

Calculation Of Mean: Utility Of Kandari's Formula

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Abstract

In this research work, a statistical formula known as Kandari's formula is devised. In comparison to traditional methods, Kandari's formula offers a faster and easier way to determine the mean of grouped data. Traditional approaches such as the assumed mean method and step-deviation method, which claim to make difficult computations simple, have a well-known flaw: they are rather lengthy. Furthermore, these procedures are rather complex. This is when Kandari's formula comes into play. In comparison to other techniques of finding the mean of grouped data, Kandari's formula is simple to learn, brief, and easy to remember. In the case of large values, Kandari's formula is also much faster and easier than the step deviation technique. Here, Kandari's method fixes this flaw and promises to cut out hard calculations that aren't needed and speed up the whole process.

Keywords: Mean, Grouped data, Complex values,

INTRODUCTION

The mean (or average) of observations, as we know, is the sum of the values of all the observations divided by the total number of observations. We can calculate the mean of grouped and ungrouped data. In this research paper, we will only take grouped data into consideration and not the ungrouped data. In most of our real-life situations, data is usually so large that to make a meaningful study it needs to be condensed as grouped data. So, we need to convert given ungrouped data into grouped data and devise some method to find its mean. There are three methods to calculate mean of grouped data, Direct method, Assumed Mean Method, and Step-Deviation Method. Direct method is used when the given data is small. Assumed mean method is used for intermediate data and Step deviation method is used for huge data. Step deviation method helps you to make the division and multiplication easier, than the Assumed mean method does. Also, these methods are quite lengthy. This is the situation where Kandari's formula is needed. Kandari's formula is easy to understand, short and very easy to execute, as compared to the other methods of calculating mean of grouped data. Kandari's formula is also a lot faster and easier in case of huge values, as compared to Step deviation method.

I. SYMBOLS USED

1. $i = i$ is a serial number for class intervals; the value of i varies as we go down through the number of class intervals.
 - a. Minimum value of $i = 1$ and maximum value of $i =$ total number of class intervals denoted by n
2. $a =$ assumed mean, which is also a class-mark
3. $k =$ it is that value of i , which defines the position of the class interval whose class-mark is taken as assumed mean (a)
4. $h =$ class size
5. $L_i =$ lower limit of class interval i
6. $U_i =$ upper limit of the class interval i
7. $L_a =$ lower limit of the class interval whose class mark is taken as the assumed mean (a). Note that, L_a can be represented in the form of $L_i + (k - i)h$, such that, $L_a = L_i + (k - i)h$
8. $U_a =$ upper limit of the class-interval whose lower limit is L_a
9. $F_i =$ it is the frequency of class interval i . Here in this paper, it is also denoted as f_i
10. $x_i =$ class mark of class interval i
11. $d_i =$ deviation of the class interval i
12. $u_i =$ it is the step-deviation of class interval i
13. $\bar{x} =$ it represents the mean
14. $\bar{u} =$ it represents the mean of step-deviations (u_i)

II. PRE-EXISTING FORMULAS:

1. $d_i = x_i - a$
2. Class mark = $\frac{\text{Upper class limit} + \text{Lower class limit}}{2}$
3. $h =$ upper class limit - lower class limit
 \Rightarrow Upper class limit = lower class limit + class size
4. $u_i = \frac{x_i - a}{h} = \frac{d_i}{h}$
5. $\bar{x} = a + h\bar{u}$
6. $\bar{u} = \frac{\sum_{i=1}^n f_i u_i}{\sum_{i=1}^n f_i}$

III. DERIVATION OF KANDARI'S FORMULA

(Part 1) We know that $d_i = x_i - a$

$$\begin{aligned}
 \Rightarrow d_i &= \frac{L_i + U_i}{2} - \frac{L_a + U_a}{2} \\
 &= \frac{L_i + (L_i + h)}{2} - \frac{L_a + (L_a + h)}{2} \\
 &= \frac{2L_i + h}{2} - \frac{2L_a + h}{2} \\
 &= \frac{2L_i + h - (2L_a + h)}{2}
 \end{aligned}$$

$$= \frac{2L_i - 2L_a}{2}$$

$$= L_i - L_a$$

Hence, $d_i = L_i - L_a$

Now, we have, $d_i = L_i - L_a$

$$= L_i - [L_i + (k - i)h]$$

$$= L_i - L_i - (k - i)h$$

$$= -(k - i)h$$

$$\therefore d_i = (i - k)h$$

(Part 2) We also know that, $u_i = \frac{x_i - a}{h} = \frac{d_i}{h}$

$$\Rightarrow u_i = \frac{(i - k)h}{h} = i - k$$

$$\therefore u_i = i - k$$

Therefore, $F_i u_i$ can be expressed as $F_i(i - k)$, i.e. $F_i u_i = F_i(i - k)$

Since, $F_i u_i = F_i(i - k)$

$$\Rightarrow \sum_{i=1}^n F_i u_i = \sum_{i=1}^n F_i(i - k)$$

$$\Rightarrow \sum_{i=1}^n F_i u_i = \sum_{i=1}^n (F_i i - F_i k)$$

$$\Rightarrow \sum_{i=1}^n F_i u_i = \sum_{i=1}^n F_i i - \sum_{i=1}^n F_i k$$

$$\Rightarrow \sum_{i=1}^n F_i u_i = [F_1(1) + F_2(2) + F_3(3) + \dots + F_n(n)] - k \left(\sum_{i=1}^n F_i \right)$$

$$= \left[\sum_{i=1}^n (F_i) + F_2(1) + F_3(2) + \dots + F_n(n - 1) \right] - k \left(\sum_{i=1}^n F_i \right)$$

$$= \left[\sum_{i=1}^n F_1 + \sum_{i=2}^n F_i + F_3(1) + \dots + F_n(n - 2) \right] - k \left(\sum_{i=1}^n F_i \right)$$

$$= \left[\sum_{i=1}^n F_1 + \sum_{i=2}^n F_i + \sum_{i=3}^n F_i + F_4(1) + \dots + F_n(n - 3) \right] - k \left(\sum_{i=1}^n F_i \right)$$

$$= \left[\sum_{i=1}^n F_i + \sum_{i=2}^n F_i + \sum_{i=3}^n F_i + \sum_{i=4}^n F_i + \dots + \sum_{i=n}^n F_i \right] - k \left(\sum_{i=1}^n F_i \right)$$

$$= \left[\sum_{i=1}^n F_i + \left(\sum_{i=1}^n F_i - \sum_{i=1}^1 F_i \right) + \left(\sum_{i=1}^n F_i - \sum_{i=1}^2 F_i \right) + \left(\sum_{i=1}^n F_i - \sum_{i=1}^3 F_i \right) + \dots \right.$$

$$\left. + \left(\sum_{i=1}^n F_i - \sum_{i=1}^{n-1} F_i \right) \right] - k \left(\sum_{i=1}^n F_i \right)$$

$$\begin{aligned}
 &= \left[n \binom{n}{i=1} F_i - \sum_{i=1}^1 F_i - \sum_{i=1}^2 F_i - \sum_{i=1}^3 F_i - \cdots - \sum_{i=1}^{n-1} F_i \right] - k \binom{n}{i=1} F_i \\
 &= \left[n \binom{n}{i=1} F_i - \left(\sum_{i=1}^1 F_i + \sum_{i=1}^2 F_i + \sum_{i=1}^3 F_i + \cdots + \sum_{i=1}^{n-1} F_i \right) \right] - k \binom{n}{i=1} F_i \\
 &= n \binom{n}{i=1} F_i - \left(\sum_{i=1}^1 F_i + \sum_{i=1}^2 F_i + \sum_{i=1}^3 F_i + \cdots + \sum_{i=1}^{n-1} F_i \right) - k \binom{n}{i=1} F_i \\
 &= n \binom{n}{i=1} F_i - k \binom{n}{i=1} F_i - \left(\sum_{i=1}^1 F_i + \sum_{i=1}^2 F_i + \sum_{i=1}^3 F_i + \cdots + \sum_{i=1}^{n-1} F_i \right) \\
 &= (n - k) \binom{n}{i=1} F_i - \left(\sum_{i=1}^1 F_i + \sum_{i=1}^2 F_i + \sum_{i=1}^3 F_i + \cdots + \sum_{i=1}^{n-1} F_i \right) \\
 &= (n - k) \binom{n}{i=1} F_i - [(n - 1)F_1 + (n - 2)F_2 + (n - 3)F_3 + \cdots + (n - (n - 1))F_{n-1}] \\
 &= (n - k) \binom{n}{i=1} F_i - [(n - 1)F_1 + (n - 2)F_2 + (n - 3)F_3 + \cdots + F_{n-1}] \\
 &= (n - k) \binom{n}{i=1} F_i \\
 &\quad - \left\{ (n - 1) \binom{n}{i=1} F_i \right. \\
 &\quad \left. - [(1)F_2 + (2)F_3 + \cdots + (n - 3)F_{n-2} + (n - 2)F_{n-1} + (n - 1)F_n] \right\} \\
 &= (n - k) \binom{n}{i=1} F_i - (n - 1) \binom{n}{i=1} F_i \\
 &\quad + [(1)F_2 + (2)F_3 + \cdots + (n - 3)F_{n-2} + (n - 2)F_{n-1} + (n - 1)F_n] \\
 &= (1 - k) \binom{n}{i=1} F_i + [(1)F_2 + (2)F_3 + \cdots + (n - 3)F_{n-2} + (n - 2)F_{n-1} + (n - 1)F_n] \\
 &= [(1)F_2 + (2)F_3 + \cdots + (n - 3)F_{n-2} + (n - 2)F_{n-1} + (n - 1)F_n] - (k - 1) \binom{n}{i=1} F_i \\
 &= \sum_{i=2}^n (i - 1)F_i - (k - 1) \binom{n}{i=1} F_i \\
 \therefore \sum_{i=1}^n F_i u_i &= \sum_{i=2}^n (i - 1)F_i - (k - 1) \binom{n}{i=1} F_i
 \end{aligned}$$

(Part 3) We know that $\bar{x} = a + h\bar{u}$

$$\begin{aligned}
 & \Rightarrow \bar{x} = a + h \left(\frac{\sum_{i=1}^n F_i u_i}{\sum_{i=1}^n F_i} \right) \\
 & = a + h \left(\frac{\sum_{i=2}^n (i-1)F_i - (k-1)(\sum_{i=1}^n F_i)}{\sum_{i=1}^n F_i} \right) \\
 & = \left(\frac{U_a + L_a}{2} \right) + h \left(\frac{\sum_{i=2}^n (i-1)F_i - (k-1)(\sum_{i=1}^n F_i)}{\sum_{i=1}^n F_i} \right) \\
 & = \left(\frac{U_a + L_a}{2} \right) + \frac{h[\sum_{i=2}^n (i-1)F_i - (k-1)(\sum_{i=1}^n F_i)]}{\sum_{i=1}^n F_i} \\
 & = \frac{(U_a + L_a)(\sum_{i=1}^n F_i) + 2h[\sum_{i=2}^n (i-1)F_i - (k-1)(\sum_{i=1}^n F_i)]}{2(\sum_{i=1}^n F_i)} \\
 & = \frac{U_a(\sum_{i=1}^n F_i) + L_a(\sum_{i=1}^n F_i) + 2h[\sum_{i=2}^n (i-1)F_i] - 2h(k-1)(\sum_{i=1}^n F_i)}{2(\sum_{i=1}^n F_i)} \\
 & = \frac{U_a(\sum_{i=1}^n F_i) + L_a(\sum_{i=1}^n F_i) + 2h[\sum_{i=2}^n (i-1)F_i] - 2(U_a - L_a)(k-1)(\sum_{i=1}^n F_i)}{2(\sum_{i=1}^n F_i)} \\
 & = \frac{U_a(\sum_{i=1}^n F_i) + L_a(\sum_{i=1}^n F_i) + 2h[\sum_{i=2}^n (i-1)F_i] - 2k(U_a - L_a)(\sum_{i=1}^n F_i) + 2(U_a - L_a)(\sum_{i=1}^n F_i)}{2(\sum_{i=1}^n F_i)} \\
 & = \frac{U_a(\sum_{i=1}^n F_i) + L_a(\sum_{i=1}^n F_i) + 2h[\sum_{i=2}^n (i-1)F_i] - 2k(U_a - L_a)(\sum_{i=1}^n F_i) + 2U_a(\sum_{i=1}^n F_i) - 2L_a(\sum_{i=1}^n F_i)}{2(\sum_{i=1}^n F_i)} \\
 & = \frac{2h[\sum_{i=2}^n (i-1)F_i] + 2U_a(\sum_{i=1}^n F_i) - 2k(U_a - L_a)(\sum_{i=1}^n F_i) + U_a(\sum_{i=1}^n F_i) - 2L_a(\sum_{i=1}^n F_i) + L_a(\sum_{i=1}^n F_i)}{2(\sum_{i=1}^n F_i)} \\
 & = \frac{2h[\sum_{i=2}^n (i-1)F_i] + 2U_a(\sum_{i=1}^n F_i) - 2k(U_a - L_a)(\sum_{i=1}^n F_i) + U_a(\sum_{i=1}^n F_i) - L_a(\sum_{i=1}^n F_i)}{2(\sum_{i=1}^n F_i)} \\
 & = \frac{2h[\sum_{i=2}^n (i-1)F_i] + 2U_a(\sum_{i=1}^n F_i) - 2k(U_a - L_a)(\sum_{i=1}^n F_i) + (U_a - L_a)(\sum_{i=1}^n F_i)}{2(\sum_{i=1}^n F_i)} \\
 & = \frac{2h[\sum_{i=2}^n (i-1)F_i] + 2U_a(\sum_{i=1}^n F_i) - (U_a - L_a)(\sum_{i=1}^n F_i)[2k-1]}{2(\sum_{i=1}^n F_i)} \\
 & = \frac{2h[\sum_{i=2}^n (i-1)F_i] + 2U_a(\sum_{i=1}^n F_i) - (U_a - L_a)(\sum_{i=1}^n F_i)[2k-1]}{2(\sum_{i=1}^n F_i)} \\
 & = \frac{h[\sum_{i=2}^n (i-1)F_i] + U_a(\sum_{i=1}^n F_i) - (U_a - L_a)[2k-1]}{\sum_{i=1}^n F_i} \\
 & = \frac{h[\sum_{i=2}^n (i-1)F_i] + U_a(\sum_{i=1}^n F_i)}{\sum_{i=1}^n F_i} - \frac{h[2k-1]}{2} \\
 & = \frac{h[\sum_{i=2}^n (i-1)F_i]}{\sum_{i=1}^n F_i} + \frac{U_a(\sum_{i=1}^n F_i)}{\sum_{i=1}^n F_i} - \frac{h[2k-1]}{2} \\
 & = \frac{h[\sum_{i=2}^n (i-1)F_i]}{\sum_{i=1}^n F_i} + U_a - \frac{h(2k-1)}{2} \\
 & = h \left(\frac{\sum_{i=2}^n (i-1)F_i}{\sum_{i=1}^n F_i} - \frac{2k-1}{2} \right) + U_a \\
 & = h \left(\frac{\sum_{i=2}^n (i-1)F_i}{\sum_{i=1}^n F_i} - \left(k - \frac{1}{2} \right) \right) + U_a \\
 & \therefore \bar{x} = h \left(\frac{\sum_{i=2}^n (i-1)F_i}{\sum_{i=1}^n F_i} - (k - 0.5) \right) + U_a
 \end{aligned}$$

It is named based on the name of the person who devised it.

$$\text{Kandari's Formula: } \bar{x} = h \left(\frac{\sum_{i=2}^n (i-1)F_i}{\sum_{i=1}^n F_i} - (k - 0.5) \right) + U_a$$

Difference between Kandari's formula and traditional method of calculating mean of any grouped data:

In the event of solving the problem using the Traditional technique, the table must contain six columns, and the formula for computing the corresponding mean must be applied using three columns. In the case of Kandari's formula, just four columns are necessary, and calculations are performed using a single column to immediately apply the formula. Clearly, the method of Kandari's formula for finding the mean of grouped data is considerably simpler, easier, and faster than the old ways.

IV. EXAMPLES OF KANDARI'S FORMULA

Each example is first solved by a traditional method and then by Kandari's formula.

Example 1: Consider the following distribution of daily wages of 50 workers of a factory. Find the mean daily wages of the workers of the factory by using an appropriate method.

Table:01 Data Of 50 Workers

Daily wages	No. of workers
100-120	12
120-140	14
140-160	8
160-180	6
180-200	10

Let us first solve this problem using the traditional method.

Solution (i): In this case, we can use **step-deviation method**.

Here, $a = 150$ and $h = 20$.

Table 02: The mean daily wages of the workers of the factory by step-deviation method.

Serial Number	Class interval	Frequency (f_i)	Class marks (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
1	100-120	12	110	-2	-24
2	120-140	14	130	-1	-14
3	140-160	8	$150 = a$	0	0
4	160-180	6	170	1	6
5	180-200	10	190	2	20
		$\sum f_i = 50$			$\sum f_i u_i = -12$

$$\therefore \text{Mean, } \bar{x} = a + h \left(\frac{\sum f_i u_i}{\sum f_i} \right)$$

$$= 150 + 20 \left(\frac{-12}{50} \right) = 150 - \frac{240}{50} = 150 - 4.8 = 145.2$$

Hence, mean daily wages of the workers is 145.2.

Let us now solve the same question using Kandari's Formula.

Solution (ii): Let $U_a = 160$, $k = 3$ and $h = 20$.

Note: Since assumed mean is randomly chosen, we can randomly choose the class interval and its serial number when using Kandari's formula. We use the upper limit of the class interval (whose class mark is assumed mean) and corresponding serial number.

Table 03: The mean daily wages of the workers of the factory by Kandari's formula.

Serial Number (i)	Class interval	Frequency (f_i)	$(i - 1)f_i$
1	100-120	12	(Not required)
2	120-140	14	$(2 - 1)14 = 14$
3	140-160	8	$(3 - 1)8 = 16$
4	160-180	6	$(4 - 1)6 = 18$
5	180-200	10	$(5 - 1)10 = 40$
		$\sum f_i = 50$	$\sum_{i=2}^n (i - 1)F_i = 88$

$$\begin{aligned} \text{Mean, } \bar{x} &= h \left(\frac{\sum_{i=2}^n (i-1)f_i}{\sum_{i=1}^n f_i} - (k - 0.5) \right) + U_a \\ &= 20 \left(\frac{88}{50} - (3 - 0.5) \right) + 160 = 20(1.76 - 2.5) + 160 = 20(-0.74) + 160 \\ &= -14.8 + 160 = 145.2 \end{aligned}$$

It is clearly visible that Kandari's formula is shorter than the Step-deviation method and easy to follow.

Example 2: The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states of a country. Find the mean percentage of female teachers.

Table 04: percentage of female teachers

Percentage of female teachers	Number of states
15-25	6
25-35	11
35-45	7
45-55	4
55-65	4
65-75	2
75-85	1

Let us first solve this problem using the traditional method.

Solution (i): In this case, we shall use Assumed mean method. Here, $a = 50$ and $h = 10$.

Table 05: Calculation of mean percentage of female teachers by traditional method

Serial Number	Percentage of female teachers	Number of states (f_i)	Class marks (x_i)	$d_i = x_i - a$	$f_i d_i$
1	15-25	6	20	-30	-180
2	25-35	11	30	-20	-220
3	35-45	7	40	-10	-70
4	45-55	4	50 = a	0	0
5	55-65	4	60	10	40
6	65-75	2	70	20	40
7	75-85	1	80	30	30
		$\sum f_i = 35$			$\sum f_i d_i = -360$

$$\begin{aligned} \therefore \text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i d_i}{\sum f_i} \right) \\ &= 50 + \left(\frac{-360}{35} \right) = 50 - \frac{360}{35} = 50 - 10.29 = 39.71 \end{aligned}$$

Therefore, the mean percentage of female teachers in the primary schools of rural areas is 39.71.

Let us now solve the same question using Kandari's Formula.

Solution (ii): Let $U_a = 55$, $k = 4$ and $h = 10$.

Table 06: Calculation of mean percentage of female teachers by using Kandari's formula

Serial Number	Percentage of female teachers	Number of states (f_i)	$(i - 1)f_i$
1	15-25	6	(Not required)
2	25-35	11	$(2 - 1)11 = 11$
3	35-45	7	$(3 - 1)7 = 14$
4	45-55	4	$(4 - 1)4 = 12$
5	55-65	4	$(5 - 1)4 = 16$
6	65-75	2	$(6 - 1)2 = 10$
7	75-85	1	$(7 - 1)1 = 6$
		$\sum f_i = 35$	$\sum_{i=2}^n (i - 1)F_i = 69$

$$\text{Mean, } \bar{x} = h \left(\frac{\sum_{i=2}^n (i-1)f_i}{\sum_{i=1}^n f_i} - (k - 0.5) \right) + U_a$$

$$\begin{aligned} &= 10 \left(\frac{69}{35} - (4 - 0.5) \right) + 55 = 10(1.971 - 3.5) + 55 = 10(-1.529) + 55 \\ &= -15.29 + 55 = 39.71 \end{aligned}$$

It is clearly visible that Kandari's formula is shorter than the Assumed mean method and easy to follow.

V. CONCLUSION

It is well known that mental calculation helps us improve our cognitive functions. Due to this reason many people, even in the age of computers, keep improving their mental math skills. Kandari's formula enables people to calculate the mean of grouped data, comprising of huge values, in their head. Kandari's formula is faster and easier than any other traditional method of calculating mean of grouped data. This method just requires one column to be calculated, which is much easier and faster as compared to other traditional methods for huge values, where you are required to calculate more than two columns.

CONFLICT OF INTEREST Author has declared that no competing interest exists.

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