ARITHMETIC NUMBER LABELLING FOR UNION GRAPHS

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Abstract

Graph labelling is an interesting and fast developing branch in graph theory. Graph labelling applications include genomics, electrical engineering, operations research, transcriptome analysis, Golomb rulers etc. In this paper, Arithmetic number labelling induced from arithmetic numbers of some standard graphs namely path, cycle, bull graph, star graph, bistar graph, jelly fish graph, tadpole graph and tree were studied. In this paper, Arithmetic number labelling of subdivided star graph S ($K_{1, n}$) and union of two subdivided star graphs S ($K_{1, n}$) $\cup$ S ($K_{1, m}$), union of subdivided star graph with star graph S ($K_{1, n}$) $\cup$ $K_{1, m}$, union of subdivided star graph with bistar graph S (K, n) $\cup$ B r, s are studied. Also, Python program code to generate the vertex labels of the union of subdivided star graph and bistar graph S (K, n) $\cup$ B r, s is given. Appropriate examples are given.

Keywords — *Arithmetic number, Arithmetic number labelling, Subdivided star graph, Union Graph.*

AMS classification: 05C78

Introduction

Assigning of integers as labels to the vertices [1]{2}[4][16][17], edges [7][18][19][20], faces [11] and blocks[10][11][13][14][23][24] to a graph is called graph labelling. The labelled graphs find its applications in image segmentation[5], communication network[3]{5}{8}[15], astrology[9], medical[22}and data science [15] and in assessing static and dynamic brain connectivity[25]. The challenging task is to obtain a label for a vertex, edge, face or block of a graph under restricted environment. Arithmetic number labelling with arithmetic numbers as basic parameter was introduced in 2022.In this paper, Arithmetic number labelling of subdivided star graph S ($K_{1, n}$) and union of two subdivided star graphs S ($K_{1, n}$) $\cup$ S ($K_{1, m}$), union of subdivided star graph with star graph S ($K_{1, n}$) $\cup$ $K_{1, m}$, union of subdivided star graph with bistar graph S (K, n) $\cup$ B r, s is studied. Also, Python program code to generate the vertex labels of the union of subdivided star graph and bistar graph S (K, n) $\cup$ B r, s is given. Appropriate examples are given.

Preliminaries

The following are the important definitions needed for this paper.

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Definition 1: **Arithmetic Number [12]**

A number which has arithmetic mean of its divisors as an integer is named as Arithmetic number. The number 22 is an arithmetic number since the arithmetic mean of its divisors 1,2,11, and 22 is 9, an integer. 1,3,5,6,7,11,13 are some of the arithmetic numbers.

Definition 2: Arithmetic number labelling [12]
An Arithmetic number labelling of a graph G is a one - to – one function.$f:\ v(G) \rightarrow W$, where W is the set of whole numbers.) that induces a bijection $f^{*}: E(G) \rightarrow (A_1, A_2, A_3, \dots, A_n)$, defined by $f^{*}(u,v) = |f(u) - f(v)|, \forall\ e = uv \in E(G)$. Arithmetic number graphs are those graphs which admit Arithmetic number labelling.
In this paper, Arithmetic number labelling is abbreviated as ANL.

Definition 3[7]: The replacement of the edge 'e' by the path (u, v, w) is the subdivision of an edge e = uv. If every edge of G is subdivided exactly once, then the resulting graph is called the subdivision graph S (G).

Definition 4[7]: Consider two graphs U_1 and U_2 which have disjointed vertex sets V_1 and V_2 and disjoint edge sets E_1 and E_2. $G = G_1 \cup G_2$ is the union of graphs G_1 and G_2. The union G has vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$. Union G has p_1+p_2 vertices and $q_1 + q_2$ edges.

Definition 5[23]: The graph obtained from K_2 by joining m pendant edges to one end of K_2 and n pendant edges to the other end of K_2 is the bistar graph $B_{m,\,n}$.

MAIN RESULTS
Theorem 1: The subdivided star graph S ($K_{1,\,n}$) admits ANL for n > 1.
Proof:

Let u_0, u_i, v_i, $1 \leq i \leq n$ be the vertices of S ($K_{1,\,n}$) and u_0u_i, u_iv_i, $1 \leq i \leq n$ be the edges of subdivided star graph S ($K_{1,\,n}$).

The function f: V (S ($K_{1,\,n}$)) $\longrightarrow$ {A_n} is defined as

$f(u_0) = 0$
$f(u_k) = A_k; 1 \leq k \leq n$
$f(v_k) = A_{n+k} + f(v_k); 1 \leq k \leq n$
The induced edge labeling f* is obtained as

$f^{*}(u_0u_k) = Ak;\ 1 \leq k \leq n$
$f^{*}(u_kv_k) = A_{n+k};\ 1 \leq k \leq n$

Clearly from the definition of f and f^*, vertex and edge labels are distinct.
Thus, it is proved that subdivided star graph S ($K_{1,n}$) admits ANL.

Example 1: Arithmetic number labelling for subdivided star graph S($K_{1,5}$).

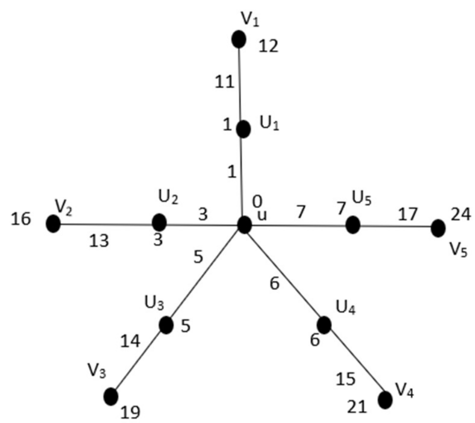

Fig 1 Arithmetic number labelling for subdivided star graph S($K_{1,5}$)

Thus, it is proved that subdivided star graph S($K_{1,5}$) admits ANL.
Theorem 2: The union of subdivided star graph and star graph, S ($K_{1,n}$) ∪ $K_{1,m}$ admits ANL for n, m > 1.

The function f: V (S ($K_{1,n}$)) ∪ $K_{1,m}$ ⟶ {A_n} is defined as

f (u) = 0
f (u_k) = A_k;1 ≤ k ≤ n
f (v_k) = A_{n+k} + Ak;1 ≤ k ≤ n
f (w) = A_{n+1}
f (w_j) = A_{n+1} + A_{2n+}j; 1 ≤ j ≤ m
The induced edge labeling f* is obtained as

f^*(uu_k) = A_k; 1 ≤ k ≤ n
f*(u_kv_k) = A_{n+k}; 1 ≤ k ≤ n
f*(ww_j) = A_{2n+j}; 1 ≤ j ≤ m
Clearly from the definition of f and f^*, vertex and edge labels are distinct.
Thus, it is proved that the union of subdivided star graph and star graph, S ($K_{1,n}$) ∪ $K_{1,m}$ admits ANL for n, m > 1.

Example 2: The union of subdivided star graph and star graph, S $(K_{1,5}) \cup K_{1,6}$ admits ANL n, m > 1.

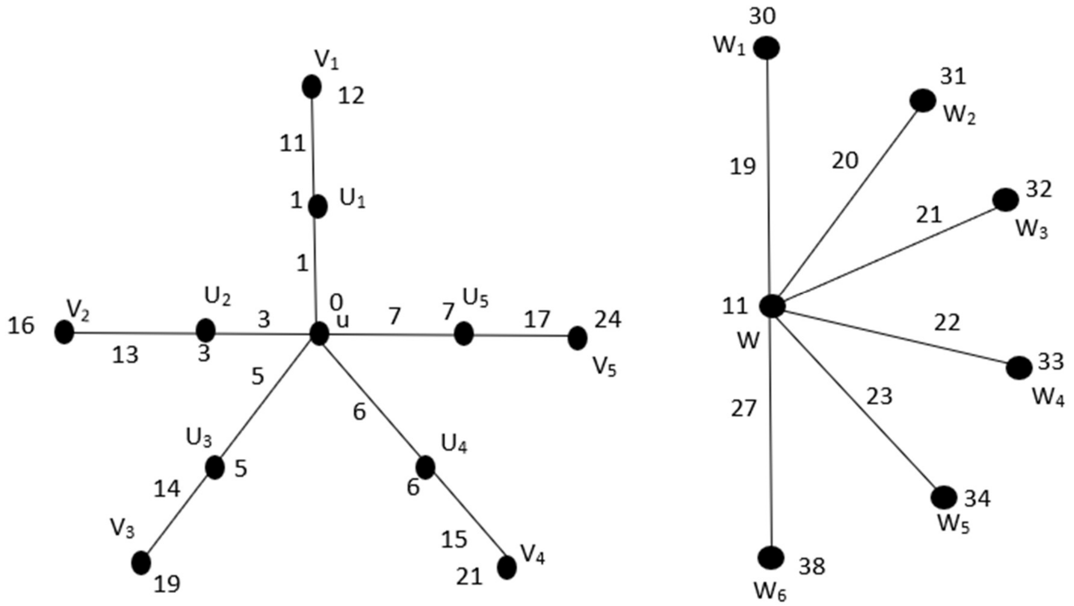

Fig 2 Arithmetic number labelling for S $(K_{1,5}) \cup K_{1,6}$

Thus, it is proved that the union of subdivided star graph and star graph, S $(K_{1,5}) \cup K_{1,6}$ admits ANL for n, m > 1.

Theorem 3: The union of subdivided star graph and bistar graph S $(K_{1,\,n}) \cup B_{r,\,s}$ admits ANL for n, r, s > 1

The function f: V (S $(K_{1,\,n}) \cup B_{r,\,s}) \longrightarrow \{A_n\}$ is defined as,

$f(u) = 0$
$f(u_k) = A_k$; $1 \leq k \leq n$
$f(v_k) = A_{n+k} + A_k$; $1 \leq k \leq n$
$f(w) = A_{n+1}$; $n > 1$
$f(w_j) = A_{n+1} + A_{2n+j}$; $1 \leq j \leq r$
$f(y) = A_{n+1} + A_{2n+r+1}$; $n, r > 1$
$f(y_j) = A_{n+1} + A_{2n+r+1} + A_{2n+r+1+j}$; $1 \leq j \leq s$
The induced edge labeling f* is obtained as

$f^*(uu_k) = A_k$; $1 \leq i \leq n$
$f^*(u_kv_k) = A_{n+k}$; $1 \leq i \leq n$
$f^*(ww_j) = A_{2n+}j$;$1 \leq i \leq r$
$f^*(wy) = A_{2n+r+1}$; $n, r > 1$
$f^*(yy_k) = A_{2n+r+1+}k$;$1 \leq k \leq s$

Clearly from the definition of f and f^*, vertex and edge labels are distinct.
Thus, it is proved that the union of subdivided star graph and bistar graph S $(K_{1,n}) \cup B_{r,s}$ admits ANL for n, r, s > 1

Example 3: The union of subdivided star graph and bistar graph S $(K_{1,4}) \cup B_{3,6}$ admits ANL.

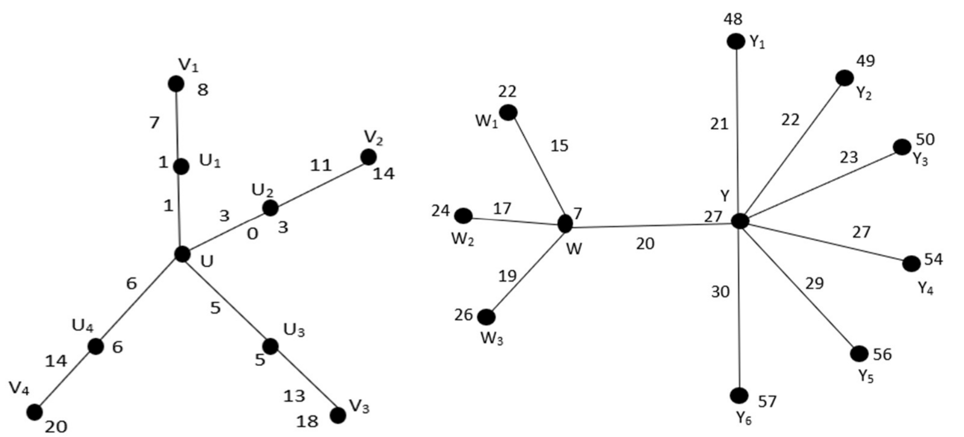

Fig 3 Arithmetic number labelling for S $(K_{1,4}) \cup B_{3,6}$

Thus, it is proved that the union of subdivided star graph and bistar graph S $(K_{1,4}) \cup B_{3,6}$ admits ANL.

Python program code

Generating the vertex labels for graphs with a greater number of vertices is a difficult task. So, in this session, we have included the Python program code to generate the vertex labels of union of subdivided star graph and bistar graph S $(K_{1,n}) \cup B_{r,s}$. Screenshot of few examples are also included. The first example has 5,3,2 as values of n, r and j. The second example has 10,7 and 4 as the values of n, r and j. The third example has 25,15 and 10 as the values of n, r, and j. This Python program code can ease the tedious process of computing the vertex labels of graphs and hence we can generate a Python program code for vertices of any graph.

The following is the Python program coding link.

https://colab.research.google.com/drive/1_MXvDI6b7RK8SHaBq3r_o6ytTa3t5A6u?usp=sharing

Python program code to generate the vertices of union of subdivided star graph and bistar graph S ($K_{1,n}$) ∪ B $_{r,s}$.

Vertex label for S (K1,n) U B r,s: $f(y_j)=A_{n+1}+A_{2n+r+1}+A_{2n+r+1+j}$

```
def findarithmetic(no):
    count=0
    sum=0
    for i in range(1,no+1):
      if(no%i==0):
        count=count+1
        sum=i+sum;
    avg=int(sum/count)
    avg1=sum/count
    if (avg1-avg==0):
          return(no)
    else:
        return(0)
def findvalue(j):
  limit=2000
  c=0
  ano =0;
  number=[]
  v=[]
  v.append(0)
  v.append(6)
  v.append(5)
  v.append(8)
  size=1000
  for i in range(1,size):
      ans=findarithmetic(i)
      if c<limit  and  ans >0 :
        number.append(ans)
```

```
        c=c+1
  for j1 in range(5,20):
    k=v[j1-2] +number[j1-1]
    #print(v[j-2])
    #print(number[j-1])
    v.append(k)
    return(number[j-1])
sbs=['o','1','2','3','4','5','6','7','8','9']

def subs(n):
  n=str(n)
  for i in n:
    print(sbs[int(i)],end='')
n=int(input("Enter value for n   : "))
r=int(input("Enter value for r   : "))
j=int(input("Enter value for j   : "))

p1=n+1
p2=(2*n)+r+1
p3=p2+j
print("f(y",end='')
subs(j)
print(")",end='')
print(" =  ",end='')
print("A",end='')
subs(p1)
print(" + ",end='')
```

```
print(" + ",end='')
print("A",end='')
subs(p2)
print(" + ",end='')
print("A",end='')
subs(p3)
a1=findvalue(p1)
a2=findvalue(p2)
a3=findvalue(p3)
print(" =  ",end='')
print(a1,end='')
print(" + ",end='')
print(a2,end='')
print(" + ",end='')
print(a3,end='')
print(" =  ",end='')
print(a1+a2+a3)
```

```
print(a1+a2+a3)
```

```
Enter value for n   : 5
Enter value for r   : 3
Enter value for j   : 2
```

$f(y_2) = A_6 + A_{14} + A_{16} = 11 + 22 + 27 = 60$

```
Enter value for n  : 10
Enter value for r  : 7
Enter value for j  : 4
f(y_4) = A_11 + A_28 + A_32 = 19 + 44 + 49 = 112
```

```
Enter value for n   : 25
Enter value for r   : 15
Enter value for j   : 10
f(y_10) =  A_26 + A_66 + A_76 =  42 + 99 + 114 =  255
```

Theorem 4: The union of two subdivided star graphs S ($K_{1,n}$) ∪ S ($K_{1,m}$) admits ANL for n, m > 1

The function f: V (S ($K_{1, n}$) ∪ S ($K_{1, m}$)) ⟶ {A_n}is defined as

$f(u) = 0$
$f(u_k) = A_k; 1 \le k \le n$
$f(v_k) = A_{n+k} + A_k; 1 \le k \le n$
$f(w) = f(v_{n-2}) - 2$
$f(w_j) = A_{2n+j} + f(w); 1 \le j \le m$
$f(x_j) = A_{2n+m+j} + f(w_j); 1 \le j \le m$
The induced edge labeling f* is obtained as

$f^*(u\, u_k) = A_k; 1 \le k \le n$
$f^*(u_k v_k) = A_{n+k}; 1 \le k \le n$
$f^*(w w_j) = A_{2n+j}; 1 \le j \le m$
$f^*(w x_j) = A_{2n+m+j}; 1 \le j \le m$
Clearly from the definition of f and f^*, vertex and edge labels are distinct.
Thus, it is proved that union of two subdivided star graphs S ($K_{1,n}$) ∪ S ($K_{1,m}$) admits ANL.

Example 4: The union of two subdivided star graphs S ($K_{1,5}$) ∪ S ($K_{1,3}$) admits ANL.

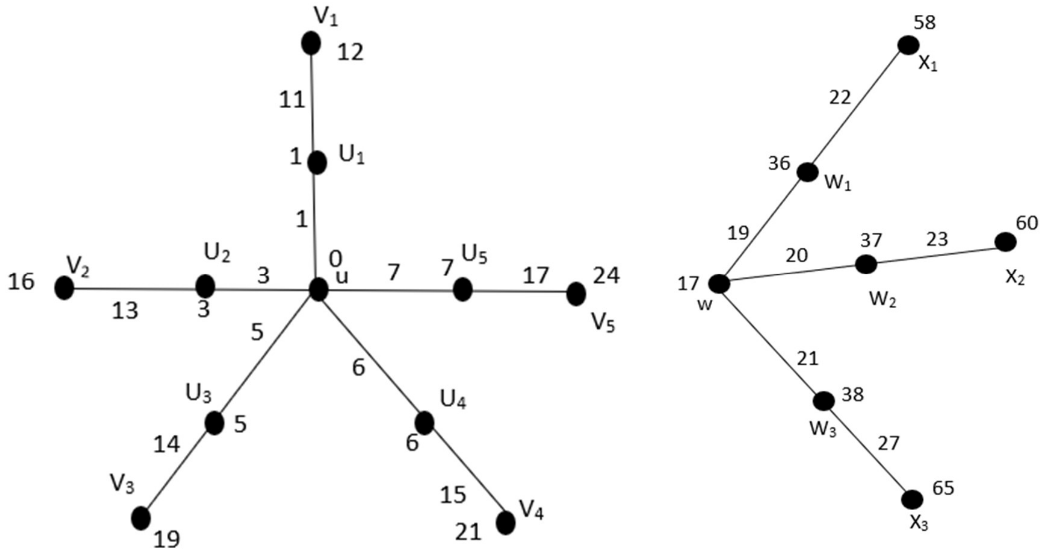

Fig 4 Arithmetic number labelling for S ($K_{1,5}$) ∪ S ($K_{1,3}$)

Thus, it is proved that the union of two subdivided star graphs S ($K_{1,5}$) ∪ S ($K_{1,3}$) admits ANL.

Conclusion:

Arithmetic number labelling for subdivision graph and union graphs are studied in this paper. Arithmetic number labelling of subdivided star graph S ($K_{1,n}$) and union of two subdivided star graphs S ($K_{1,n}$) ∪ S ($K_{1,m}$), union of subdivided star graph with star graph S ($K_{1,n}$) ∪ $K_{1,m}$, union of subdivided star graph with bistar graph S (K, n) ∪ B r, s are studied. Also, Python program code to generate the vertex labels of the union of subdivided star graph and bistar graph represented by S (K, n) ∪ B r, s is given. Appropriate examples are given. There is further scope of establishing arithmetic number labelling for more graphs with application to heterogenous fields.

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