"A STUDY OF UNIFORM CONVERGENCE OF SERIES AND SEQUENCE OF FUNCTION"

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Abstract
In the current paper auth or gives the relation between uniform convergence and series of fuzzy value function in which he the find the uniform convergence of fuzzy valued function sequence and it's series by giving some examples for that he first evaluation some differentiation and Hen stock integration of some function especially. Which was fuzzy valued and discuss some important inferences in this he also discusses some series including power series having coefficient of fuzzy function and again he gives brief description on radius of convergence related to power series and by doing all this at finally he proposed the interaction between uniform convergence and value of series function. In that arithmetic properties he included Addition of sequences, Subtraction, Multiplication and division of Sequences in this property he showed that how that properties work at end further more he focused on convergent sequence boundless.

Keyword – learnability uniform convergence, function, convergence sequence, statistical convergence, fuzzy number.

Introduction –

In ancient time in the field of uniform convergence there was a study based on contributions of stokes theorem, Cauchy Integral formal serial theorem even though the contributions in the mathematics separately with the concept of uniform convergence is discussed in very large extent whose study is depends on popular article but there was one mathematician known as watercress searched strong foundations of different function especially elliptical and analytical function in the uniform convergence history the important lectures of watercress which he gave in the university of Bedim in the mid of 1880's gave separate branched in the development of uniform convergence so that's why watercress was consider as the 1st person who gave introduction about uniform convergence.

Sequence is important for the infinite counting of the objects. It is considered as the one of the keys used for the dialysis which generally seen everywhere we can used if for explaining the function. That is the function is continuous or not also to can able to find closeness of sets in case of metric space. If we find the limit of sequence it explained us. What is the end of the list especially I to its value if it in some sense approaches? This value this is consider as one of the applications of convergence there is one another natural application of convergence called limit of function or a limit of the sequence of function.

Submitted: 17/12/2022 **Accepted**: 10/01/2023

So, in this way we can described the uniform convergence which was nothing but one of the ways of convergence of a function which have stranger that the other convergence specially points wise convergence and the sequence of the function which we can denote by in is only uniformly converges when there is a limiting truncation of to the group of E if there is any positive small integer or a number like E then a value of N. We can get from each of those functions of f. Which was change less or equal to value of E for each point in the group of E if we described it in another way it will be like in converges to f uniformly then the role of f is approaches to f(x). Which was uniformly converges thorough the domain.
In terms of generalization someone can directly expand the characteristics of functions. Which for warded from E – M in which M is the Metric space. The usual setting in the form of general term of uniform convergence of nets of function. Which goes from E to X so we can say that the net fa uniformly converges with a limit function f compounds from E to X if for entourage vin X there exist α0 such that for each X in E for every $\alpha > a^0$ (fa^x) in this type of functions. The limit of uniform convergence is seen to be continuous which remains instant?

On the basis of this we studied one theorem in which of there eruct any unique solution s(x) to second order derivative of second case in fact they given respectively by the series.

$$\sum_{j=0}^{00} \frac{(-1)^1 x^{2j}}{(2j)^1} = \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + - - - - -$$

And

$$\sum_{j=0}^{00} \frac{(-1)^{j-1} x^{2j-1}}{(2j-1)^1} = \text{x} \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - - - - -$$

Where the convergence is uniform on any bounded interval (x) & s(x) and smooth on R.
Cauchy sequence gives

$$C(x) = \sum_{j=0}^{00} \frac{(-1)^1 x^{2j}}{(2j)^1}$$

$$s(x) = \sum_{j=1}^{00} \frac{(-1)^{j-1} x^{2j+1}}{(2j+1)^1}$$

on [-M, M] , M70 we claim that there series converge uniformly for c(x) we have

$$\left|\frac{(-1)^j x^{2j}}{(2j)!}\right| < \text{aj} = \frac{m^{2j}}{(2j)!}$$

It is know that$\sum jaj$ <00 by M – hrt the conclusion follows in participate

$X = \sum \frac{(-1)^j x^{2j}}{(2j)!}$ is convergent for every x ϵ R same result hold for s(n).

Formal definition and notation

In a different direction, Mursaleen introduced a more general version of statistical convergence namely λ-statistical convergence. For a non-decreasing sequence λ = (λ_n) n ∈ ω of positive numbers with λn+1 ≤ λ_n+ 1, λ_1 = 1, $\lambda_n \to \infty$ as n → ∞ if we denote In = [n − λ_n + 1, n], then a real sequence (x_k)k∈ω is said to be λ statistically convergent to x if

$$\lim_{n\to\infty} \frac{1}{\lambda n} |\{k \in In : |x_k - x| \geq \varepsilon\}| = 0$$

for every ε > 0. The set of all λ-statistically convergent sequences is denoted by S_λ. Subsequently a lot of interesting investigations have been done by various authors on several related notions of this convergence. On the other hand in the notion of strongly almost convergence was introduced. A real sequence (x_n)n∈ω is said to be strongly almost convergent if there exists a real number x such that

We know that the following holds for sequences in a metric space, if f is continuous: $\lim_{n\to\infty} f(x_n)$= f($\lim_{n\to\infty} x_n$)
The weakest form of convergence is pointwise convergence.
Definition: $f_n \to f$ pointwise if for every x and for every $\epsilon > 0$, there exists N so that if n≥ N then d$(f_n(x), f(x)) < \epsilon$

Theorem :

Let [a, b][a, b] be an interval and f_n be differentiable functions with continuous derivative on the interval. Suppose the derivatives $f_n{}'$ converge uniformly to a function g:[a,b]→R.
Suppose also that there exists a point x_0 so that the limit $\lim_{n\to\infty} f_n(x_0)$ exists (i.e. we have pointwise convergence at some point). Then the functions f_n converge uniformly to a differentiable function ff such that the derivative f′=g.

Corollary

Let [a, b] be an interval and f_n be differentiable functions with continuous derivative on the interval. Suppose the series ∑n=1∞|f′n|∑n=1∞|fn′| converges absolutely.
Suppose also that there exists a point X_0 so that ∑n=1∞fn(X_0)∑n=1 f_n (X_0) converges. Then the series ∑n→∞f_0 ∑n→∞ f_n converges uniformly on [a,b] to a differentiable function ff such that:
dx ∑n→∞ f_n (x)=∑n→∞dx f_n (x)

Ideal Convergent Sequence Spaces of Fuzzy Star–
A fuzzy star–shaped number is a fuzzy set u : R_n −→ [0, 1], satisfying the following conditions:
(a) u is normal, that is, there exists t 0 ∈ R_n such that u(t0) = 1,
(b) u is upper semicontinuous,
(c) supp u = cl{t ∈ R_n : u(t) > 0} is compact,
(d) u is fuzzy star–shaped with respect to t i.e., if there exists t ∈R_n such that for any t0 ∈ R_n and
λ ∈ [0, 1], u(λt0 + (1 − λ)t) ≥ u(t0).

The set of all fuzzy star–shaped numbers is denoted by S^n . To define algebraic operations on S^n , the notion of alpha level set needs to be stated which is defined for a fuzzy star–shaped number u ∈ S n and for each α ∈ [0, 1] as,
α = {t ∈ R_n : u(t) ≥ α}, if α ∈ (0, 1] supp u, if α = 0.

The linear structure on the set of all fuzzy star–shaped numbers S n structured by addition u + v and scalar multiplication λ_u, λ ∈ R, is induced by α–level sets for each α ∈ [0, 1] via the relation,

[u + v] α = [u] α + [v] α , [λ_u] α = λ[u] α .

Both the addition u + v and scalar multiplication λ_u∈ S^n. To advance and enhance the properties of this space a metric namely d_p or Lp metric was defined in to study the distance between two fuzzy star–shaped numbers and analyze the space with this metric in terms of metric space. The metric was defined for each 1 ≤ p < ∞, as

and d_∞ (u, v) = sup 0≤α≤1 d_H([u] α , [v] α),

where d_H is a Hausdorff metric.

Thus d_∞(u, v) = $\lim_{p\to\infty}$ d_p (u, v) with $d_p \le d_q$ if p ≤ q, for all u, v ∈ S^n .

The Hausdorff metric is taken into consideration on account of it making the space of all compact sets in Rn a complete and a separable space, that is to say for any two sets A, B ∈ K_n where K_n denotes the class of all nonempty compact sets in R_n , the Hausdorff distance between A and B is given by

d_H(A, B) = max{sup a∈A inf b∈B ‖a − b‖,

sup b∈B in a∈A ‖a − b‖}.

such that (K_H , d_H) is a complete and separable space. Therefore this would give an edge over the other metrics since the support of each fuzzy star–shaped numbers is compact in itself.

theorem 1. A sequence u = (u_K) ∈ ω∗ (S^n) is convergent to u_0 ∈ ω∗ (S^n)

if there exists a positive integer N = N(ϵ) such that d_p (u_K , u_0) < ϵ for all k ≥ N. In this case, we write $\lim_{k\to\infty}$ d_p (u_K , u_0) = 0 and u_0 is called the limit of the sequence (u_K). If u_0 = 0¯ then the sequence (u_K) is called null sequence. Following are sequence spaces associated with fuzzy star–shaped numbers, c(S^n) = The space of all convergent sequences of fuzzy star–shaped numbers in R_n .

c_0 (S^n) = The space of all null sequences of fuzzy star–shaped numbers in R_n

Theorem 2.- Consider the metric space (S^n , d_p). Let u ∈ S n and r > 0 then the open ball cantered at u with radius r is defined as B

d_p (u, r) = {v ∈ S^n : d_p (v, u) < r}

Theorem 3. A subset U of a metric space (S^n , d_p) is said to be bounded if there exists a positive real number r > 0 such that U ⊆ B d_p (u,r) for some u ∈ S^n

Theorem 4. A nonempty subset U of (S^n , d_p) is said to be open if, for every u ∈ U there exists r > 0 such that Bd_p (u, r) ⊆ U

Corollary 1. A convergent sequence (U_k) ∈ ω∗ (S^n) has a unique limit.

Theorem 5. Every open ball in the metric space (S^n , d_p) is an open set.

Proof. Consider Bd_p (u ,r) to be an open ball.

Let v ∈ Bd_p (u, r) =⇒ d_p (u, v) < r.

Now put r1 = r − d_p (u, v) > 0.

To show that Bd_p (v,r1) ⊆ Bd_p (u, r). Let w ∈ Bd_p (v,r1)
this implies that d_p (w, v) < r1 = r − d_p (u, v)
⇒ d_p (u, v) + d_p (w, v) < r
⇒ d_p (u, w) < r
⇒ w ∈ Bd_p (u, r). Thus, B d_p (u, r) is an open set

Ideal Convergent Sequence Spaces of Fuzzy Star–

Shaped Numbers In 1965, fuzzy set theory was introduced initially by Zadeh as an extension of crisp set theory. He mainly focused on the convexity property of fuzzy sets because of its importance in metric definitions and their topological properties, The convex fuzzy set which is normal, upper semi-continuous and whose support set is compact is called a fuzzy number. Fuzzy number plays a significant part in the appositeness of fuzzy mathematics, and references therein. Due to the importance of the convexity and that the star–shapedness can be seen as a natural extension to this property, it has been introduced in several ways. Diamond initiated the conceptualisation of fuzzy star–shaped numbers and analyzed the properties of Lp–metric for 1 ≤ p < ∞ on the same. The usual convergence of the sequences of fuzzy star–shaped numbers was defined in previous section and this section deals in generalizing the notion of this standard convergence to ideal convergence.

The structure was the introduction to a new type of convergence known as the ideal convergence (I–convergence) and was presented by Kostyrko et al.1999. Later on, the notion of ideal convergence was applied on the sequences of fuzzy numbers by Kumar and Kumar as a generalization of ordinary convergence provided by Matloka . Some applications of I–convergence can be found in. In this fragment, we intend to put forward the sequence spaces c I (S^n), c I 0 (S^n) and ℓ I ∞(S^n) of fuzzy star–shaped numbers in R_n with respect to the Lp–metric. Let I be an admissible ideal in N and u = (U_k) be a sequence of fuzzy star–shaped .

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