COMPARATIVE STUDY OF LINEAR AND NON LINEAR OPTIMIZATION MODEL FOR MIXTURE CONTENTS IN COMPOUND

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Abstract: - The concepts of linear and non-linear programming are used for solve the optimization problems. The paper will discuss the linear programming method and then convert problem in quadratic to find the minimize function. We will discuss the modeling of the cost minimize function for a Pharmacy company which can also apply to another manufacturing unit, where the products are based on some conditions. The paper will present the comparative result with both linear and converted non-linear programming method.
Keywords: - **Algorithm, productivity, feasible, Gradient, Optimization, Vitamin, Cost, Minimization**

1. I. INTRODUCTION

The computational procedure requires at most m non-zero variables in the solution at any step. In the case of less than m non-zero variables at any stage of computations, the degeneracy arises in the LP problem. Operations Research has gained significance in applications like world-class Manufacturing systems (WCM), Lean production, Six-sigma quality management, Benchmarking, and Just-in-time (JIT) inventory techniques. The growth of global markets and the resulting increase in competition have highlighted the need for Operation Research. One of the essential managerial skills is the ability to allocate and utilize resources appropriately in the efforts of achieving optimal performance efficiently. In some cases, such as small-scale low complex environments, decisions based on intuition with minimal quantitative basis may be reasonably acceptable and practical in achieving the goal of the organization. [1] Discussed the technique and philosophy leading the art of modeling and solved the real life challenging issues. They described that MINTO is useful software to find the feasible solution with modeling. [2] Used the black –box optimizer and discussed with practical explanation. They gave some ideas about different algorithms that will allow her put them in use. They discussed the variant of gradient descent, summarize challenges, introduce the most common optimization algorithms, review architectures in a parallel and distributed setting, and investigate additional strategies for optimizing gradient descent. They investigated algorithms which are used in SGD. [3] Used the n-dimensional space variant and applied the operation equation of reflection, contraction and expansion. According to [4] Gradient descent is foundation of neural networks and optimization heuristics have accelerated the progress in deep

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learning. They proposed a frame to understand the method of different gradient descent optimization methods by analyzing several adaptive methods and other learning rate methods. They verified the proposed strategy with massive deep learning network experiments. They investigated an optimization method combination framework and a multistage combination strategy for gradient descent optimization methods to improve the training of DNNs. Their combined work has improved with compare to non combined MNIST. According to [5] machine learning consists of finding values of unknown weights in a cost function by minimizing the cost function based on learning data. In existence find the first derivative of cost function to find out the minimum value of cost function. At the local minima, the first derivative become zero and cannot found the global minimum. They modified in the adaptive momentum estimation scheme with some new term. They added convergence condition for proposed method and convergence value are also analyzed. The paper will be discussed the Simplex method also for find the mixture quantities optimization. [6] Considered application of linear programming in solves optimization with constraints. They used Simplex method for find maximum value of function. They used LingPro function and MATLAB for solution. They analyzed the complexity to applied Simplex method on real life problems with computer time result and number of variables. They represented the problem with a mathematical model which involves an objective function and linear inequalities. They concluded that Simplex Simplex method provides a systematic way of examining the vertices of the feasible region to determine the optimal value of the objective function. According to [7] Simplex method is the most popular and successful method for solving linear programs. The objective function of linear programming problem (LPP) involves in the maximization and minimization problem with the set of linear equalities and inequalities constraints. There are different methods to solve LPP, such as simplex, dual-simplex, Big-M and two phase method. They presented approach to solve LPP with seven steps process choosing Key element rule. They used the elementary row transformation to reduce the complexity. By the proposed technique elementary transformation has completely avoid the complexity and achieve the result in considerable duration. They presented seven steps involved in Simplex method and steps give us overview of the procedure. In this proposed used the seven steps algorithm, after 1st iteration the columns for the non-basic and leaving variables change. When 0 is found in the key column, the row always will be same in the new tableau and vice-versa. Thy used for solving maximization problem with constraints of the form of $\leq$, $\geq$ and = constraints. [8] Presented Mathematical view of gradient descent in form of optimization algorithm. They found gradient descent as the important optimization strategies for machine learning models and get the variants to minimize the error between predicted functions and empirical data. Gradient descent functions can be direct used in Matlab, Python, Excel, and Weka. They presented how this algorithm is working. [9] Developed the python program to solve the Simplex problem and get the optimal result. It can help to optimize the objective function for minimization as well as maximization and linear constraints with n number of variables. . The simplex algorithm using python can extend to solve real world problems like least cost formulation and maximization of profit. They developed completely functional windows application which works for n no. of equations and variables and shows the output in an excel sheet on the same system. The excel sheet contains each iteration formed during the course of finding the solution and the key column value and key row value are also show in a red and green boxes respectively. [10] Presented a model of Bintang Bakery home industry which produced three types of bread for maximum profit. The purpose of this research is to optimize the benefits of the Bintang Bakery home industry. Profit optimization calculations performed used Lindo tools. [11] They presented a comparative study of stochastic, momentum, Nesterov, AdaGrad, RMSProp, AdaDelta, Adam, AdaMax and Nadam gradient descent algorithms based on the speed of convergence of these different algorithms, as well as the mean absolute error of each algorithm in the generation of an optimization solution. [12] Introduced the smoothed analysis of worst case and average case algorithms. In smoothed analysis, they measured the maximum over inputs of the expected performance of an algorithm under small random perturbations of that input. [13] Presented a modified conjugate gradient method. An interesting feature of the presented method is that the direction is always a descent direction for the objective function. [14] Presented to solve a fuzzy transportation problem with linear fractional fuzzy objective function. They proposed approach the fractional fuzzy transportation problem is decomposed into two linear fuzzy transportation problems. [15] Attempted to get an insight about the various application of optimization techniques in business. They studied based on different cases applied on selected sectors, viz., industrial, financial, resource allocation, agriculture, marketing and personnel management area.

II. Preliminary

The Simplex method provides a systematic algorithm that consists of moving from one basic feasible solution to another in a prescribed manner so that the value of the objective function is improved. The simplex algorithm is an iterative procedure for solving LP problems. It consists of

1. Having a trial basic feasible solution to constraint equations
2. Testing whether it is an optimal solution
3. Improving the first trial solution by a set of rules, and repeating the process till an optimal solution is obtained.

The computational procedure requires at most m non-zero variables in the solution at any step. In the case of less than m, none zero variables at any stage of computations the degeneracy arises in the LP problem. It is very interesting to note that a feasible solution at any iteration is related to the feasible solution at the Successive iteration in the following way. One of the then on-basic variables at one iteration becomes basic in the following way and is called an entering variable.

State the LP problem in standard form:

$$Max\ Z = c_1x_1 + c_2x_2 + c_3x_3 \ldots\ldots\ldots c_nx_n$$

Subject to the constraints:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \ldots\ldots\ldots a_{1n}x_n + x_{n+1} = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \ldots\ldots\ldots a_{2n}x_n + x_{n+2} = b_2$$

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$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 \ldots \ldots \ldots a_{mn}x_n + x_{m+n} = b_m$$

And $x_1 \geq 0, \quad x_2 \geq 0 \ldots \ldots \ldots \ldots \ldots \ldots .. x_n \geq 0$

$$x_{n+1} \geq 0, \quad x_{n+2} \geq 0 \ldots \ldots \ldots \ldots \ldots \ldots .. x_{n+m} \geq 0$$

Linear Programming problems in which constraints may also have ≥ and = after ensuring that all b_i are ≥ 0 are considered.

In this paper, we will discuss the Pharmacy company issue which manufactures vitamin tablets. The company manufactures the two types of vitamin tablets A and B. The common is used to make the tablets are in small and different quantities (grams). To make a sample tablet it is required to mix up the compound in a testing manner and perfect quantity.

III. Methodology

The cost of each compound is different and the company has the target to optimize the compound quantity and minimize the cost of production. Here are two types of tablets X and Y. The X contains α units of vitamin A per gram and β units of vitamin B per gram. The cost of vitamin A c_1 per gram and the cost of vitamin B c_2 per gram. Table Y contains σ units of vitamin A and vitamin B is ρ units. Let x_1a gram of tablet X and x_2 grams of Y to be produced or purchased. Now the problem can be formulated by

$$MinZ = c_1x_1 + c_2x_2$$

The daily requirement requirements of vitamin A and B are b_1 unit and b_2 units. We have the challenge to minimize the product cost So the production rate can be increased. Now the problem can be written in the form of formulation: -

$$MinZ = c_1x_1 + c_2x_2$$

With the restriction

$$\alpha x_1 + \beta x_2 \geq b_1$$

$\sigma x_1 + \rho x_2 \geq b_2$ With $x_1, x_2\ are\ \geq 0$

Now we introduce the surplus variables $x_3\ and\ x_4\ are\ \geq 0$ for converting the constraint in equal form and using the artificial variables $a_1, a_2\ are\ \geq 0$ now the constraint becomes

$$\alpha x_1 + \beta x_2 - x_3 + a_1 \geq b_1$$
$$\sigma x_1 + \rho x_2 - x_3 + a_2 \geq b_2$$

Now the objective function becomes:

$$Max\ Z^1 = -c_1x_1 - c_2x_2 + 0x_3 + 0x_4 - Ma_1 - Ma_2$$

Now this linear problem is converted intoan optimization problem so we can apply the Simplex table or Method to solve the Pharmacy company issue

	C_b	X_b	X_1	X_2	S_1	S_2	A_1	A_2	Min Ratio (X_b/X_k)
a_1	-M	b_1	α	β	-1	0	1	0	$\frac{b_1}{\alpha}$
a_2	-M	b_2	σ	ρ	0	-1	0	1	-$\frac{b_2}{\rho}$
		$-Mb_1 - Mb_2$	$-M\alpha - M\sigma$	$-M\beta - M\rho$	M	M	0	0	

Now they continuously follow the steps and get

Since all$\Delta j \geq 0$, an optimal solution is attained. Now we introduce the new concept of Gradient descent as AX- b =0 reformulated as a quadratic minimization problem. If the system matrix A is real symmetric and positive definite, an objective function can be defined as a non-linear function with minimization of

$$F(x) = X^t AX - 2X^t b$$

So $$\nabla F(x) = 2(Ax - b)$$

For a general real matrix A, linear least square define

$$F(x) = |Ax - b|^2$$

In traditional linear least square for real A and b the Euclidean norm is used

$$\nabla F(x) = 2A^T(Ax - b)$$

The line search minimization finding the locally optimal step size γ in every iteration can be performed analytically for quadratic functions and explicit formulas for the locally optimal γ are known. Let for the real symmetric and positive definite matrix A, a simple algorithm

$$r \coloneqq b - Ax$$

$$\gamma \coloneqq \frac{r^T r}{r^T A r}$$

$x \coloneqq x + \gamma r$ if $r^T r$ is sufficiently small; then exit the loop.

Return X as the result:

This method we use to solve the problem which already done by the Simplex method. Now we take

$$MinZ = c_1 x_1 + c_2 x_2$$

With the restriction $$\alpha x_1 + \beta x_2 \geq b_1$$

$$\sigma x_1 + \rho x_2 \geq b_2$$

$$F(x) = [x_1, x_2]^{\alpha \quad \beta x_1}_{\sigma \quad \rho x_2} - 2[x_1 x_2]^{b_1}_{b_2}$$

$$= (\alpha x_1 + \sigma x_2)x_1 + (\beta x_1 + \rho x_2)x_2 - 2(b_1 x_1 + b_2 x_2)$$

$$= \alpha x_1^2 + \sigma x_1 x_2 + \beta x_1 x_2 + \rho x_2^2 - 2b_1 x_1 - 2b_2 x_2$$

$$= \alpha x_1^2 + (\alpha + \beta)x_1x_2 + \rho x_2^2 - 2b_1x_1 - 2b_2x_2$$

Now the gradient descent can be applied

$$\frac{\partial F(x)}{\partial x_1} = 2\alpha\, x_1 + (\alpha + \beta)x_2 - 2b_1 = 0$$
$$\frac{\partial F(x)}{\partial x} = 2\rho\, x_2 + (\alpha + \beta)x_1 - 2b_2 = 0$$

Here the equations can be consider in the form

$\alpha\, x_1 + (\alpha + \beta)x_2 - c_1 = 0$ And the second be represent as

$$\rho\, x_2 + (\alpha + \beta)x_1 - c_2 = 0$$

Now by solving both equations:

$$[(\alpha + \beta)^2 - \rho\alpha]x_2 = c_2\alpha - c_1(\alpha + \beta)$$

$x_2 = \frac{c_2\alpha - c_1(\alpha+\beta)}{[(\alpha+\beta)^2 - \rho\alpha]}$ And $x_1 = \frac{1}{\alpha}[c_1 - (\alpha + \beta)\left\{\frac{c_2\alpha - c_1(\alpha+\beta)}{(\alpha+\beta)^2 - \alpha\rho}\right\}$

These are desiresto minimize the cost of the function. There can replace the value of all coefficient of variables than easily can find the value of variables. With replacement, these values in the cost function get the minimum cost of the product.

IV. Results and discussion

This paper discussed the Pharmacy company issue which manufactures vitamin tablets. The company manufactures the two types of vitamin tablets A and B. The components used to make the tablets are in small and different quantities (grams). To make a sample tablet it is required to mix up the compound in a testing manner and perfect quantity. The X contains α units of vitamin A and σ units of vitamin B. The paper discussed the Simplex method and introduces a new concept to change the linear form of the equation intoa quadratic form. In the quadratic form of the equation can apply the gradient descent method to find the maximum and minimum value of the function. So In this paper,first we changed the linear equation problem of a Pharmacy company and then convert it into the quadratic equation form. Here we found the modeling for the cost minimization function.

The value of variables is directly calculated with

$= \frac{1}{\alpha}[c_1 - (\alpha + \beta)\left\{\frac{c_2\alpha - c_1(\alpha+\beta)}{(\alpha+\beta)^2 - \alpha\rho}\right\}$ And $x_2 = \frac{c_2\alpha - c_1(\alpha+\beta)}{[(\alpha+\beta)^2 - \rho\alpha]}$

These cost variables can be applied in other cost functions with changes in the cost coefficients and the conditions of the production

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VI. References

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