Since C^TC is right m-regular NSFM, By Definition (2.2),

$(C^TC)^m(C^TC)^-(C^TC) = (C^TC)^m$ for some right m-g-inverse $(C^TC)^-$ of C^TC.
Since R $(C^m)\subseteq$ R $((C^TC)^m)$, by Lemma(2.6), $C^m = U(C^TC)^m$ for some $U\in N_n$ and tame $Y=(C^TC)^-C^T$.

$$
\begin{aligned}
C^mYC &= C^m (YC) = (U(C^TC)^m)((C^TC)^- C^TC)\\
&= U((C^TC)^m (C^TC)^- C^TC))\\
&= U(C^TC)^m\\
&= C^m.
\end{aligned}
$$

Tame $Z = (C^TC)^- (C^m)^T$.

$$
\begin{aligned}
C^mZ = (C^m)Z &= (U(C^TC)^m)((C^TC)^- (C^m)^T)\\
&= U(C^TC)^m (C^TC)^- (C^TC)^m U^T\\
&= U(C^TC)^m (C^TC)^- (C^TC)(C^TC)^{m-1}U^T\\
&= U(C^TC)^m (C^TC)^{m-1} U^T\\
&= U(C^TC)^{2m-1} U^T\\
&= (U(C^TC)^{2m-1} U^T)^T\\
&= (C^mZ)^T
\end{aligned}
$$

Hence C has a $\{1_r^m, 3^m\}$ inverse. For m=1, $Y= (A^T C)^-C^T$ is C {1, 3} inverse of C.

Theorem 3.14: For $C \in N_n$, if C^TC is a left m-regular NSFM and $C(C^m) \subseteq C((CC^T)^m)$ then C has a $\{1_l^m, 4^m\}$ inverse. For m=1, $Y = A^T (C^TC)^-$ is a {1, 4} inverse of C.

Proof: Proof is similar to Theorem(3.13) and hence omitted.

Theorem 3.15 : For $C \in N_n$ be a right m-regular NSFM and $R((C^TC)^m) \subseteq R(C^m)$ then C^TC has a $\{3^m\}$ inverse.

Proof:

Since C is right m-regular NSFM. By Definition (2.3),

$C^mUC= C^m$ for some right m-g-inverse $U\in N_n$, of C.

Since $R((C^TC)^m)\subseteq R(C^m)$, by Lemma(2.6),

$(C^TC)^m = ZC^m$ for some $Z \in N_n$ and take $Y = UC$.

$$
\begin{aligned}
(C^TC)^m Y &= (ZC^m)(UC)\\
&= ZC^m UC = ZC^m = (C^TC)^m = ((C^TC)^m)^T = ((C^TC)^mY)^T
\end{aligned}
$$

Hence C^TC has a (3^m) inverse.

Theorem 3.16: Let $C \in N_n$, be a left m-regular NSFM and $C((CC^T)^m)\subseteq C(C^m)$ then, CC^T has a $\{4^m\}$ inverse.

Proof: This can be proved in the same manner as that of Theorem (3.15) and hence omitted.

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