A BOX- COX CONTROL CHART FOR MODIFIED BURR III DISTRIBUTION

Sivaranjini R* and Vijayashankar N

1 Research scholar, Department of Statistics, Annamalai University, Annamalai Nagar, Tamil Nadu, India.

2 Assistant Professor of Statistics Department of Statistics, Annamalai University, Annamalai Nagar, Tamil Nadu, India

Corresponding author : shreeranjini9396@gmail.com

ABSTRACT

The Modified Burr III (MBIII) distribution is a new exponential family of distributions that we have introduced in the process of control chart. The proposed model is positively skewed, with a declining or unimodal form (depending on its parameter values). The MBIII distribution's statistical features are investigated and derived to evaluate the process of control chart. We've created Box-Cox control charts in mean and variance, for 3 sigma level as we assume the data is non-normal. Finally, as an example, an application to a simulation data set that is fit in the derived equation of the 3 sigma level to test the control of the selected distribution is shown.

KEY WORDS: Box-Cox,Control Chart, Mean, Variance, and MBIII distribution.

1. INDRODUCTION

A sigma rating can be used to define the maturity of a manufacturing process by expressing its yield, or the percentage of defect-free products it produces, and how many standard deviations of a normal distribution the proportion of defect-free outcomes corresponds. Researchers are frequently drawn to a flexible approach for analyzing lifetime data. Regardless of the fact that several continuous univariate distributions have indeed been built in recent years, numerous sets of data from reliability, economics, insurance, medicine, and other disciplines do not reflect similar distributions. As a reason, finding solutions within those fields with modified, extended, and generalized distributions is justifiable[14,15].

The Box-Cox chart can be used to determine where the sigma minimum value is located. Two more lines appear in the Box-Cox plot: “the upper and lower confidence limits”. Potential standards of λ minimize the variation are found in the region between the confidence bounds[16]. Based on creating the Pearson differential equation,[2] constructed a system of twelve types of distribution functions. The density function comes in a variety of shapes that can be used in a variety of situations. Inside the Burr distribution network, the “Burr III distribution” has been the most widely used approach. This is also referred as the Dagum distribution[3] in assessments of income, and prosperity distribution. The Inverse Burr distribution[8] is recognized in the actuarial field, while the kappa distribution is notable inclimatological profession [10]. The number of distributions hazard function has only growing, decreasing, or stable shapes. As a function, they can't be used to depict lifetime data such as human mortality or equipment life

Submitted: 08/11/2022 **Accepted**: 30/11/2022

cycles with a bathtub-shaped hazard function. For decades, statisticians have worked on numerous modifications and various extensions of probability distributions.

Non-monotonic forms, such as the bathtub shape, the unimodal or modified unimodal shape, and others, are required for certain longevity statistics.Many academics have spent years developing numerous modified variants of the parent distribution using various strategies to generate monotonic and non-monotonic geometries. Due to the variation, the MBIII distribution has periodic decreasing, rising, bathtub, unimodal,and essentially constant hazard rate shapes[4].

In reliability, survival, and selection of sampling applications, the Burr III distribution is being used to design health data in finance and economics, along with failure data information.There are various modified, extended, and generalized variations of the Burr III distribution in the research, namely the inverse Burr III distribution[7,8]. The modified, extended, and generalized distributions are created by applying a transformation to the base distribution or by adding one or more parameters. Frequently, these newly generated distributions fit data better than sub- and competitive models. Burr[2] proposed the Burr family of twelve distributions for fitting cumulative frequency functions on frequency data.The density function comes in a variety of shapes that can be used in a variety of situations.

In this research, we assumed that the quality variate follows the MBIII distribution and established control limits for that data in the same way as Shewhart control limits are developed. The online method of such a quality can be monitored using the theory of the MBIII distribution if a process quality characteristic is believed to follow the MBIII distribution. In the absence of any such population model specification, we use the normal distribution and any related constants that are available. Although it is extremely difficult to verify and corroborate sample data that follows a normal distribution. The normality assumption cannot be sustained without the use of a goodness - of - fit test if the sample group is tiny[13].

Many studies have been published literature that seek to solve the problem using a control chart. Wu and Jiao[17] suggested an attribute control chart to regulate mean of a variable where the run length between two consecutive non-conforming samples is validated, and data is considered non-conforming if it contains a predetermined number of non-conforming items.

2. BACKGROUND OF THE STUDY

Control chart is a Statistical ProcessControl (SPC)tool for monitoring and improving quality of products in every manufacturing process.Shewhart A. Walter, who worked at Bell Telephone Laboratories in the 1920s, came up with the concept of a control chart. Since its inception, some changes have been made, but the core concept of presenting the statistic on a graph with lower and higher bounds has remained intact. It is vital for quality engineers to assess whether the control chart in use is capable of detecting an out-of-control process early enough. The primary goal of creating control charts [2017][11] is to recognize the assignable causes of the online process as soon as possible.

In SPC, control charts provide effective tools towards assessing industrial processes. An intensive monitoring of the production process is required to enhance the high quality of the made items, to manufacture the product according to the supplied requirements, and to reduce the inspection cost. As a result, control charts are a tool for achieving excellent product quality. The central line or the “Central Limit” (CL) signifies the average value for the process under control, while the other two horizontal lines, well-known as“Upper Control Limit (UCL)”and “Lower Control Limit (LCL)”, remainsignified in such a technique that when a process is under control, almost all of the data points fall within these limits.The control charts can be used to investigate both variable and attribute features[6].

A control chart keeps track of a manufacturing process using data from individual items or subgroups of things in the process. A measurable quality attribute, such as length, diameter, or thickness, is associated with each object. The control chart will issue a signal indicating that the process has altered if the control chart statistic exceeds the control limitations. Following such a signal, the SPC approach requires that a search be conducted to identify the particular causes that are to blame.Process engineers must apply their process knowledge to assist in the identification of the special cause[9].

Table-1.Related existing distribution in Statistical Quality Control

Author’s (Year)	Distribution	Description
Muhammad Aslam, Osama H. Arif and Chi-Hyuck Jun [2017][11]	Weibull distribution (WD)	An attribute control chart built on accelerated hybrid censoring logic has been provided based on the observation of defective items whose life follows a WD. An acceleration factor can be applied to various pressured circumstances such as stress, load, strain, temperature, and so on to evaluate the product.
Muhammad Aslam, Osama H. Arif and Chi-Hyuck Jun [2017][12]	Gamma Distribution (GD)	A control chart based on multiple dependent (or postponed) states sampling for GDquality characteristic is proposed using the gamma to normal transformation. Two sets of control limits are given in the control chart, which can be derived by looking at the average run length in control (ARL). The shift in the scale parameter of a GD is used to compute the out-of-control ARL.
GaddeSrinivasa Rao [2018][5]	Exponentiated Half Logistic Distribution (EHLD)	EHLD was studied to produce an attribute control chart for time truncated life tests with known or unknown shape parameter. The performance of

		the suggested chart in terms of ARL is evaluated using the Monte Carlo Simulation (MCS).
GaddeSrinivasa Rao and Edwin Paul [2020][6]	Log Logistic distribution (LLD)	A LLD is examined while building an attribute control chart for time reduced life tests with known or unknown shape parameter. Performance of suggested chart in terms of ARL is evaluated through the MCS.

3. DESIGN STRUCTURE OF THEMODIFIED BURR III DISTRIBUTION

The Cumulative Distribution Function (CDF) of MBIII distribution with three parameters[1]specified as

$$F(x;\alpha,\beta,\theta)=\left[1+\theta x^{-\beta}\right]^{-\frac{\alpha}{\theta}} \qquad x>0,\alpha,\beta,\theta>0 \qquad \ldots(1)$$

The Probability Density Function (PDF)specified as

$$f(x;\alpha,\beta,\theta)=\alpha\beta x^{-\beta-1}\left[1+\theta x^{-\beta}\right]^{-\frac{\alpha}{\theta}-1} \qquad x>0 \qquad \ldots(2)$$

Where $\alpha,\beta\ and\ \theta$ are the shape parameters of MBIII distribution. The restrictive distribution for $\theta\to 0$; MBIII distribution is a generalized Inverse Weibull distribution.

4. PERFORMANCE MEASURES OF THE MODIFIED BURR III DISTRIBUTION

The statistical properties include;Mean, Variance and Ordinary Moments,. The r^{th} moments of MBIII distribution are obtained as

$$\mu_r'=\int_0^\infty x^r\,f(x)dx=\alpha\theta^{\frac{r}{\beta}-1}\;Q(P_r,Q_r), \qquad r=1,2,3 \qquad \ldots(3)$$

Where $Q(P_r,Q_r)$ is a beta function, $P_r=1-\frac{r}{\beta}$ and $Q_r=\frac{\alpha}{\theta}+\frac{r}{\beta}$ $\qquad r<\beta$

Central moments:

$$\mu_r=\sum_{j=0}^{r}\binom{r}{j}(-1)^j\,\alpha^{j+1}\theta^{\frac{r}{\beta}-j-1}[Q(P_1,Q_1)]^jQ(P_{r-j,}Q_{r-j}) \qquad \ldots(4)$$

The$E(X)$ mean and $Var(X)$variance of the Modified Burr III distribution are

$$E(X) = \frac{\alpha}{\theta^{\frac{1}{\beta}+1}} Q \left(\frac{1}{\beta} + 1 \frac{\alpha}{\theta} - \frac{1}{\beta}\right) \quad \ldots (5)$$

$$Var(X) = \frac{\alpha}{\theta^{\frac{2}{\beta}+1}} \left[Q \left(\frac{2}{\beta} + 1 \frac{\alpha}{\theta} - \frac{2}{\beta}\right) - \frac{\alpha}{\theta} Q^2 \left(\frac{1}{\beta} + 1 \frac{\alpha}{\theta} - \frac{1}{\beta}\right)\right] \quad \ldots (6)$$

5. CONTROL LIMITS USING 3 σTHE MODIFIED BURR III DISTRIBUTION

$$UCL = \frac{\alpha}{\theta^{\frac{1}{\beta}+1}} + 3\theta \sqrt{\frac{\alpha}{\theta^{\frac{2}{\beta}+1}} \left[Q \left(\frac{2}{\beta} + 1 \frac{\alpha}{\theta} - \frac{2}{\beta}\right) - \frac{\alpha}{\theta} Q^2 \left(\frac{1}{\beta} + 1 \frac{\alpha}{\theta} - \frac{1}{\beta}\right)\right]} \quad \ldots (7)$$

$$Center\ Line = \frac{\alpha}{\theta^{\frac{1}{\beta}+1}} \quad \ldots (8)$$

$$LCL = \frac{\alpha}{\theta^{\frac{1}{\beta}+1}} - 3\theta \sqrt{\frac{\alpha}{\theta^{\frac{2}{\beta}+1}} \left[Q \left(\frac{2}{\beta} + 1 \frac{\alpha}{\theta} - \frac{2}{\beta}\right) - \frac{\alpha}{\theta} Q^2 \left(\frac{1}{\beta} + 1 \frac{\alpha}{\theta} - \frac{1}{\beta}\right)\right]} \quad \ldots (9)$$

NUMERICALILLUSTRATION

The Box-Cox transformation is used to estimate the value, which is then compared to known values.To validate the applicability of the model, an example on design of Box-Cox control chart is considered. Using a simulated data set, the control chart of the MBIII distribution is determined. The table 1 along with figure 1-3; is constructed using α, β and θparameters, and being random variables, also the UCL and LCL are reported.

Table 1: Box-Cox plot of control chart for 3 sigma levels

α	θ	$\beta = 1$			$\beta = 5$			$\beta = 10$		
		UCL	CL	LCL	UCL	CL	LCL	UCL	CL	LCL

1	3	0.42	0.11	-0.19	1.02	0.27	-0.59	1.15	0.30	-0.71
	5	0.22	0.04	-0.14	0.83	0.15	-0.54	0.97	0.17	-0.69
	7	0.15	0.02	-0.11	0.73	0.09	-0.54	0.89	0.12	-0.67
	9	0.11	0.01	-0.09	0.68	0.07	-0.53	0.85	0.09	-0.66
	10	0.1	0.01	-0.08	0.65	0.06	-0.53	0.83	0.07	-0.66
5	3	2.19	0.56	-1.08	5.17	1.34	-2.79	5.76	1.49	-3.36
	5	1.15	0.2	-0.75	4.16	0.73	-2.73	4.89	0.85	-3.35
	7	0.77	0.10	-0.56	3.69	0.48	-2.72	4.49	0.59	-3.31
	9	0.58	0.06	-0.45	3.40	0.36	-2.68	4.25	0.45	-3.19
	10	0.51	0.05	-0.41	3.29	0.32	-2.66	4.16	0.39	-3.07
10	3	4.62	1.11	-2.40	10.47	2.68	-5.52	11.58	2.99	-6.74
	5	2.39	0.4	-1.59	8.39	1.45	-5.49	9.82	1.70	-6.73
	7	1.59	0.20	-1.18	7.42	0.97	-5.49	9.00	1.18	-6.65
	9	1.18	0.12	-0.94	6.84	0.72	-5.40	8.51	0.89	-6.41
	10	1.05	0.1	-0.85	6.62	0.63	-5.35	8.33	0.79	-5.61

Figure 1: Control limit forBox-Plot when $\beta = 1$

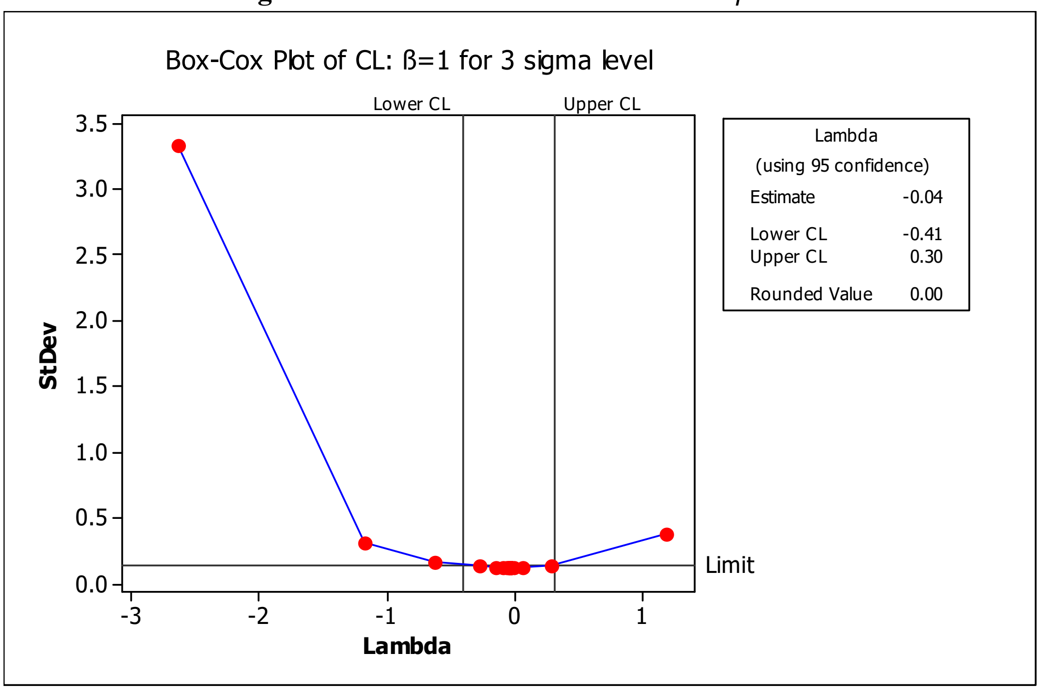


Figure 2: Control limit forBox-Plot when $\beta = 5$

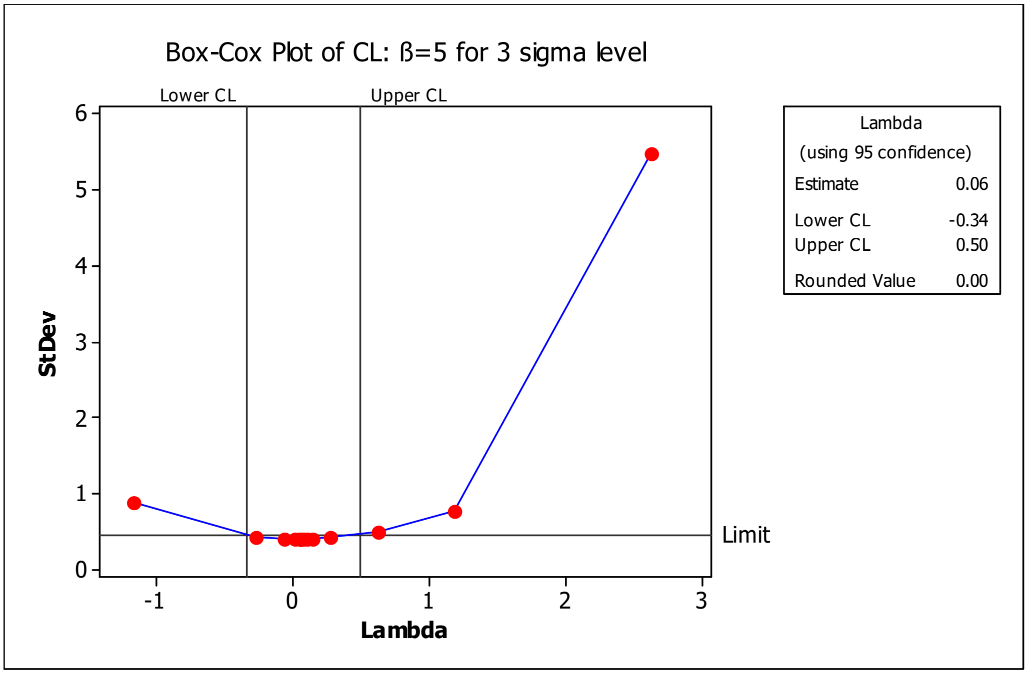
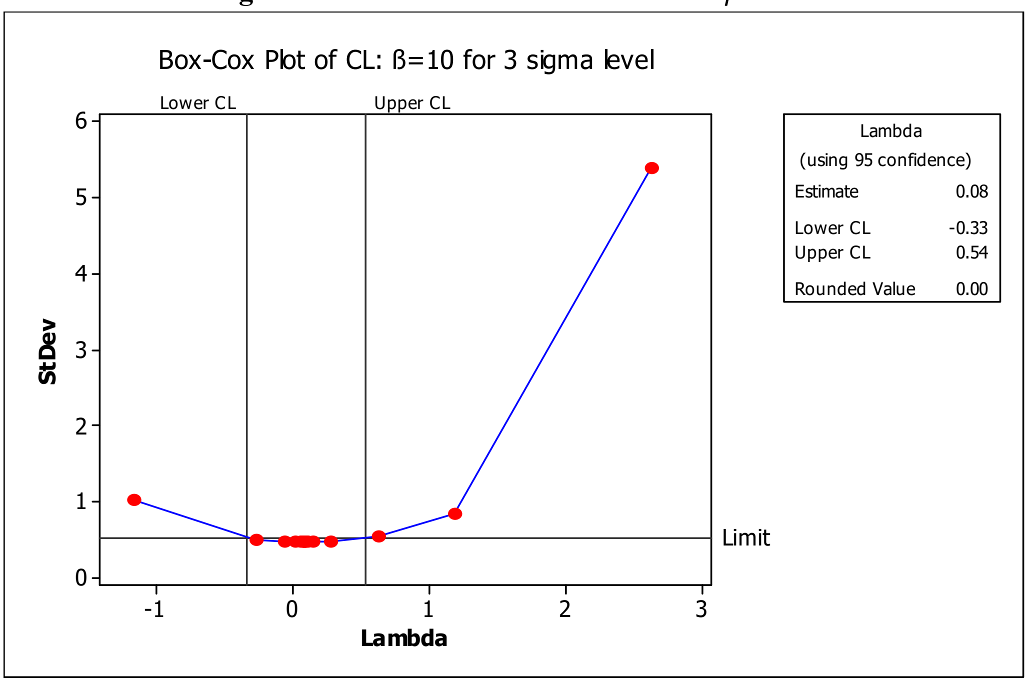


Figure 3: Control limit forBox-Plot when $\beta = 10$



CONCLUSION

Based on the MBIII distribution, we created a Box-Cox control chart. The control process is sentenced using a new algorithm because to the non-normal nature of the process. Table 1 along with figure 1-3; shows a Box-Cox plot of a control chart for a 3 sigma level with appropriate parameter values. The LCL was found to be greater when $\beta = 1$, whereas the UCL was found to be larger when $\beta = 5$ and 10. This demonstrates that the estimated value increased as the β value increased.It is possible to use 6-sigma to further refine the model.

REFERENCES

1. Azeem Ali, Syed AnwerHasnain, Munir Ahmad. (2015). "Modified Burr III distribution properties and applications." *Pakistan journal of Statistics*, Vol.31 (6), pp.697-708.
2. Burr, I. W. (1942), "Cumulative frequency functions", *The Annals of Mathematical Statistics*, Vol.13, pp.215–232.
3. Dagum, C.,(1977). "New model of personal income-distribution-specification and estimation", *Economieappliquée*, Vol.30 (3). pp.413-37.
4. Farrukh Jamal, Muhammad A. Nasir, Ali H. Abuzaid, M.H.Tahir, Wali Khan Mashwani. (2021). "New Modified Burr III Distribution, Properties and Applications." *HAL open science*.
5. GaddeSrinivasa Rao, Edwin Paul. (2020). "Time truncated control chart using log logistic distribution". *Biometrics & Biostatistics International Journal*. Vol.9 (2), pp.76-82.
6. GaddeSrinivasa Rao. (2018). "A Control Chart for Time Truncated Life Tests Using Exponentiated Half Logistic Distribution". *Applied Mathematics & Information Sciences*. Vol.12(1), pp.125-131.
7. Gove, J. H., Ducey, M. J., Leak, W. B., and Zhang, L. (2008), "Rotated sigmoid structures in managed uneven-aged northern hard work stands: a look at the Burr Type III distribution", *Foresty*, Vol.81(2), pp.161-176.
8. Kleiber, C. and Kotz, S. (2003), "Statistical size distribution in economics and actuarial sciences", *John Wiley and Sons, New York*.
9. Mang Zhang. (2009). "Performance of Control Charts for Weibull Processes." *Florida State University Libraries*.
10. Mielke, Jr, PW. (1973). "Another family of distributions for describing and analyzing precipitation data" *Journal of Applied Meteorology*. Vol. 12(2). pp.275-80.
11. Muhammad Aslam, Osama H. Arif and Chi-Hyuck Jun (2017). "An attribute control chart for a Weibull distribution under accelerated hybrid censoring." PLOS ONE, Vol.3, pp.1-13.
12. Muhammad Aslam, Osama-H. Arif, Chi-Hyuck Jun (2017). "A Control Chart for Gamma Distribution using Multiple Dependent State Sampling." *Industrial Engineering & Management Systems*. Vol.16(1), pp.109-117.
13. Rosaiah,K. B. Srinivasa Rao and M. Rami Reddy. (2019). "Variable Control Charts based on Percentiles of Exponentiated Inverse Kumaraswamy Distribution." *Journal of Information and Computational Science*. Vol.9(10), pp.841-854.

14. Tennant, Geoff (2001). Six Sigma: SPC and TQM in Manufacturing and Services. Gower Publishing, Ltd. p. 6. ISBN 0-566-08374-4.
15. The Inventors of Six Sigma". Archived from the original on 2005-11-06. Retrieved 2006-01-29.
16. Velilla, S. (1993). “Quantile-based estimation for the Box-Cox transformation in random samples”. *Statistics and Probability Letters* Vol.16, pp.137-145.
17. Wu Z, Jiao J. (2008). “A control chart for monitoring process mean based on attributes inspection”. *Int J Prod Res*. Vol.46 (15). pp.4331–4347.