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ROUGH SPHERICAL FUZZY BI-IDEAL IN NEAR RINGS

V.S.Subha[1],S.Lavanya [2], C.B.Aswini[3]

Assistant professor (deputed),PG and Research department of mathematics ,Govt Arts college C.Multur Chidambaram[1]

Assistant professor,Department of mathematics,Bharathi womens college chennai[2]

Department of mathematics ,Annamalai university,Annamalai nagar,608002[3]

ABSTRACT

This paper studies relationship between spherical fuzzy set, rough set and near ring. We define spherical fuzzy bi-ideal in near ring and pro-pose rough spherical fuzzy bi-ideal in near ring.Investigate some interesting properties of rough spherical fuzzy bi-ideal.

Keywords: Spherical fuzzy set, Spherical fuzzy bi-ideal, Near-ring, Rough set, rough fuzzy set, Rough spherical fuzzy set, Rough spherical fuzzy bi-ideal..

1.INTRODUCTION

Fuzzy [14] sets have a great progress in every scientific research area. After the introduction of ordinary fuzzy sets, new extensions have appeared one by one in the literature. Among these extensions, picture fuzzy sets [2], neutrosophic sets [10] and spherical fuzzy sets [5] are the members of the same class since any element in these sets is represented by a membership degree, a non-membership degree and a hesitancy degree assigned by independently. Spherical fuzzy sets have been proposed by Gundogdu and Kahraman [5]. The notion of rough sets was introduced by Pawlak [9] in the year 1982. The algebraic approach of rough sets was studied by many researchers. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set. The lower and upper approximation operators are based on equivalence relation. The rest of the paper is organized as follows: Section 2 is full of information about the basics . In Section 3, we introduced the spherical fuzzy bi-ideal in near ring and discussed some interesting properties. At last in section 4 we combine rough set and spherical fuzzy set. Also we define rough spherical fuzzy bi-ideal in near ring.

2.PRELIMINARIES

We review some definitions that will be useful in our results. Throughout this paper let us denote ℵ as near ring. Let Π be an equivalence relation on ℵ. A congruence relation Π on S is said to be complete if $[a]_\Pi[b]_\Pi = [ab]_\Pi$.

Let (ℵ, Π) be an approximation space. Let A be any nonempty subset of ℵ. The sets

$$\Pi^{-}(A) = \{x \in \aleph / [x]_\Pi \subseteq A\} \text{ and}$$

$$\Pi^{+}(A) = \{x \in \aleph / [x]\Pi \cap A \neq \varphi\}$$

are called the lower and upper approximations of A. Then $\Pi(A) = (\Pi^{-}(A), \Pi^{+}(A))$ is called rough set in (S, Π), iff $\Pi^{-}(A) \neq \Pi^{+}(A)$. A fuzzy subset of a nonempty set X is defined as a function $\beta : X \to [0, 1]$ Let Λ be a fuzzy subset of ℵ. The fuzzy subsets of ℵ defined by

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$$\Pi^{+}(\Lambda)(x) = \bigvee_{a\in[x]_\Pi} \Lambda(a) \quad \text{and} \quad \Pi^{-}(\Lambda)(x) = \bigwedge_{a\in[x]_\Pi} \Lambda(a)$$

are called respectively, the Π-upper and Π-lower approximations of the fuzzy set Λ.

Then $\Pi(\Lambda) = (\Pi^{-}(\Lambda), \Pi^{+}(\Lambda))$ is called a rough fuzzy set of Λ with respect to Π if $\Pi^{-}(\Lambda) \neq \Pi^{+}(\Lambda)$.

Definition 2.1. [1] An intuitionistic fuzzy set defined on ℵ is an object having the form

$I = (\langle i, I_m(i), I_n(i)\rangle$: i ∈ ℵ)

of each element i∈ ℵ to the set I respectively, and satisfies

$$0 \le I_m + I_n \le 1.$$

Definition 2.2. [6] Let be $\mathcal{E}$ the SF set of the universe of U is defined by $\mathcal{E} = \{l, \langle \mathcal{E}_m(l), \mathcal{E}_n(l), \mathcal{E}_h(l)\rangle\}$

Where $\mathcal{E}_m(l)$: U → [0, 1], $\mathcal{E}_n(l)$: U → [0, 1], $\mathcal{E}_h(l)$: U → [0, 1] and

$$0 \le \mathcal{E}_m^2(l) + \mathcal{E}_n^2(l) + \mathcal{E}_h^2(l) \le 1 \text{ for every } l \in U$$

for each l, the numbers $\mathcal{E}_m(l)$, $\mathcal{E}_n(l)$and $\mathcal{E}_h(l)$ are the degree of membership, non membership and hesitancy of l to Ɛ, respectively.

Example 2.3. Let ℵ = {p, q, r, s} be the universe. A SF set Ɛ of ℵ is defined by $\mathcal{E}_m(i)$ = {0.6, 0.4, 0.2, 0.4}, $\mathcal{E}_n(i)$= {0.6, 0.5, 0.3, 0.6} and $\mathcal{E}_h(i)$ = {0.3, 0.4, 0.5, 0.1} where i = p, q, r, s.

3. SPHERICAL FUZZY BI-IDEAL (SF*BI*) IN NEAR RINGS

This section deals with notion ofSF*BI* in near-ring ℵ. Also we prove the intersection of two SF*BI* is also a SF*BI* in near ring ℵ.

Definition 3.1. A SF set $\mathcal{E} = \langle \mathcal{E}_m(l), \mathcal{E}_n(l), \mathcal{E}_h(l)\rangle$in ℵ is called a SF*BI* of ℵ if the resulting conditions are true:

(1)$\mathcal{E}_m(i - j) \ge \mathcal{E}_m(i) \wedge \mathcal{E}_m(j)$

$\mathcal{E}_n(i - j) \ge \mathcal{E}_n(i) \wedge \mathcal{E}_n(j)$

$\mathcal{E}_h(i - j) \le \mathcal{E}_h(i) \vee \mathcal{E}_h(j)$

(2)$\mathcal{E}_m(ijk) \ge \mathcal{E}_m(i) \wedge \mathcal{E}_m(k)$

$\mathcal{E}_n(ijk) \ge \mathcal{E}_n(i) \wedge \mathcal{E}_n(k)$

$\mathcal{E}_h(ijk) \le \mathcal{E}_h(i) \vee \mathcal{E}_h(k)$ for all i, j, k ∈ℵ

Example 3.2. Let ℵ = {z0, z1, z2, z3} be a near-ring with the following multiplication table

TABLE 1

+	z_0	z_1	z_2	z_3
z_0	z_0	z_1	z_2	z_3
z_1	z_1	z_0	z_3	z_2
z_2	z_2	z_3	z_1	z_0
z_3	z_3	z_2	z_0	z_1

TABLE 2

·	z_0	z_1	z_2	z_3
z_0	z_0	z_0	z_0	z_0
z_1	z_0	z_0	z_0	z_0

z_2	z_0	z_0	z_0	z_0
z_3	z_0	z_1	z_2	z_3

Let $\mathcal{E}$ be a SF set of ℵ defined by,
$\mathcal{E}_m$ (z_0) = $\mathcal{E}_m$ (z_1) = $\mathcal{E}_m$ (z_1) = $\mathcal{E}_m$ (z_2) = 0.3
$\mathcal{E}_n$ (z_0) = $\mathcal{E}_n$ (z_2) = 0.6 , $\mathcal{E}_n$ (z_1) = $\mathcal{E}_n$ (z_3) = 0.5
$\mathcal{E}_h$ (z_0) = 0.4 , $\mathcal{E}_h$ (z_1) = $\mathcal{E}_h$ (z_2) = $\mathcal{E}_h$ (z_3) = 0.6
Then Ɛ is a SF*BI* of ℵ

Theorem 3.3. Let $\mathcal{L} = \langle \mathcal{L}_m, \mathcal{L}_n, \mathcal{L}_h \rangle$ and $\mathcal{K} = \langle \mathcal{K}_m, \mathcal{K}_n, \mathcal{K}_h \rangle$are two 𝕊𝔽*BI*s of ℵ. If $\mathcal{L} \subset \mathcal{K}$ then $\mathcal{L} \cap \mathcal{K}$ is S𝕊𝔽*BI* of ℵ

Proof. Since $\mathcal{L}$ and $\mathcal{K}$ are two SF*BI*s of ℵ. Let i, j, k ∈ ℵ. Then we first prove for membership function

$(\mathcal{L}_m \cap \mathcal{K}_m)(\mathrm{i}-\mathrm{j}) = \mathcal{L}_m(\mathrm{i}-\mathrm{j}) \wedge \mathcal{K}_m(\mathrm{i}-\mathrm{j})$
$\geq (\mathcal{L}_m(\mathrm{i}) \wedge \mathcal{L}_m(\mathrm{j})) \wedge (\mathcal{K}_m(\mathrm{i}) \wedge \mathcal{K}_m(\mathrm{j}))$
$= (\mathcal{L}_m(\mathrm{i}) \wedge \mathcal{K}_m(\mathrm{i})) \wedge (\mathcal{L}_m(\mathrm{j}) \vee \mathcal{K}_m(\mathrm{j}))$
$= (\mathcal{L}_m \cap \mathcal{K}_m)(\mathrm{i}) \wedge (\mathcal{L}_m \cap \mathcal{K}_m)(\mathrm{j})$

Also
$(\mathcal{L}_m \cap \mathcal{K}_m)(\mathrm{ijk}) = \mathcal{L}_m(\mathrm{ijk}) \wedge \mathcal{K}_m(\mathrm{ijk})$
$\geq (\mathcal{L}_m(\mathrm{i}) \wedge \mathcal{L}_m(\mathrm{k})) \wedge (\mathcal{K}_m(\mathrm{i}) \wedge \mathcal{K}_m(\mathrm{k}))$
$= (\mathcal{L}_m(\mathrm{i}) \wedge \mathcal{K}_m(\mathrm{i})) \wedge (\mathcal{L}_m(\mathrm{k}) \vee \mathcal{K}_m(\mathrm{k}))$
$= (\mathcal{L}_m \cap \mathcal{K}_m)(\mathrm{i}) \wedge (\mathcal{L}_m \cap \mathcal{K}_m)(\mathrm{k})$

Consequently we can prove for non-membership function
$(\mathcal{L}_n \cap \mathcal{K}_n)(\mathrm{i}-\mathrm{j}) \geq (\mathcal{L}_n \cap \mathcal{K}_n)(\mathrm{i}) \wedge (\mathcal{L}_n \cap \mathcal{K}_n)(\mathrm{j})$
$(\mathcal{L}_n \cap \mathcal{K}_n)(\mathrm{ijk}) \geq (\mathcal{L}_n \cap \mathcal{K}_n)(\mathrm{i}) \wedge (\mathcal{L}_n \cap \mathcal{K}_n)(\mathrm{k})$
Similarly we can prove for hesitancy function
$(\mathcal{L}_h \cap \mathcal{K}_h)(\mathrm{i}-\mathrm{j}) \leq (\mathcal{L}_h \cap \mathcal{K}_h)(\mathrm{i}) \vee (\mathcal{L}_h \cap \mathcal{K}_h)(\mathrm{j})$
$(\mathcal{L}_h \cap \mathcal{K}_h)(\mathrm{ijk}) \leq (\mathcal{L}_h \cap \mathcal{K}_h)(\mathrm{i}) \wedge (\mathcal{L}_h \cap \mathcal{K}_h)(\mathrm{k})$
Thus intersection of two 𝕊𝔽*BI* is 𝕊𝔽*BI*

Theorem 3.4 Arbitrary intersection of 𝕊𝔽BI is also 𝕊𝔽BI.

Proof. Let $\{\mathcal{P}^i = \langle \mathcal{P}^i_m, \mathcal{P}^i_n, \mathcal{P}^i_h \rangle$, i∈ I} be a family of 𝕊𝔽BI of ℵ.
For any α, β, γ ∈ ℵ. We have
$\bigcap_{i \epsilon I} \mathcal{P}^i_m(\mathrm{k}) = \bigcap_{i \epsilon I} \mathcal{P}^i_m(\mathrm{k})$, $\bigcap_{i \epsilon I} \mathcal{P}^i_n(\mathrm{k}) = \bigcap_{i \epsilon I} \mathcal{P}^i_n(\mathrm{k})$, $\bigcap_{i \epsilon I} \mathcal{P}^i_h(\mathrm{k}) = \bigcap_{i \epsilon I} \mathcal{P}^i_h(\mathrm{k})$
consider $\bigcap_{i \epsilon I} \mathcal{P}^i_m(\alpha - \beta) = inf_{i \epsilon I} \mathcal{P}^i_m(\alpha - \beta)$
$\geq inf_{i \epsilon I}(\mathcal{P}^i_m(\alpha) \Lambda \mathcal{P}^i_m(\beta))$
$= inf_{i \epsilon I}(\mathcal{P}^i_m(\alpha)\ \Lambda inf_{i \epsilon I} \mathcal{P}^i_m(\beta))$
$= \bigcap_{i \epsilon I} \mathcal{P}^i_m(\alpha)\ \Lambda\ \bigcap_{i \epsilon I} \mathcal{P}^i_m(\beta)$
Also $\bigcap_{i \epsilon I} \mathcal{P}^i_m(\alpha j \beta) = inf_{i \epsilon I} \mathcal{P}^i_m(\alpha j \beta)$
$\geq inf_{i \epsilon I}(\mathcal{P}^i_m(\alpha) \Lambda \mathcal{P}^i_m(\beta))$
$= inf_{i \epsilon I}(\mathcal{P}^i_m(\alpha)\ \Lambda inf_{i \epsilon I} \mathcal{P}^i_m(\beta))$
$= \bigcap_{i \epsilon I} \mathcal{P}^i_m(\alpha)\ \Lambda\ \bigcap_{i \epsilon I} \mathcal{P}^i_m(\beta)$

Consequently we can prove for

$$\bigcap_{i\epsilon I} \mathcal{P}_n^i\,(\alpha-\beta) \geq inf_{i\epsilon I}\mathcal{P}_n^i(\alpha)\ \Lambda inf_{i\epsilon I}\mathcal{P}_n^i(\beta)$$

$$\bigcap_{i\epsilon I} \mathcal{P}_h^i\,(\alpha-\beta) \leq inf_{i\epsilon I}\mathcal{P}_h^i(\alpha)\ \Lambda inf_{i\epsilon I}\mathcal{P}_h^i(\beta)$$

And

$$\bigcap_{i\epsilon I} \mathcal{P}_n^i\,(\alpha j\beta) \geq \bigcap_{i\epsilon I} \mathcal{P}_n^i\,(\alpha)\ \Lambda \bigcap_{i\epsilon I} \mathcal{P}_n^i\,(\beta)$$

$$\bigcap_{i\epsilon I} \mathcal{P}_h^i\,(\alpha j\beta) \leq \bigcap_{i\epsilon I} \mathcal{P}_h^i\,(\alpha)\ \Lambda \bigcap_{i\epsilon I} \mathcal{P}_h^i\,(\beta)$$

Definition 3.5. Let Θ be a mappings from a set A to B and $\mathcal{F}$ be a 𝕊𝔽 on B. Then the preimage of $\mathcal{F}$ under Θ denoted by Θ−1 ($\mathcal{F}$ (x)) is defined by

Θ−1 ($\mathcal{F}_m$ (x)) = $\mathcal{F}_m$ (Θ(x)), Θ−1 ($\mathcal{F}_n$ (x)) = $\mathcal{F}_n$ (Θ(x)) and Θ−1 ($\mathcal{F}_h$ (x)) = $\mathcal{F}_h$ (Θ(x)) for all x ∈ ℵ.

Theorem 3.6. If Θ : 𝔄 −→ 𝔅 be an onto homomorphism of ℵ. Let 𝔓 be a 𝕊𝔽 of B then Θ−1 (𝔓) is a 𝕊𝔽BI of A.

Proof : Let $\alpha, \beta \epsilon \mathfrak{A}$. Then

$$\begin{aligned}
\Theta{-}1(\mathfrak{P}_m)(\alpha-\beta) &= \mathfrak{P}_m(\Theta(\alpha-\beta))\\
&= \mathfrak{P}_m(\Theta(\alpha){-}\Theta(\beta))\\
&\geq min\{\mathfrak{P}_m(\Theta(\alpha)),\ \mathfrak{P}_m(\Theta(\beta))\}\\
&= min\{\Theta^{-1}(\mathfrak{P}_m)(\alpha), \Theta^{-1}(\mathfrak{P}_m)(\beta)\}
\end{aligned}$$

Also

$$\begin{aligned}
\Theta^{-1}(\mathfrak{P}_m)(\alpha k\beta) &= \mathfrak{P}_m(\Theta(\alpha k\beta))\\
&= \mathfrak{P}_m(\Theta(\alpha)\Theta(k)\Theta(\beta))\\
&\geq min\{\mathfrak{P}_m(\Theta(\alpha)),\ \mathfrak{P}_m(\Theta(\beta))\}\\
&= min\{\Theta^{-1}(\mathfrak{P}_m)(\alpha), \Theta^{-1}(\mathfrak{P}_m)(\beta)\}
\end{aligned}$$

Hence proved

4.ROUGH SPERICAL FUZZY BI-IDEALS(ℝ𝕊𝔽*BI*) IN NEAR -RINGS

In this section we introduce the new idea ℝ𝕊𝔽*BI* in near ring ℵ. Throughout this section let us denote Π the complete congruence relation on ℵ.

Definition 4.1. Let $\mathcal{E} = \langle j/\ \mathcal{E}_m(j), \mathcal{E}_n(j),\ \mathcal{E}_h(j)\rangle$ be a 𝕊𝔽 set in ℵ and Π be a congruence relation on ℵ. Then ℝ𝕊𝔽 set of $\mathcal{E}$ with respect to the approximation space (Π, ℵ) is defined by Π($\mathcal{E}$) = (Π⁻($\mathcal{E}$), Π⁺($\mathcal{E}$)).

The lower approximation of $\mathcal{E}$ is denoted by Π⁻($\mathcal{E}$) and defined as

$\Pi^-(\mathcal{E}) = \langle j, \Pi^-(\mathcal{E}_m)(j), \Pi^-(\mathcal{E}_n)(j), \Pi^-(\mathcal{E}_h)(j) | j \in \aleph \rangle$

Where

$\Pi^-(\mathcal{E}_m)(l) = \Lambda_{s\in[l]_\Pi}\ \mathcal{E}_m(s)$, $\Pi^-(\mathcal{E}_n)(l) = \Lambda_{s\in[l]_\Pi}\ \mathcal{E}_n(s)$, $\Pi^-(\mathcal{E}_h)(l) = \vee_{s\in[l]_\Pi}\ \mathcal{E}_h(s)$

With the condition that

$0 \leq (\Pi^-(\mathcal{E}_m))^2 + (\Pi^-(\mathcal{E}_n))^2 + (\Pi^-(\mathcal{E}_h))^2 \leq 1$

and the upper approximation of $\mathcal{E}$ is denoted by $\Pi^+(\mathcal{E})$ and defined as

$\Pi^+(\mathcal{E})=\langle j, \Pi^+(\mathcal{E}_m)(j), \Pi^+(\mathcal{E}_n)(j), \Pi^+(\mathcal{E}_h)(j) | j \in \aleph \rangle$

$\Pi^+(\mathcal{E}_m)(l)=\vee_{s\in[l]_\Pi} \mathcal{E}_m(s)$, $\Pi^+(\mathcal{E}_n)(l)=\wedge_{s\in[l]_\Pi} \mathcal{E}_n(s)$, $\Pi^+(\mathcal{E}_h)(l)=\wedge_{s\in[l]_\Pi} \mathcal{E}_h(s)$

With the condition that

$0\leq (\Pi^+(\mathcal{E}_m))^2+ (\Pi^+(\mathcal{E}_n))^2+ (\Pi^+(\mathcal{E}_h))^2\leq 1$

Example 4.2.Let $\aleph = \{L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8\}$ be the universe set and Π be the congruence relation on ℵ. The equivalence classes of ℵ are defined by

$\aleph/\Pi = \{\{L_1, L_6, L_8\}, \{L_2\}, \{L_3\}, \{L_4, L_5, L_7\}\}$

Let $\mathcal{E}$ be the 𝕊𝔽 set of ℵ defined by

$\mathcal{E}_m(x) = \{L_1 = 0.5$
$L_2 = P_7 = 0.4$
$L_3 = P_6 = 0.8$
$L_4 = 0.6$
$L_5 = 0.7$
$P_8 = 0.3$

$\mathcal{E}_n(x) = \{ L_1 = L_5 = L_8 = 0.1$
$L_2 = 0.6$ $L_3 = L_6 = 0.2$
$L_4 = 0.3$
$L_7 = 0.4$

$\mathcal{E}_h(x) = \{L_1 = 0.4$
$L_2 = L_8 = 0.2$
$L_3 = 0.5$
$L_4 = L_6 = 0.4$
$L_5 = 0.3$
$L_7 = 0.7$

Then the lower approximation of $\mathcal{E}$ for all $x\in \aleph$ is given by

$\Pi^-(\mathcal{E}_m) = \{L_1 = L_6 = L_8 = 0.3$
$L_2 = L_4 = L_5 = L_7 = 0.4$
$L_3 = 0.8$

$\Pi^-(\mathcal{E}_n) = \{L_1 = L_4 = L_5 = L_6 = L_7 = L_8 = 0.1$
$L_2 = 0.6$
$L_3 = 0.2$

$\Pi^-(\mathcal{E}_h) = \{L_1 = L_6 = L_8 = 0.6$
$L_2 = 0.2$
$L\ 3 = 0.5$
$L_4 = L_5 = L_7 = 0.7$

Then the upper approximation of $\mathcal{E}$ for all $x\in \aleph$ is given by

$\Pi^+(\mathcal{E}_m) = \{L_1 = L_3 = L_6 = L_8 = 0.8$
$L_2 = 0.4$
$L_4 = L_5 = L_7 = 0.7$

$\Pi^+(\mathcal{E}_n) = \{L_1 = L_3 = L_6 = L_8 = 0.2$

$L_2 = 0.6$
$L_4 = L_5 = L_7 = 0.4$
$\Pi^+(\mathcal{E}_h)$={$L_1 = L_2 = L_6 = L_8 = 0.2$
$L_3 = 0.5$
$L_4 = L_5 = L_7 = 0.5$
Then $\Pi(\mathcal{E}) = (\Pi^-(\mathcal{E}), \Pi^+(\mathcal{E}))$ is a ℝ𝕊𝔽 set of ℵ

Example 4.3. Let ℵ = {$P_1,P_2,P_3,P_4,P_5,P_6,P_7,P_8$} be the universe set and Π be the congruence relation on ℵ. The equivalence classes of ℵ are defined by
ℵ/Π = {{P_1,P_4}, {P_2,P_3,P_6}, {P_5}, {P_7,P_8}}
Let $\mathcal{E}$ be the 𝕊𝔽 set of ℵ defined by
$\mathcal{E}_m(x)$ ={$P_1 = 0.2$
$P_2 = 0.5$
$P_3 = P_8 = 0.3$
$P_4 = P_6 = 0.3$
$P_5 = P_7 = 0.4$
$\mathcal{E}_n(x)$ ={$P_1 = 0.2$
$P_2 = P_6 = P_8 = 0.4$
$P_3 = 0.2$
$P_4 = 0.3$
$P_5 = 0.5$
$P_7 = 0.7$
$\mathcal{E}_h(x)$ ={$P_1 = P_3 = 0.4$
$P_2 = 0.7$
$P_4 = 0.2$
$P_5 = P_7 = 0.5$
$P_6 = 0.3$
$P_8 = 0.8$
Then the lower approximation of $\mathcal{E}$ for all $x \in$ ℵ is given by
$\Pi^-(\mathcal{E}_m)$={$P_1 = P_4 = 0.2$
$P_5 = 0.4$
$P_2 = P_3 = P_6 = P_7 = P_8 = 0.3$
$\Pi^-(\mathcal{E}_n)$={$P_1 = P_4 = 0.1$
$P_2 = P_3 = P_6 = 0.2$
$P_5 = 0.5$
$P_7 = P_8 = 0.4$
$\Pi^-(\mathcal{E}_h)$={$P_1 = P_4 = 0.4$
$P_2 = P_3 = P_6 = 0.7$
$P_5 = 0.5$
$P_7 = P_8 = 0.8$
Then the upper approximation of $\mathcal{E}$ for all $x \in$ ℵ is given by
$\Pi^+(\mathcal{E}_m)$={$P_1 = P_2 = P_3 = P_4 = P_6 = 0.6$

$P_5 = P_7 = P_8 = 0.4$

$\Pi^+(\mathcal{E}_n)=\{P_1 = P_4 = 0.3$

$P_2 = P_3 = P_6 = 0.4$

$P_5 = 0.5$

$P_7 = P_8 = 0.7$

$\Pi^+(\mathcal{E}_h)=\{P_1 = P_4 = 0.2$

$P_2 = P_3 = P_6 = 0.3$

$P_5 = P_7 = P_8 = 0.5$

Then $\Pi(\mathcal{E}) = (\Pi^-(\mathcal{E}), \Pi^+(\mathcal{E}))$ is a ℝ𝕊𝔽 set of ℵ

Definition 4.4. A 𝕊𝔽*BI* of ℵ is said to be a ℝ𝕊𝔽*BI* of ℵ if it is both Π-lower ℝ𝕊𝔽*BI* and Π-upper ℝ𝕊𝔽*BI* of ℵ.

A 𝕊𝔽*BI* of ℵ is called a Π-lower (upper) ℝ𝕊𝔽*BI* of ℵ if its lower(upper) approximation is 𝕊𝔽*BI* of ℵ.

Example 4.5. Let ℵ = $\{z_0, z_1, z_2\}$ be a near-ring with the following multiplication table.

Table 3

+	z_0	z_1	z_2
z_0	z_0	z_1	z_2
z_1	z_1	z_2	z_0
z_2	z_2	z_1	z_0

Table 4

•	z_0	z_1	z_2
z_0	z_0	z_0	z_0
z_1	z_0	z_1	z_2
z_2	z_0	z_2	z_1

Let ϒ be the congruence relation on ℵ. The equivalence classes of ℵ are defined by

ℵ/ϒ = $\{\{z_0\}, \{z_1, z_2\}\}$

Let $\mathcal{E}$ be a 𝕊𝔽 set of ℵ defined by

$\mathcal{E}_m(z_0) = \mathcal{E}_m(z_1) = \mathcal{E}_m(z_2)=0.2$

$\mathcal{E}_n(z_0)=0.5$, $\mathcal{E}_n(z_1)= \mathcal{E}_n(z_2)=0.4$

$\mathcal{E}_h(z_1)=0.5$, $\mathcal{E}_h(z_0)= \mathcal{E}_h(z_2)=0.3$

Then the lower approximation of $\mathcal{E}$ is

$\underline{\Pi}(\mathcal{E}_m)(z_0) = \underline{\Pi}(\mathcal{E}_m)(z_1)= \underline{\Pi}(\mathcal{E}_m)(z_2) =0.2$

$\underline{\Pi}(\mathcal{E}_n(z_0)=0.5, \underline{\Pi}\,\mathcal{E}_n(z_1)=\underline{\Pi}(\mathcal{E}_n(z_1)=0.4$

$\underline{\Pi}(\mathcal{E}_h(z_1)=0.3, \underline{\Pi}(\mathcal{E}_h(z_1) =\underline{\Pi}(\mathcal{E}_h(z_1)=0.5$

Then the upper approximation of $\mathcal{E}$ is

$\overline{\Pi}(\mathcal{E}_m)(z_0)=\overline{\Pi}(\mathcal{E}_m)(z_1) = \overline{\Pi}(\mathcal{E}_m)(z_2)=0.2$

$\overline{\Pi}(\mathcal{E}_n)(z_0)=0.5$, $\overline{\Pi}(\mathcal{E}_n)(z_1)=\overline{\Pi}(\mathcal{E}_n)(z_1)=0.4$

$\overline{\Pi}(\mathcal{E}_h)(z_0)=\overline{\Pi}(\mathcal{E}_n)(z_1)=\overline{\Pi}(\mathcal{E}_n)(z_1)=0.3$

then $\underline{\Pi}(\mathcal{E})$ and $\overline{\Pi}(\mathcal{E})$ are 𝕊𝔽*BI* of ℵ . hence $\mathcal{E}$ is a ℝ𝕊𝔽*BI* of ℵ

Theorem 4.6. Let $\mathcal{E}$ be a 𝕊𝔽*BI* of ℵ. Then $\mathcal{E}$ be a ℝ𝕊𝔽*BI* of ℵ

Proof : Assume that $\mathcal{E}$ be a $\mathbb{SF}BI$ of $\aleph$. we want to prove $\mathcal{E}$ is $\mathbb{RSF}BI$. For that we have to prove
$\Pi^{-}(\mathcal{E})$ and $\Pi^{+}(\mathcal{E})$ are $\mathbb{SF}BI$ of $\aleph$. Let i, j, p $\in \aleph$
Then we consider

$\Pi^{-}(\mathcal{E}_m)(i-j) = \wedge_{s\in[i-j]_\Pi} \mathcal{E}_m(s)$
$= \wedge_{s\in[i]_\Pi+[-j]_\Pi} \mathcal{E}_m(s)$
$=\wedge_{k-q\in[i]_\Pi+[-j]_\Pi} \mathcal{E}_m(k\text{-}q)$
$= \wedge_{k\in[i]_\Pi, q\in[-j]_\Pi} \mathcal{E}_m(k) \wedge \mathcal{E}_m(q)$
$=(\wedge_{k\in[i]_\Pi} \mathcal{E}_m(k)) \wedge (\wedge_{q\in[-j]_\Pi} \mathcal{E}_m(q))$
$= \Pi^{-}(\mathcal{E}_m)(i) \wedge \Pi^{-}(\mathcal{E}_m)(-j)$
$\geq \Pi^{-}(\mathcal{E}_m)(i) \wedge \Pi^{-}(\mathcal{E}_m)(j)$

Moreover

$\Pi^{-}(\mathcal{E}_m)(ijk)=\wedge_{s\in[ijk]_\Pi} \mathcal{E}_m(s)$
$=\wedge_{s\in[i]_\Pi[j]_\Pi[k]_\Pi} \mathcal{E}_m(s)$
$=\wedge_{pqr\in[i]_\Pi[j]_\Pi[k]_\Pi} \mathcal{E}_m(pqr)$
$\geq \wedge_{p\in[i]_\Pi r\in[k]_\Pi} \min\{\mathcal{E}_m(p), \mathcal{E}_m(r)\}$
$= \min\{\wedge_{p\in[i]_\Pi} \mathcal{E}_m(p), \wedge_{r\in[k]_\Pi} \mathcal{E}_m(r)\}$
$\min\{\Pi^{-}(\mathcal{E}_m)(i), \Pi^{-}(\mathcal{E}_m)(k)\}$

similarly we can prove the other case

$\Pi^{-}(\mathcal{E}_n)(i-j) \geq \Pi^{-}(\mathcal{E}_n)(i) \wedge \Pi^{-}(\mathcal{E}_n)(j)$
$\Pi^{-}(\mathcal{E}_n)(ijk) \geq \min\{\Pi^{-}(\mathcal{E}_n)(i), \Pi^{-}(\mathcal{E}_n)(k)\}$

and

$\Pi^{-}(\mathcal{E}_h)(i-j) \geq \Pi^{-}(\mathcal{E}_h)(i) \wedge \Pi^{-}(\mathcal{E}_h)(j)$
$\Pi^{-}(\mathcal{E}_h)(ijk) \geq \min\{\Pi^{-}(\mathcal{E}_h)(i), \Pi^{-}(\mathcal{E}_h)(k)\}$

Hence $\Pi^{-}(\mathcal{E})$ is $\mathbb{SF}BI$ of $\aleph$. consequently we can prove $\Pi^{+}(\mathcal{E})$ is $\mathbb{SF}BI$ of $\aleph$. Thus $\mathcal{E}$ is a $\mathbb{RSF}BI$ of $\aleph$.

Theorem 4.7. If $\mathcal{E}$ *and* $\mathcal{F}$ are $\mathbb{RSF}BI$ of $\aleph$. Then $\mathcal{E} \cap \mathcal{F}$ is $\mathbb{RSF}BI$ of $\aleph$.

Proof. Since $\mathcal{E}$ and $\mathcal{F}$ are $\mathbb{RSF}BI$ of $\aleph$. Then for all i, j $\in \aleph$ we consider

$(\Pi^{-}(\mathcal{E}_m) \cap \Pi^{-}(\mathcal{F}_m))(i-j)$
$=(\Pi^{-}(\mathcal{F}_m))(i-j)\wedge(\Pi^{-}(\mathcal{E}_m)(i-j))$
$\geq (\Pi^{-}(\mathcal{E}_m)(i)\wedge \Pi^{-}(\mathcal{E}_m)(j))\wedge (\Pi^{-}(\mathcal{F}_m)(i) \wedge\Pi^{-}(\mathcal{F}_m)(j))$
$=(\Pi^{-}(\mathcal{E}_m)(i)\wedge \Pi^{-}(\mathcal{F}_m)(i))\wedge (\Pi^{-}(\mathcal{E}_m)(j) \wedge\Pi^{-}(\mathcal{F}_m)(j))$
$\geq (\Pi^{-}(\mathcal{E}_m) \cap (\mathcal{F}_m))(i) \wedge (\Pi^{-}(\mathcal{E}_m) \cap (\mathcal{F}_m))(j)$

Moreover

$(\Pi^{-}(\mathcal{E}_m) \cap \Pi^{-}(\mathcal{F}_m))(ijk)$
$=(\Pi^{-}(\mathcal{E}_m))(ijk)\wedge(\Pi^{-}(\mathcal{F}_m)(ijk))$
$\geq (\Pi^{-}(\mathcal{E}_m)(i)\wedge \Pi^{-}(\mathcal{E}_m)(k))\wedge (\Pi^{-}(\mathcal{F}_m)(i) \wedge\Pi^{-}(\mathcal{F}_m)(k))$
$\geq (\Pi^{-}(\mathcal{E}_m) \cap (\mathcal{F}_m))(i) \wedge (\Pi^{-}(\mathcal{E}_m) \cap (\mathcal{F}_m))(k)$

For nonmembership function

$(\Pi^{-}(\mathcal{E}_n) \cap \Pi^{-}(\mathcal{F}_n))(i-j)$

$=(\Pi^{-}(\mathcal{F}_n))(\mathrm{i-j})\wedge(\Pi^{-}(\mathcal{E}_n)(i-j))$
$\geq(\Pi^{-}(\mathcal{E}_n)(i)\wedge\Pi^{-}(\mathcal{E}_n)(j))\wedge(\Pi^{-}(\mathcal{F}_n)(i)\wedge\Pi^{-}(\mathcal{F}_n)(j))$
$=(\Pi^{-}(\mathcal{E}_n)(i)\wedge\Pi^{-}(\mathcal{F}_n)(i))\wedge(\Pi^{-}(\mathcal{E}_n)(j)\wedge\Pi^{-}(\mathcal{F}_n)(j))$
$\geq(\Pi^{-}(\mathcal{E}_n)\cap(\mathcal{F}_n))(i)\wedge(\Pi^{-}(\mathcal{E}_n)\cap(\mathcal{F}_n))(j)$
Also
$(\Pi^{-}(\mathcal{E}_n)\cap\Pi^{-}(\mathcal{F}_n))(\mathrm{i}jk)$
$=(\Pi^{-}(\mathcal{E}_n))(\mathrm{ijk})\wedge(\Pi^{-}(\mathcal{F}_n)(ijk))$
$\geq(\Pi^{-}(\mathcal{E}_n)(i)\wedge\Pi^{-}(\mathcal{E}_n)(k))\wedge(\Pi^{-}(\mathcal{F}_n)(i)\wedge\Pi^{-}(\mathcal{F}_n)(k))$
$\geq(\Pi^{-}(\mathcal{E}_n)\cap(\mathcal{F}_n))(i)\wedge(\Pi^{-}(\mathcal{E}_n)\cap(\mathcal{F}_n))(k)$
Finally for hesitancy function
$(\Pi^{-}(\mathcal{E}_h)\cap\Pi^{-}(\mathcal{F}_h))(\mathrm{i-j})$
$=(\Pi^{-}(\mathcal{E}_h))(\mathrm{i-j})\wedge(\Pi^{-}(\mathcal{F}_h)(i-j))$
$\geq(\Pi^{-}(\mathcal{E}_h)(i)\vee\Pi^{-}(\mathcal{E}_h)(j))\wedge(\Pi^{-}(\mathcal{F}_h)(i)\vee\Pi^{-}(\mathcal{F}_h)(j))$
$=(\Pi^{-}(\mathcal{E}_h)(i)\wedge\Pi^{-}(\mathcal{F}_h)(i))\vee(\Pi^{-}(\mathcal{E}_h)(j)\wedge\Pi^{-}(\mathcal{F}_h)(j))$
$\geq(\Pi^{-}(\mathcal{E}_h)\cap(\mathcal{F}_h))(i)\vee(\Pi^{-}(\mathcal{E}_h)\cap(\mathcal{F}_h))(j)$
Also
$(\Pi^{-}(\mathcal{E}_h)\cap\Pi^{-}(\mathcal{F}_h))(\mathrm{i}jk)$
$=(\Pi^{-}(\mathcal{E}_h))(\mathrm{ijk})\wedge(\Pi^{-}(\mathcal{F}_h)(ijk))$
$\geq(\Pi^{-}(\mathcal{E}_h)(i)\vee\Pi^{-}(\mathcal{E}_h)(k))\wedge(\Pi^{-}(\mathcal{F}_h)(i)\vee\Pi^{-}(\mathcal{F}_h)(k))$
$\geq(\Pi^{-}(\mathcal{E}_h)\cap(\mathcal{F}_h))(i)\vee(\Pi^{-}(\mathcal{E}_h)\cap(\mathcal{F}_h))(k)$
Therefore $\Pi^{-}(\mathcal{E}_m)\cap\Pi^{-}(\mathcal{F}_m)$ is $\mathbb{SF}BI$ of $\aleph$. Similarly we can prove for $\Pi^{+}(\mathcal{E}_m)\cap\Pi^{+}(\mathcal{F}_m)$.Consequently
we prove for the remaining cases .Hence $\mathcal{E}\cap\mathcal{F}$ is $\mathbb{RSF}BI$ of $\aleph$.

5.CONCLUSION

Spherical fuzzy sets are an attempt to provide a general view to three dimensional fuzzy sets. To investigate the structure of an algebraic system, we see that the spherical fuzzy ideals on near ring with special properties always play a central role. The purpose of this paper is to initiated the concept of spherical fuzzy bi-ideals and rough spherical fuzzy bi-ideals on near rings. Some characterizations of rough spherical fuzzy bi-ideals are obtained on near ring. Our future work is to extend this idea to other algebraic domain such as semi hyper near ring, semigroup etc.

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