“A STUDY OF UNIFORM CONVERGENCE OF SERIES AND SEQUENCE OF
FUNCTION”

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Abstract
In the current paper auth or gives the relation between uniform convergence and series of fuzzy
value function in which he the find the uniform convergence of fuzzy valued function sequence
and it’s series by giving some examples for that he first evaluation some differentiation and
Hen stock integration of some function especially. Which was fuzzy valued and discuss some
important inferences in this he also discusses some series including power series having
coefficient of fuzzy function and again he gives brief description on radius of convergence
related to power series and by doing all this at finally he proposed the interaction between
uniform convergence and value of series function. In that arithmetic properties he included
Addition of sequences, Subtraction, Multiplication and division of Sequences in this property
he showed that how that properties work at end further more he focused on convergent
sequence boundless.

Keyword – learnability uniform convergence, function, convergence sequence, statistical
convergence, fuzzy number.

Introduction –
In ancient time in the field of uniform convergence there was a study based on contributions
of stokes theorem, Cauchy Integral formal serial theorem even though the contributions in the
mathematics separately with the concept of uniform convergence is discussed in very large
extent whose study is depends on popular article but there was one mathematician known as
watercress searched strong foundations of different function especially elliptical and analytical
function in the uniform convergence history the important lectures of watercress which he gave
in the university of Bedim in the mid of 1880’s gave separate branched in the development of
uniform convergence so that’s why watercress was consider as the 1st person who gave
introduction about uniform convergence.

Sequence is important for the infinite counting of the objects. It is considered as the one of the
keys used for the dialysis which generally seen everywhere we can used if for explaining the
function. That is the function is continuous or not also to can able to find closeness of sets in
case of metric space. If we find the limit of sequence it explained us. What is the end of the list
especially I to its value if it in some sense approaches? This value this is consider as one of the
applications of convergence there is one another natural application of convergence called limit
of function or a limit of the sequence of function.
So, in this way we can describe the uniform convergence which was nothing but one of the ways of convergence of a function which have stranger that the other convergence specially points wise convergence and the sequence of the function which we can denote by $\text{in}$ is only uniformly converges when there is a limiting truncation of to the group of $E$ if there is any positive small integer or a number like $E$ then a value of $N$. We can get from each of those functions of $f$. Which was change less or equal to value of $E$ for each point in the group of $E$ if we described it in another way it will be like in converges to $f$ uniformly then the role of $f$ is approaches to $f(x)$. Which was uniformly converges thorough the domain.

In terms of generalization someone can directly expand the characteristics of functions. Which for warded from $E – M$ in which $M$ is the Metric space. The usual setting in the form of general term of uniform convergence of nets of function. Which goes from $E$ to $X$ so we can say that the net $f_\alpha$ uniformly converges with a limit function $f$ compounds from $E$ to $X$ if for entourage $\lambda(x)$ there exist $\alpha_0$ such that for each $x$ in $E$ for every $\alpha > \alpha_0$ ($f_\alpha (x)$) in this type of functions.

The limit of uniform convergence is seen to be continuous which remains instant?

On the basis of this we studied one theorem in which of there eruct any unique solution $s(x)$ to second order derivative of second case in fact they given respectively by the series.

$$\sum_{j=0}^{\infty} \frac{(-1)^j x^{2j}}{(2j)!} = \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

And

$$\sum_{j=0}^{\infty} \frac{(-1)^j x^{2j+1}}{(2j+1)!} = \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

Where the convergence is uniform on any bounded interval $(x) & s(x)$ and smooth on $R$.

Cauchy sequence gives

$$C(x) = \sum_{j=0}^{\infty} \frac{(-1)^j x^{2j}}{(2j)!}$$

$$s(x) = \sum_{j=1}^{\infty} \frac{(-1)^j x^{2j+1}}{(2j+1)!}$$

on $[-M, M]$, $M \geq 0$ we claim that there series converge uniformly for $c(x)$ we have

$$\left|\frac{(-1)^j x^{2j}}{(2j)!}\right| < aj = \frac{m^{2j}}{(2j)!}$$

It is know that $\sum ja_j <00$ by $M – hrt$ the conclusion follows in participate

$$X = \sum \frac{(-1)^j x^{2j}}{(2j)!}$$

is convergent for every $x \in R$ same result hold for $s(n)$.

**Formal definition and notation**

In a different direction, Mursaleen introduced a more general version of statistical convergence namely $\lambda$-statistical convergence. For a non-decreasing sequence $\lambda = (\lambda_n) n \in \omega$ of positive numbers with $\lambda_{n+1} \leq \lambda_n + 1, \lambda_1 = 1, \lambda_n \to \infty$ as $n \to \infty$ if we denote $In = [n - \lambda_n + 1, n]$, then a real sequence $(x_k)_{k \in \omega}$ is said to be $\lambda$ statistically convergent to $x$ if

$$\lim_{n \to \infty} \frac{1}{\lambda_n} \left|\{k \in In : |x_k - x| \geq \epsilon\}\right| = 0$$

for every $\epsilon > 0$. The set of all $\lambda$-statistically convergent sequences is denoted by $S_x$. Subsequently a lot of interesting investigations have been done by various authors on several
related notions of this convergence. On the other hand in the notion of strongly almost convergence was introduced. A real sequence \((x_n)_{n \in \omega}\) is said to be strongly almost convergent if there exists a real number \(x\) such that

\[
\lim_{n \to \infty} f(x_n) = f(\lim_{n \to \infty} x_n)
\]

The weakest form of convergence is pointwise convergence.

Definition: \(f_n \to f\) pointwise if for every \(x\) and for every \(\varepsilon > 0\), there exists \(N\) so that if \(n \geq N\) then \(d(f_n(x), f(x)) < \varepsilon\)

**Theorem:**

Let \([a, b][a, b]\) be an interval and \(f_n\) be differentiable functions with continuous derivative on the interval. Suppose the derivatives \(f_n'\) converge uniformly to a function \(g : [a, b] \to \mathbb{R}\).

Suppose also that there exists a point \(x_0\) so that the limit \(\lim_{n \to \infty} f_n(x_0)\) exists (i.e. we have pointwise convergence at some point). Then the functions \(f_n\) converge uniformly to a differentiable function \(f\) such that the derivative \(f' = g\).

**Corollary**

Let \([a, b]\) be an interval and \(f_n\) be differentiable functions with continuous derivative on the interval. Suppose the series \(\sum n=1^{\infty}|f'|\sum n=1^{\infty}|f_n'|\) converges absolutely.

Suppose also that there exists a point \(X_0\) so that \(\sum n=1^{\infty}f_n(X_0)\sum n=1^{\infty}f_n(X_0)\) converges. Then the series \(\sum n \to \infty f_n \sum n \to \infty f_n\) converges uniformly on \([a, b]\) to a differentiable function \(f')\) such that:

\[
\frac{dx}{\sum n \to \infty f_n (x) = \sum n \to \infty dx f_n (x)}
\]

**Ideal Convergent Sequence Spaces of Fuzzy Star--**

A fuzzy star–shaped number is a fuzzy set \(u : R_n \to [0, 1]\), satisfying the following conditions:

(a) \(u\) is normal, that is, there exists \(t_0 \in R_n\) such that \(u(t_0) = 1\),
(b) \(u\) is upper semicontinuous,
(c) \(\text{supp } u = \text{cl}\{t \in R_n : u(t) > 0\}\) is compact,
(d) \(u\) is fuzzy star–shaped with respect to \(t\) i.e., if there exists \(t \in R_n\) such that for any \(t_0 \in R_n\) and \(\lambda \in [0, 1]\), \(u(\lambda t_0 + (1 - \lambda)t) \geq u(t_0)\).

The set of all fuzzy star–shaped numbers is denoted by \(S^n\). To define algebraic operations on \(S^n\), the notion of alpha level set needs to be stated which is defined for a fuzzy star–shaped number \(u \in S^n\) and for each \(\alpha \in [0, 1]\) as,

\[
\alpha = \{t \in R_n : u(t) \geq \alpha\}, \text{ if } \alpha \in (0, 1] \text{ supp } u, \text{ if } \alpha = 0.
\]
The linear structure on the set of all fuzzy star–shaped numbers $S^n$ structured by addition $u + v$ and scalar multiplication $\lambda u$, $\lambda \in \mathbb{R}$, is induced by $\alpha$–level sets for each $\alpha \in [0, 1]$ via the relation,

$$[u + v]_\alpha = [u]_\alpha + [v]_\alpha, \quad [\lambda u]_\alpha = \lambda [u]_\alpha.$$ 

Both the addition $u + v$ and scalar multiplication $\lambda u$ $\in S^n$. To advance and enhance the properties of this space a metric namely $d_p$ or $L^p$ metric was defined in to study the distance between two fuzzy star–shaped numbers and analyze the space with this metric in terms of metric space. The metric was defined for each $1 \leq p < \infty$, as

$$d_\infty (u, v) = \sup_{0 \leq \alpha \leq 1} d_H ([u]_\alpha, [v]_\alpha),$$

where $d_H$ is a Hausdorff metric.

Thus $d_\infty (u, v) = \lim_{p \to \infty} d_p (u, v)$ with $d_p \leq d_q$ if $p \leq q$, for all $u, v \in S^n$.

The Hausdorff metric is taken into consideration on account of it making the space of all compact sets in $\mathbb{R}^n$ a complete and a separable space, that is to say for any two sets $A, B \in K_n$, the Hausdorff distance between $A$ and $B$ is given by

$$d_H (A, B) = \max \{\sup a \in A \inf b \in B \|a - b\|, \sup b \in B \inf a \in A \|a - b\|\}.$$ 

such that $(K_n, d_H)$ is a complete and separable space. Therefore this would give an edge over the other metrics since the support of each fuzzy star–shaped numbers is compact in itself.

**Theorem 1.** A sequence $u = (u_k) \in \omega^* (S^n)$ is convergent to $u_0 \in \omega^* (S^n)$ if there exists a positive integer $N = N(\epsilon)$ such that $d_p (u_k, u) < \epsilon$ for all $k \geq N$. In this case, we write $\lim_{k \to \infty} d_p (u_k, u_0) = 0$ and $u_0$ is called the limit of the sequence $(u_k)$. If $u_0 = 0^*$ then the sequence $(u_k)$ is called null sequence. Following are sequence spaces associated with fuzzy star–shaped numbers, $c(S^n)$ = The space of all convergent sequences of fuzzy star–shaped numbers in $R^n$.

$c_0 (S^n)$ = The space of all null sequences of fuzzy star–shaped numbers in $R^n$

**Theorem 2.-** Consider the metric space $(S^n, d_p)$. Let $u \in S^n$ and $r > 0$ then the open ball cantered at $u$ with radius $r$ is defined as $B$

$$d_p (u, r) = \{v \in S^n : d_p (v, u) < r\}.$$ 

**Theorem 3.** A subset $U$ of a metric space $(S^n, d_p)$ is said to be bounded if there exists a positive real number $r > 0$ such that $U \subseteq B d_p (u, r)$ for some $u \in S^n$

**Theorem 4.** A nonempty subset $U$ of $(S^n, d_p)$ is said to be open if, for every $u \in U$ there exists $r > 0$ such that $B d_p (u, r) \subseteq U$

**Corollary 1.** A convergent sequence $(u_k) \in \omega^* (S^n)$ has a unique limit.

**Theorem 5.** Every open ball in the metric space $(S^n, d_p)$ is an open set.

Proof. Consider $B d_p (u, r)$ to be an open ball.

Let $v \in B d_p (u, r) \Rightarrow d_p (u, v) < r$.

Now put $r_1 = r - d_p (u, v) > 0$. 

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To show that $B_{d_p}(v,r1) \subseteq B_{d_p}(u,r)$. Let $w \in B_{d_p}(v,r1)$, this implies that $d_p(w,v) < r1 = r - d_p(u,v)$

$$
\Rightarrow d_p(u,v) + d_p(w,v) < r
$$

$\Rightarrow d_p(u,w) < r$

$\Rightarrow w \in B_{d_p}(u,r)$. Thus, $B_{d_p}(u,r)$ is an open set

**Ideal Convergent Sequence Spaces of Fuzzy Star–Shaped Numbers**

In 1965, fuzzy set theory was introduced initially by Zadeh as an extension of crisp set theory. He mainly focused on the convexity property of fuzzy sets because of its importance in metric definitions and their topological properties. The convex fuzzy set which is normal, upper semi-continuous and whose support set is compact is called a fuzzy number. Fuzzy number plays a significant part in the appositeness of fuzzy mathematics, and references therein. Due to the importance of the convexity and that the star–shapedness can be seen as a natural extension to this property, it has been introduced in several ways. Diamond initiated the conceptualisation of fuzzy star–shaped numbers and analyzed the properties of $L_p$–metric for $1 \leq p < \infty$ on the same. The usual convergence of the sequences of fuzzy star–shaped numbers was defined in previous section and this section deals in generalizing the notion of this standard convergence to ideal convergence.

The structure was the introduction to a new type of convergence known as the ideal convergence (I–convergence) and was presented by Kostyrko et al.1999. Later on, the notion of ideal convergence was applied on the sequences of fuzzy numbers by Kumar and Kumar as a generalization of ordinary convergence provided by Matloka. Some applications of I–convergence can be found in. In this fragment, we intend to put forward the sequence spaces $c_I(S^n)$, $c_I 0 (S^n)$ and $\ell_I \infty(S^n)$ of fuzzy star–shaped numbers in $\mathbb{R}^n$ with respect to the $L_p$–metric. Let $I$ be an admissible ideal in $\mathbb{N}$ and $u = (U_k)$ be a sequence of fuzzy star–shaped.

**References**: