# APPLICATION OF STOCHASTIC MODEL WITH RECRUITMENT PROCESS IN A MANPOWER ORGANIZATION 

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#### Abstract

Stochastic models are essential for organizing manpower resources. In this study, a stochastic model with a non-stationary recruiting process is developed and examined. Here, a shock process defines the time-dependent recruitment. The Weighted Exponential Distribution is one of the special situations for particular parameter values in the shock process. Additionally, it is presumable that the process of getting promoted or terminated follows a weighted exponential distribution. We calculate the anticipated time to employee recruitment in the organization using the renewal process and the survival function. The model analysis indicates that the shock process in recruiting greatly affects the traits of manpower organization.


Keywords: Distribution, Manpower, Organization, Parameter, Recruitment

## Introduction

The main goal of recruitment is to find "the number of minimum-cost and quality personnel necessary to satisfy the organization's human resource demands" ${ }^{1}$. Recruitment entails seeking out and obtaining qualified candidates for open positions in sufficient numbers and quality so that the business can choose the best candidates to fill open positions ${ }^{2}$. According to Osoian and Zaharie (2014) ${ }^{3}$, identifying recruitment sources is an essential first step for any business. However, there is no one source that is always the best or most appropriate because it depends on the organization's needs, the characteristics of the positions, the size, reputation, and budget of the business, as well as the availability of labor.
Mallikharjuna Rao et al. (2015) designed and studied a manpower model under the assumption that the recruitment process is nonhomogeneous compound Poisson, with mean recruitment rate $\lambda(t)=a+b t$, i.e., recruitment rate is linearly dependent on time, and inter recruitment times are distributed exponentially. However, a detailed examination of the recruitment methods reveals that the recruitment rates may be rising, falling, or stable. The Duane process, in which the inter-recruitment times follow, is a good way to describe the time-varying recruitment rates following weibull distribution ${ }^{4}$.

A staffing strategy, according to Heneman and Judge (2008), is the interaction of an organization's Human resource strategy with important decisions pertaining to the recruitment and growth of its employees, such as decisions on recruitment, selection, and employment programs ${ }^{5}$. According to Ravichandran (2011), an organization must be set up such that it may best meet its strategic objectives, functional demands, and environmental constraints as they relate to providing the required services. Additionally, if the company wants to get a competitive edge, this is vital ${ }^{6}$.

## WEIGHTED EXPONENTIAL DISTRIBUTION (WED)

Gupta and Kundu (2009) ${ }^{7}$ to create a new class of WED using Azzalini (1985) ${ }^{8}$ idea, and its definition is as follows: The following probability density function (pdf) indicates that a random variable $X$ should have a WED with the shape and scale parameters.

$$
f(x, \lambda, \alpha)=\frac{\alpha+1}{\alpha} \lambda e^{(-\lambda)}\left[1-e^{(-\alpha \lambda x)}\right] \quad x>0, \quad \lambda, \alpha
$$

A random variable $X$ tracks Weighted Exponential $(\alpha, \lambda)$ if it has the pdf (1). For all values of, the PDF of the WED is unimodal and displays an increasing hazard function It is feasible to use the hazard function to represent lifetime information that takes stress and strain into account as a result of the hazard function's continuing expansion. The corresponding distribution function of X is

$$
\begin{align*}
F(\wp, \lambda, \alpha)=1 & +\frac{1}{\alpha} e^{\{-\lambda(\alpha+1) \wp\}} \\
& -\frac{\alpha+1}{\alpha} e^{(-\lambda \wp)} \tag{2}
\end{align*}
$$

The WED has a variety of significant characterisitcs ${ }^{8}$, despite not belonging to this family of distributions, the weighted exponential class can be used to create the limiting distribution, which is the exponential distribution. Recent literature ${ }^{9,10,11,12}$ showed the adaptability and simplicity of the model.

## MODEL DESCRIPTION AND SOLUTION

$$
\begin{align*}
& \bar{H}(\wp)=\frac{\alpha+1}{\alpha} e^{(-\lambda \wp)} \\
&-\frac{1}{\alpha} e^{\{-\lambda(\alpha+1) \wp\}}  \tag{3}\\
& P\left(\wp_{i}<Y\right)= \int_{0}^{\infty} g_{k}(\wp) \bar{H}(\wp) d x
\end{align*}
$$

$$
\begin{align*}
& =\int_{0}^{\infty} g_{k}(\wp)\left(\frac{\alpha+1}{\alpha} e^{(-\lambda \wp)}-\frac{1}{\alpha} e^{\{-\lambda(\alpha+1) \wp\}} d \wp\right) \\
& =\int_{0}^{\infty} g_{k}(\wp) \frac{\alpha+1}{\alpha} e^{(-\lambda \wp)} d \wp-\int_{0}^{\infty} g_{k}(\wp) \frac{1}{\alpha} e^{\{-\lambda(\alpha+1) \wp\}} d \wp \\
& =\frac{\alpha+1}{\alpha} g_{k}^{*}(\lambda) \\
& \quad-\frac{1}{\alpha} g_{k}^{*} \lambda(\alpha+1)  \tag{4}\\
& P(T>t)=\sum_{k=0}^{\infty} V_{k}(t) P\left(\not \wp_{i}\right. \\
& <Y)
\end{align*}
$$

The number of decisions made in $(0, t]$ from a renewal process $V_{i}(t)\left[F_{k}(t)-F_{k+1}(t)\right]$

$$
\begin{gather*}
=\sum_{k=0}^{\infty}\left[F_{k}(t)-F_{k+1}(t)\right]\left[\frac{\alpha+1}{\alpha} g_{k}^{*}(\lambda)-\frac{1}{\alpha} g_{k}^{*} \lambda(\alpha+1)\right] \\
=\frac{\alpha+1}{\alpha} \sum_{k=0}^{\infty}\left[F_{k}(t)-F_{k+1}(t)\right] g_{k}^{*}(\lambda) \\
\quad-\frac{1}{\alpha} \sum_{k=0}^{\infty}\left[F_{k}(t)-F_{k+1}(t)\right] g_{k}^{*}[\lambda(\alpha+1)] \\
=\frac{\alpha+1}{\alpha} \sum_{k=0}^{\infty}\left[F_{k}(t)-F_{k+1}(t)\right]\left[g^{*}(\lambda)\right]^{k} \\
\quad-\frac{1}{\alpha} \sum_{k=0}^{\infty}\left[F_{k}(t)-F_{k+1}(t)\right]\left\{g^{*}[\lambda(\alpha+1)]\right\}^{k} \tag{5}
\end{gather*}
$$

Taking Laplace transformation $L(t)$ we get,

$$
L(t)=1-\frac{\alpha+1}{\alpha} \sum_{k=0}^{\infty}\left[F_{k}(t)-F_{k+1}(t)\right]\left[g^{*}(\lambda)\right]^{k}+\frac{1}{\alpha} \sum_{k=0}^{\infty}\left[F_{k}(t)-F_{k+1}(t)\right]\left\{g^{*}[\lambda(\alpha+1)]\right\}^{k}
$$

On simplification we get

$$
\begin{align*}
& L(t)=1-\frac{\alpha+1}{\alpha}+\frac{\alpha+1}{\alpha}\left[1-g^{*}(\lambda)\right] f^{*}(s) \sum_{k=0}^{\infty}\left\{g^{*}(\lambda) f^{*}(s)\right\}^{k-1}+\frac{1}{\alpha} \\
& \quad-\frac{1}{\alpha}[1 \\
&\left.\quad-g^{*}(\lambda)(\alpha+1)\right] f^{*}(s) \sum_{k=0}^{\infty}\left\{g^{*}(\lambda)(\alpha+1) f^{*}(s)\right\}^{k-1} \\
&=\frac{\alpha+1}{\alpha}\left[1-g^{*}(\lambda)\right] f^{*}(s) \sum_{k=0}^{\infty}\left\{g^{*}(\lambda) f^{*}(s)\right\}^{k-1} \\
& \quad-\frac{1}{\alpha}[1 \\
&\left.\quad-g^{*}(\lambda)(\alpha+1)\right] f^{*}(s) \sum_{k=0}^{\infty}\left\{g^{*}(\lambda)(\alpha+1) f^{*}(s)\right\}^{k-1} \tag{6}
\end{align*}
$$

Laplace-Stieltjes transform can be used to demonstrate that

$$
\begin{aligned}
l^{*}(s)=\frac{\alpha+1}{\alpha} & \frac{\left[1-g^{*}(\lambda)\right] f^{*}(s)}{\left[1-g^{*}(\lambda)\right] f^{*}(s)} \\
& -\frac{1}{\alpha} \frac{\left[1-g^{*}(\lambda)(\alpha+1)\right] f^{*}(s)}{\left[1-g^{*}(\lambda)(\alpha+1)\right] f^{*}(s)}
\end{aligned}
$$

The random variable representing the inter-arrival time should be exponential.
Now $f^{*}(s)=\left(\frac{c}{c+s}\right)$, by substituting in the following equation we get equation (7).

$$
\begin{gather*}
=\frac{\alpha+1}{\alpha} \frac{\left[1-g^{*}(\lambda)\right] \frac{c}{c+s}}{\left[1-g^{*}(\lambda) \frac{c}{c+s}\right]}-\frac{1}{\alpha} \frac{\left[1-g^{*}(\lambda)(\alpha+1)\right] \frac{c}{c+s}}{\left[1-g^{*}(\lambda)(\alpha+1) \frac{c}{c+s}\right]} \\
=\frac{\alpha+1}{\alpha} \frac{\left[1-g^{*}(\lambda)\right] c}{\left[c+s-g^{*}(\lambda) c\right]} \\
-\frac{1}{\alpha} \frac{\left[1-g^{*}(\lambda)(\alpha+1)\right] c}{\left[c+s-g^{*}(\lambda)(\alpha+1) c\right]} \tag{7}
\end{gather*}
$$

$$
\begin{aligned}
& E(T)=-\frac{d}{d s} l^{*}(s) \text { Given } s=0 \quad E(T)=\frac{\alpha+1}{\alpha c\left[1-g^{*}(\lambda)\right]}-\frac{1}{\alpha c\left[1-g^{*}(\lambda)(\alpha+1)\right]} \\
& g^{*}(\lambda)=\frac{\mu}{\mu+\lambda}, \quad g^{*}(\lambda)(\alpha+1)=\frac{\mu}{\mu+\lambda(\alpha+1)} \\
& =\frac{\alpha+1}{\alpha c\left[1-\frac{\mu}{\mu+\lambda}\right]}-\frac{1}{\alpha c\left[1-\frac{\mu}{\mu+\lambda(\alpha+1)}\right]}
\end{aligned}
$$

On simplification we get the expected time

$$
\begin{align*}
& E(T)=\frac{1}{c}\left[\frac{(\alpha+1)(\mu+\lambda)}{\alpha \lambda}\right. \\
& \left.-\frac{(\alpha+1)(\mu+\lambda)}{\alpha \lambda(\alpha+1)}\right] \tag{8}
\end{align*}
$$

## Results

Table 1. Expected recruitment at different stages

| c | $\alpha=0.3, \mu$ <br> $=0.5, \lambda=0.1$ | $\alpha=0.5, \mu$ <br> $=0.8, \lambda=0.1$ | $\alpha=0.7, \mu$ <br> $=0.9, \lambda=0.1$ |
| :---: | :---: | :---: | :---: |
| 1 | 6 | 9 | 10 |
| 2 | 3 | 4.5 | 5 |
| 3 | 2 | 3 | 3.333 |
| 4 | 1.5 | 2.25 | 2.5 |
| 5 | 1.2 | 1.8 | 2 |
| 6 | 1 | 1.5 | 1.667 |
| 7 | 0.857 | 1.286 | 1.429 |
| 8 | 0.75 | 1.125 | 1.25 |
| 9 | 0.667 | 1 | 1.111 |
| 10 | 0.6 | 0.9 | 1 |
| 20 | 0.3 | 0.45 | 0.5 |
| 30 | 0.2 | 0.3 | 0.333 |
| 50 | 0.12 | 0.18 | 0.2 |



Figure 1. Observed expected recruitment at different stages

## Discussion

As in the table 1, inter-arrival time increases at different period and different stage of recruitment is observed. In all the three stages of the parametric ( $\alpha, \mu$ and $\lambda$ ) change and fix, i.e.,
$\alpha=0.3, \mu=0.5, \lambda=0.1, \alpha=0.5, \mu=0.8, \lambda=0.1$ and $\quad \alpha=0.7, \mu=0.9, \lambda=0.1, \quad$ we observed the recruitment time to decrease as the inter-arrival time increases. This result of WED is found similar in the study ${ }^{13}$ where the authors used three-parameter generalized Rayleigh distribution. Another study ${ }^{14}$ where the threshold level is a random variable following generalized exponentiated gamma distribution for recruitment in organization.

Keeping in mind that measures for inclusion and equal employment opportunity are interwoven into the recruitment process. It is important to plan ahead for staffing demands so that properly qualified candidates may be found and trained.

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