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REGULARIZATION FOR POLYHARMONIC FUNCTIONS FOR SOME REGIONS IN R^m

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In this work, the polygarmonious functions of the nth order satisfying the condition given in some unlimited set of m-dimensional space are considered, having received an integral representation with the help of it, fragmen-Lindelof theorems are obtained and the solution of the regularization problem is considered. $2n \geq m$

Annotation

In this work, some properties and regulation of the Carlemann function are studied to determine the integral formula of n-th order polygarmonic functions (D ^ n u (y) = 0) and their properties that satisfy the condition 2n≥m in certain unbounded areas of real m-dimensional Euclidean space.

Resume

In this article we consider Carleman's functions, to find integral representation for the polygarmonious functions (Δ^n u(y)=0)defined in unbounded domain of Euclidean space which satisfies 2n≥m.
Having obtained an integral representation with the help of it, we obtain theorems of the Phragmen - Lindelöf type.

Let be an m-dimensional real Euclidean space, , -bounded one-connected region lying in a layer with a boundary where , has bounded first-order partial derivatives, assume that the area of the boundary satisfies the growth condition$R^m x = (x_1, x_2, x_3, \dots, x_m)$, $y = (y_1, y_2, y_3, \dots, y_m), x \in R^m, y \in R^m x' = (x_1, x_2, \dots, x_{m-1}, 0)$, $y' = (y_1, y_2, \dots, y_{m-1}, 0)$, $r = |x - y|$, $s = |x' - y'|$, $\alpha^2 = s, D\left\{y: y = (y_1, y_2, \dots, y_m), y_i \in R, i = \overline{1, m-1}, 0 < y_m < h, h = \frac{\pi}{\rho}, \rho > 0,\right\} \partial D = L \cup S, L = \{y: y_m = 0\}$, $S = \{y: y_m = f(y_1, \dots, y_{m-1})\} f(y_1, \dots, y_{m-1}) \partial D$,

$$\int_{\partial D} exp(-b_0 ch\rho_0 |y'|)\, ds < \infty, b_0 > 0, 0 < \rho_0 < \rho,$$

Consider the following Cauchy problem.

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$u \in C^{2n}(D)$ and (1) (2)

$$\Delta^n u(y) = 0, y \in D \left\{ \begin{array}{l} u(y) = F_0(y),\ \Delta u(y) = F_1(y),\ \ldots,\quad \Delta^{n-1}u(y) = F_{n-1}(y),\quad y \in \partial D, \\ \frac{\partial u(y)}{\partial n} = G_0(y), \frac{\partial \Delta u(y)}{\partial n} = G_1(y), \ldots, \frac{\partial \Delta^{n-1}u(y)}{\partial n} = G_{n-1}(y), \qquad y \in \partial D, \end{array} \right.$$

where given to continuous functions, -external normal to .Need to find ,satisfying (1) and (2).$F_i(y), G_i(y) \partial D \partial D u \in C^{2n}(D)$

With arbitrary initial data, the problem is unsolvable. If part of the boundary and the initial data are analytical and can be analytically continued inwards to the region, then the continuation exists only but not sustainably. Therefore, it belongs to the number of incorrectly set tasks.

In 1943, Tikhonov pointed out the practical importance of unstable problems and showed that if you narrow the class of possible solutions to a compact, the problem becomes stable [2], [3].

In 1926, Carlemann constructed an integral formula for a class of bounded functions. Andthe idea of introducing into the integral Cauchy formula an additional function that depends on the positive parameter and allows by the marginal transition to extinguish the influence of integrals along the part of the boundary where the value of the continuing function is not set.

Based on these studies, M.M. Lavrentyev introduced an important concept- the Carleman function and with its help built a regularization of the problem. With the help of the method of M.M. Lavrentyev, Sh. Yarmukhamedov obtained the regularization and solvability of the Cauchy problem for the Laplace equation in bounded areas [3]. In 2009, Juraeva N (second author) of the article obtained the regularization and solvability of the Cauchy problem for polyharmonic equations of order in some unlimited regions (with arbitrary odd odd regions). and even when) [4] - [8]. $nmm\ \ 2n < m$

Let's assume that the solution of the problem (1) - (2) exists and is continuously differentiable, up to the end points of the boundary and satisfies a certain growth condition (correctness class), which ensures the uniqueness of the solution.$u(y) 2n - 1$

Having solved the problem (1),(2) with the help of its solution we obtain theorems of the Fragman-Lindelof type. Fragman-Lindelof type theorems were the subject of research on the works of M.A. Evgrafov, I.A. Chegis and A.F. Lavrentyev,I.S. Arshon and others.

In 1960, M.A. Evgrafov and I.A. Chegis in the article - Generalization of the Fragman-Lindelof type theorem for analytic functions to harmonic functions in space - (DAN USSR, volume 134, number 2, 252-262) proved

Theorem 1. Let be the harmonic function in the cylinder , , . If the conditions are met $u(r, \phi, x) 0 \le r \le a 0 \le \phi < 2\pi - \infty < x < \infty$

$$u(a, \phi, x) = 0, \left| \frac{\partial u}{\partial r}(a, \varphi, x) \right| < c, \qquad \max_{(r,\phi)} |u(r, \phi, x)| < c \exp e^{\frac{\pi |x|}{2(a+\varepsilon)}}, \varepsilon > 0$$

then $u(r, \phi, x) \equiv 0$

Theorem 2. Let the harmonic function in the cone , ,. If the conditions are met $u(r, \theta, \phi) 0 < r < \infty 0 \le \phi < 2\pi 0 \le \theta \le \theta_0 < \pi$

$u(r,\theta_0,\phi)=0,\left|\frac{\partial u}{\partial\theta}(r,\theta_0,\varphi)\right|<c,\quad \max_{(\theta,\phi)}|u(r,\theta,\phi)|<c\,exp(r+\frac{1}{r})^{\frac{\pi}{2\theta_0}-\varepsilon},\varepsilon>0$

then $u(r,\theta,\phi)\equiv 0$

In 1961, I.A. Chegis in the article - The theorem of the Fragman-Lindelof type for harmonic functions in a rectangular cylinder - (Dokl. AS USSR, volume 136, number 3, 556-9) proved

Theorem 1. - Harmonic function in a cylinder over a rectangle , ; . If the conditions are met $u(x,y,t)0\le \mathrm{x}\le \mathrm{a}0\le \mathrm{y}\le b-\infty\le t\le\infty$

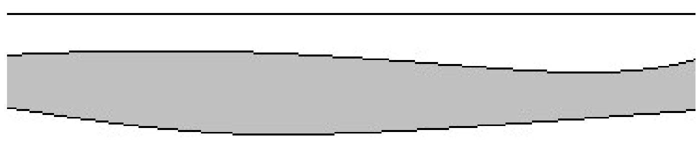
$u(x,y,t)|_\Gamma=0$, where is the G-surface of the cylinder;

$\left|\frac{\partial u(x,0,t)}{\partial y}\right|<c,\left|\frac{\partial u(x,b,t)}{\partial y}\right|<c\ ,\ \max_{(x,y)} u(x,y,t)<c\,exp\,e^{\pi|t|/b+\varepsilon},\varepsilon>0$

then.$u(x,y,t)\equiv 0$

It should be noted that the theorem uses the condition of bounding the normal derivativeof tolk 'o on two opposite faces of a rectangular cylinder. For an infinite layer, i.e., a region of view, the statement of the theorem remains valid.$-\infty<x<\infty,0\le y\le b,-\infty<t<\infty$

E.M. Landis in the book -Equations of the second order of elliptical and parabolic types. Moscow, 1971 g.55 p.) - set the problem in the form - Let the cylinder contain an area that goes to infinity (in one or both directions - all the same) in the boundary of G of this region as smooth as you like $0\le\sum_{k=0}^{n-1}\mathrm{x}_l^2<1$



Let the solution and equation be defined in the region as smooth as possible up to the boundary and . Does it follow from this that unlimited (exponentially grows when going to infinity).) $\Delta\Delta u=0u|_\Gamma=0,\quad \left.\frac{\partial u}{\partial n}\right|_\Gamma=0$

Defining the functions and the following equals: $\phi_\sigma(y,x)\quad \Phi_\sigma(y,x)(s>0,\sigma\ge 0,m-$ размерность пространство)

if $m=2k+1,k=2,3,\ldots$

$$\phi_\sigma(y,x)=c_1\frac{\partial^{k-1}}{\partial s^{k-1}}\int_0^\infty Im\frac{\varsigma(i\sqrt{s+u^2}+y_m)}{i\sqrt{s+u^2}+y_m-x_m}\frac{du}{\sqrt{u^2+s}},\qquad (3)$$

if then $m=2k,\quad k=2,3,\ldots$

$$\phi_\sigma(y,x)=c_1\frac{\partial^{k-2}}{\partial s^{k-2}}Im\left(\frac{\zeta(i\sqrt{s}+y_m)}{\sqrt{s}(i\sqrt{s}+y_m-x_m)},\right)\qquad (4)$$

Where is$\varsigma(\omega)=\quad exp\big(\sigma\omega^2-achi\rho_1(\omega-h/2)-bchi\rho_0(\omega-h/2)\big),$

$$c_1=c_0\,exp\left(-\sigma x_m^2+achi\rho_1(x_m-\frac{h}{2})+bchi\rho_0\left(x_m-\frac{h}{2}\right)\right),\quad b>b_0\left(cos\frac{\rho_0 h}{2}\right)^{-1}$$

For all odd , as well as even with the condition we assume $m \geq 3 m 2n < m$

$$\Phi_\sigma(y,x) = C_{n,m} r^{2(n-1)} \varphi_\sigma(y,x) \quad , C_{n,m} = (-1)^{\frac{m}{2}-1} \left(\Gamma(n - \frac{m}{2} + 1) 2^{2n-1} \pi^{\frac{m}{2}} \Gamma(n)\right)^{-1} \tag{5}$$

And for even with the condition defining the function at , : $m 2n \geq m \Phi_\sigma(y,x) s > 0, \sigma \geq 0, a \geq 0 2n \geq m$

$$\Phi_\sigma(y,x) = C_{n,m} \int_{\sqrt{s}}^{\infty} Im \left[\frac{exp(\sigma w + w^2) - achi \;\; _1\!\left(w - \frac{h}{2}\right)}{\omega - x_1} \right] (u^2 - s)^{n-k} du , \;\; \omega = iu + y_1 \tag{6}$$

$$C_{n,m} = (-1)^{\frac{m}{2}-1} \left(\Gamma(n - \frac{m}{2} + 1) 2^{2n-1} \pi^{\frac{m}{2}} \Gamma(n)\right)^{-1}$$ Get

Theorem 1. For the function is the place $\Phi_\sigma(y,x)$

$$\Phi_\sigma(y,x) = \begin{cases} C_{n,m} r^{2n-m} \ln r + G_\sigma(y,x), & 2n \geq m, m - \text{чётное число}, \\ C_{n,m} r^{2n-m} + G_\sigma(y,x), & \text{в остальных случаях}, \end{cases}$$

Where is

$$C_{n,m} = (-1)^{\frac{m}{2}-1} \left(\Gamma(n - \frac{m}{2} + 1) 2^{2n-1} \pi^{\frac{m}{2}} \Gamma(n)\right)^{-1}$$

where regular by variable y and continuously differentiated by $.G_\sigma(y,x) D \cup \partial D = \overline{D}$

Theorem 2. The function , defined by formula (3) is a polyharmonic function of order n po at $\Phi_\sigma(y,x) y$s > 0.

Theorem 3. With a fixed function, satisfies$x \in D \Phi_\sigma(y,x)$

$$\sum_{k=0}^{n-1} \int_{\partial D \setminus S} \left[|\Delta^k \Phi_\sigma(y,x)| - \left| \frac{\partial \Delta^k \Phi_\sigma(y,x)}{\partial \overline{n}} \right| \right] ds_y \leq C(x) \varepsilon(\sigma),$$

where the constant depends on $C(x)$x and the -external normal k when $.\bar{n} \partial D \varepsilon(\sigma) \to 0 \sigma \to \infty$

Consequence 3. The function , defined by formula (3) is a Carlemann function for the point and part of $.\Phi_\sigma(y,x) x \in D \partial D \setminus S$

Theorem 4. Let the solution of problems and , having continuous partial derivatives of order up to the end points of the boundary. If the growth condition is met for anyone $u(x)(1) - (2) 2n - 1 \partial D y \in D$

$$\sum_{k=0}^{n-1} |\Delta^k u(y)| + |grad \Delta^{n-k-1} u(y)| \leq c_0 \, exp(exp(\rho_1 |y'|))$$

and for any fulfilled the condition of growth$y \in \partial D$

$$\sum_{k=0}^{n-1} |\Delta^k u(y)| + \left| \frac{\partial}{\partial n} \Delta^{n-k-1} u(y) \right| \leq c_0 \, exp\left(a \cos \rho_2 \left(y_1 - \frac{h}{2} \right) exp(\rho_2 |y'|) \right)$$ Where is

$\rho_1 < \rho_2 < \rho_3 < \rho$

Then for anyone the integral representation is true.$x \in D$

$$u(x) = \sum_{k=0}^{n-1} \int_{\partial D} \left[\Delta^k \Phi_\sigma(y,x) \frac{\partial \Delta^{n-k-1} u(y)}{\partial n} - \Delta^{n-k-1} u(y) \frac{\partial \Delta^k \Phi_\sigma(y,x)}{\partial n} \right] ds. \quad x \in D$$

Theorem 5. Let the solution of the problem , having continuous partial derivatives of order up to the end points of the boundary . If the growth condition is met for any $u(x)(1) - (2) 2n - 1 \partial D y \in D$

$$\sum_{k=0}^{n-1}|\Delta^k u(y)| + |grad\Delta^{n-k-1}u(y)| \le c_0\, exp(exp(\rho_1|y'|))$$

and if the growth condition is met $\forall y \in \partial D$

$$\sum_{k=0}^{n-1}\int_{\partial B}\Delta^k u \cdot \exp\left(-\rho_2|y'|\right)ds < \infty, \qquad \text{where } \rho_2 < \rho_1 < \rho .$$
$$\sum_{k=0}^{n-1}\frac{\partial \Delta^k u}{\partial n}\cdot \exp\left(-\rho_2|y'|\right)ds < \infty$$

Then the integral representation is true. $\forall x \in D$

$$u(x) = \sum_{k=0}^{n-1}\int_{\partial D}\left[\Delta^k \Phi_\sigma(y,x)\frac{\partial \Delta^{n-k-1}u(y)}{\partial n} - \Delta^{n-k-1}u(y)\frac{\partial \Delta^k \Phi_\sigma(y,x)}{\partial n}\right]ds.$$

Theorem 6. Let the solution of the problem , having continuous partial derivatives of order up to the end points of the boundary . If the growth condition is met for any $u(x)(1) - (2)2n - 1\partial Dy \in D$

$$\sum_{k=0}^{n-1}|\Delta^k u(y)| + |grad\Delta^{n-k-1}u(y)| \le c_0\, exp(exp(\rho_1|y'|))$$

And if the growth condition is met $\forall y \in \partial D$

$$\Delta^k u = 0, \quad \forall k \in [0, n-1], \quad \forall y \in \partial D,$$
$$\sum_{k=0}^{n-1}\frac{\partial \Delta^k u}{\partial n}\cdot \exp\left(-\rho_2|y'|\right)ds < \infty, \quad \forall y \in \partial D, \qquad \text{where } \rho_2 < \rho_1 < \rho .$$

then the integral representation is fair $\forall x \in D$

$$u(x) = \sum_{k=0}^{n-1}\int_{\partial D}\Delta^{n-k-1}\Phi_\sigma(y,x)\frac{\partial \Delta^k u(y)}{\partial n}ds.$$

Consequence. Let the solution of the problem , having continuous partial derivatives of order up to the end points of the boundary . If the growth condition is met for any $u(x)(1) - (2)2n - 1\partial Dy \in D$

$$\sum_{k=0}^{n-1}|\Delta^k u(y)| + |grad\Delta^{n-k-1}u(y)| \le c_0\, exp(exp(\rho_1|y'|))$$

If the growth condition is met $\forall y \in \partial D$

$$\sum_{k=0}^{n-1}|\Delta^k u(y)| + \left|\frac{\partial \Delta^{n-k-1}u(y)}{\partial y}\right| \le c_0 \quad \text{where } \rho_2 < \rho_1 < \rho .$$

Тогда справедливо in $\forall x \in D$ (x)=0

Consequence**.** Let the solution of the problem , having continuous partial derivatives of order up to the end points of the boundary . If the growth condition is met for any$u(x)(1) - (2)2n - 1\partial D y \in D$

$$\sum_{k=0}^{n-1} |\Delta^k u(y)| + |grad\Delta^{n-k-1}u(y)| \le c_0\, exp(exp(\rho_1|y'|))$$

If the growth condition is met $\forall y \in \partial D$

$$\Delta^k u = 0, \quad \forall k \in [0, n-1], \quad \forall y \in \partial D,$$

$$\sum_{k=0}^{n-1} \frac{\partial \Delta^k u}{\partial n} \cdot \exp(-\rho_2|y'|)ds < \infty, \quad \forall y \in \partial D,$$

Where is. $\rho_2 < \rho_1 < \rho$

Then u(x)=0 is true $\forall x \in D$

These results are in a sense a response to the problems posed by E.M. Landis for polyharmonic functions of order . n

Theorem 7. Let be a solution to the problem having continuous partial derivatives of order up to the endpoints of the boundary . If the growth condition is met for anyone $u(x)(1) - (2)2n - 1\partial D y \in D$

$$\sum_{k=0}^{n-1} |\Delta^k u(y)| + |grad\Delta^{n-k-1}u(y) \le c_0\, exp(exp(\rho_1|y'|))|$$

and

$\forall y \in \partial D \backslash S \left|\frac{\partial \Delta^{n-1-k} u(y)}{\partial \bar{n}}\right| + |\Delta^{n-1-k}u(y)| \le 1$,

then fair

$$|u(x) - u_\sigma(x)| \le MC(\sigma)\, exp(-\sigma x_m), \quad \sigma \ge \sigma_0 > 0, \quad x \in D.$$

Where is $u_\sigma(x) = \sum_{k=0}^{n-1} \int_S \left[G_{n-k-1}(y)\Delta^k \Phi_\sigma(y,x) - F_{n-k-1}(y)\frac{\partial \Delta^k \Phi_\sigma(y,x)}{\partial \bar{n}}\right] ds$

$C(\sigma)$- A polygamy with well-defined coefficients.

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