ON CERTAIN TYPES OF GROUPOIDS AND TOPOLOGICAL GROUPOID

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Abstract:
In this work, we introduce new kinds of topological groupoid which are source proper groupoid , submersive groupoid , and use them to construct a new kind of groupoid space which are source proper group space and submersive group space . The main objective of this paper is to find new relationships between these types written as proposition and can be usedin the field of algebraic topology.

Keywords: groupoid, topological space, topological groupoid, source proper groupoid.

1. Introduction:

The main objective of this research is to study certain types of topological groupoid, which is source proper groupoid, denoted by (SC-groupoid), submersive groupoid,denoted by (SSC-groupoid),source proper group-space ,denoted by(SC $\mathcal{T}$-space)and submersive group-space denoted by (SSC $\mathcal{T}$-space)and also some properties of these groupoids are studied. The category C contain for:(i) The class for objects. (ii) If $\mathfrak{r} \in$ $morphism$ (S, L) with domain S and range L, we write $\mathfrak{r}: S \to L$ for all arranged pair of things S and L. (iii) A function that associates two morphisms $\mathfrak{r}: S \to L$ and $\mathfrak{r}_1: L \to H$ their composite $\mathfrak{r}_1 o \mathfrak{r} : S \to H$ for all ordered triple of objects S, L, and H. This satisfies the following axioms:(1) The associative axiom: let $\mathfrak{r}: S \to L, \mathfrak{r}_1: L \to H$, $\mathfrak{r}_2: H \to K$ then $\mathfrak{r}_2(\mathfrak{r}_1\mathfrak{r}) = (\mathfrak{r}_2\ \mathfrak{r}_1)\ \mathfrak{r}$.(2) the identity axiom of all objects L there is the morphism $I_L: L \to L$ where let $\mathfrak{r}: S \to L$, implies $I_L\mathfrak{r} = \mathfrak{r}$, and if $\mathfrak{r}_1: L \to H$, then $\mathfrak{r}_1 I_L = \mathfrak{r}_1$[5].The category of continuous maps and topological spaces that is denoted by **T**[3].A groupoid be the pair of sets (N,M) where be get: (1) onto functions $\alpha: N \to M$, $\beta: N \to M$ they are called respectively, a source function , a target function . (2) one-to-one function $w: M \to N$ known as the object inclusion with $\alpha o w = I_M$, $\beta o w = I_M$ where $I_M : M \to M$.(3) A partial composition λ in N . A compositional rule for the set $N * N$ is defined as $N * N = \{(n_1, n_2) \in N \times N | \alpha\ (n_1) = \beta(n_2)\}$ "fiber product of β and α over M" s.t :(i) $\lambda(n, \lambda(n_1, n_2)) = \lambda(\lambda(n_1, n_2), n_2), \forall\ (n, n_1), (n_1, n_2) \in N * N$.(ii) $\alpha\ (\lambda\ (n_1, n_2)) = \alpha\ (n_2), \beta(\lambda\ (n_1, n_2)) = \beta(n_1)$ for each $(n_1, n_2) \in N * N$.
(iii) $\lambda\ (n_1, w(\alpha(n_1))) = n_1$ and $\lambda\ (w(\beta(n_1)), n_1) = n_1$, for all $n_1 \in N$. (4) A bijection $\delta: N \to N$ known as the inversion of N satisfying:(a)$\alpha(\delta(n_1)) = \beta(n_1), \beta(\delta(n_1)) = \alpha(n_1))$, for all $n_1 \in N$.(b)$\lambda(\delta(n_1), n_1) = w(\alpha\ (n_1))$, $\lambda(n_1, \delta(n_1)) = w(\beta(n_1))$, for all $n_1 \in N$. We they note $\delta(n_1) = (n_1)^{-1}$, known an inverse for element $n_1 \in N$,$w(x) = x$ known a unit for element on N associated into an element $x \in M$. We will take notes $(n_1, n_2) = n_1 n_2$. We say that N is a groupoid on M or N is known a groupoid M be known of base.

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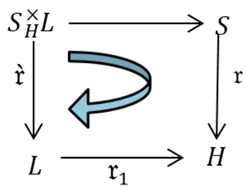
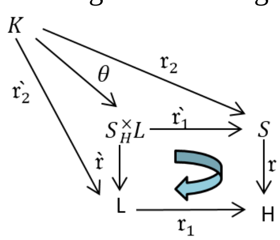
We call say this is N be the groupoid in M [7].see[8] For every s ∈ S, $\prod_{s\in S} xs \xrightarrow{Ps} xs$ such that $Ps(x) = xs$, for ever $s \in \prod_{s\in S} xs$, Ps is called the projection map[7]. The morphism for groupoids be the pair for function $(\mu, \mu_o): (N, M) \to (\grave{N}, \grave{M})$ where $\grave{\alpha} o\, \mu = \mu_o o \alpha$, $\grave{\beta} o\, \mu = \mu_o o \beta$, $\mu(\lambda(n, \grave{n})) = \grave{\lambda}(\mu(n), \mu(\grave{n}))$ for all $(n, \grave{n}) \in N * N$ [3]
If $(\mu, \mu_o): (N, M) \to (\grave{N}, \grave{M})$ is the morphism for groupoids implies a kernal of μ be a set ker $\mu = \{n \in N \mid \mu(n) \in \grave{w}(\grave{M})\}$[2].

2. On Topological groupoid: Definition(1):[1]Suppose $r: S \to H$, $\mathfrak{r}_1: L \to H$ is continuous maps, when S , L and H be topological spaces .Then the fiber product of r and $\mathfrak{r}_1$ is $S{}^{\times}_{H}L = \{(s, l): \mathfrak{r}(s) = \mathfrak{r}_1(l)\}$ which is a sub space of $S{}^{\times}_{H}L$. i.e, the next diagram :



(1)…is commutative in **T** where

$\grave{\mathfrak{r}}_1 = \mathrm{pr}_1 |_{S{}^{\times}_{H}L}$, $\grave{\mathfrak{r}} = \mathrm{pr}_2 |_{S{}^{\times}_{H}L}$,and ***T*** the category of topological spaces and continuous maps. The shape (1) result an universal property ,i.e, let K denotes any topological space. and $\mathfrak{r}_2$: K→S, $\grave{\mathfrak{r}}_2$: K→L both continuous functions in ***T*** s.t $\mathfrak{r} \circ \mathfrak{r}_2 = \mathfrak{r}_1 \circ \grave{\mathfrak{r}}_2$ then there exist a unique continuous function $\theta : K \to S \times L$ making the following diagram:



The definition of the function θ is $\theta(b) = (\mathfrak{r}_2(b), \grave{\mathfrak{r}}_2(b))$ for every $b \in K$. In(1), if r is injective or surjective map so is $\grave{\mathfrak{r}}$ and the same thing applies to $\mathfrak{r}_1$ and $\grave{\mathfrak{r}}_1$.

Definition (2):[2]
Suppose S , L is topological space Then $\mathfrak{r}: S \to L$ be call proper ,let a function $\mathfrak{r} \times I_H : S \times H \to L \times H$ is closed for all topological space H and $\mathfrak{r}$ is continuous .

Proposition (3):[4]
Let $\mathfrak{r}: S \to L$ be continuous injective function then $\mathfrak{r}$ is proper function if and only if $\mathfrak{r}$ is closed function and $\mathfrak{r}$ is homeomorphism of S on to a closed subspace of L.

Proposition (4):[2]
If we define a proper function $\mathfrak{r}: S \to L$, implies a restriction for $\mathfrak{r}$ into closed of subset B for S be the proper function of B into L.

Remark (5):[3]
If (N, M) is any groupoid, then:
(1)The subset of N denoted by $N_x = \alpha^{-1}(x)$ is known as the α-fiber at $x \in M$.
(2)The subset of N denoted by ${}_y\text{N} = \beta^{-1}(y)$ is called the β -fiber at $\in M$.
(3) ${}_yN_x = N_x \cap {}_yN$ a set for elements in N s.t have y as a target and
x as a source
(4) The function $\tau: N \to M \times M$; $\tau(n) = (\beta(n), \alpha(n))$ is known as the transitor of N and ${}_yN_x = \tau^{-1}(y, x)$, for every $x, y \in M$.

Definition (6):[2]
The topological group spaces be the set $\mathcal{T}$containing structures:
(1) $\mathcal{T}$ be the topological space .
(2) $\mathcal{T}$ is a group.
The inversion law $\nu: \mathcal{T} \to \mathcal{T}$and the composition law $\gamma: \mathcal{T} \times \mathcal{T} \to \mathcal{T}$ are both continuous.
Definition (7):[2]
If $\mathcal{S}$ is a topological space , $\mathcal{T}$ is the topological group. The left action for $\mathcal{T}$ into $\mathcal{S}$ be the continuous function $\pi: \mathcal{T} \times \mathcal{S} \to \mathcal{S}$ with the following properties:
(1) $\pi(e, u) = u$, for all $u \in \mathcal{S}$ where e is the element of identity in $\mathcal{T}$.
(2) $\pi(a, \pi(h, u)) = \pi(\gamma(a, h), u)$, $\forall u \in \mathcal{S}$, where γ is the law of composition of $\mathcal{T}$.
The action π and the space $\mathcal{S}$ is known as group space and indicated by $\mathcal{T}$ -space more specifically (left $\mathcal{T}$ - space).

Definition (8):[4]
If $\mathcal{S}$ be a $\mathcal{T}$ -space then:
(1)The orbit of $u \in \mathcal{S}$ is defined as $orb(u) = \pi(u, \mathcal{T}) = \{\pi(u, a): a \in \mathcal{T}\}$ and the collection of $\mathcal{S}$ orbits is known to as orbit space and represented by $\mathcal{S}/\mathcal{T}$.
(2) The stabilizer of $u \in \mathcal{S}$ is the set of
(2) The stabilizer of $u \in \mathcal{S}$ is the set of elements in $\mathcal{T}$ that fix u. $stab(u) = \mathcal{T}_u = \{a \in \mathcal{T} \mid \pi(a, u) = u\}$.
(3) $\mathcal{S}$ is free $\mathcal{T}$-space if the action of $\mathcal{T}$ on $\mathcal{S}$ is free.

Definition (9):[6]
Let $\mathcal{S}$ be a $\mathcal{T}$ -space. An action π of $\mathcal{T}$ on $\mathcal{S}$ is said to be:
(1) Transitive if $orb(u) = \mathcal{S}$ for all $\in \mathcal{S}$.
(2) Trivial if $\ker = \mathcal{T}$.
(3) Free if the stabilizer of every element is trivial, i.e. $stab(u) = \{e\}$, for all $u \in \mathcal{S}$.

Theorem (10):[1]
If $\mathcal{S}$ is Hausdorff space and $\mathcal{S}$ be $\mathcal{T}$ -space with $\mathcal{T}$ compact and then:
(1) $\mathcal{S} / \mathcal{T}$ is Hausdorff .
(2) The law of action $\pi: \mathcal{S} \times \mathcal{T} \to \mathcal{S}$ is a closed map.

(3) $\mathcal{S}\ /\ \mathcal{T}$ is compact if and only if $\mathcal{S}$ is compact
(4) The map $\varphi: \mathcal{S} \rightarrow \mathcal{S}\ /\ \mathcal{T}$ is proper.

Definition (11):[6]
The topological groupoid be the groupoid (N, M) with topologies onto , M s.t a functions $\beta: N \rightarrow M$, $\alpha: N \rightarrow M$, $w: M \rightarrow N$, $\lambda: N * N \rightarrow N$ and $\delta: N \rightarrow N$ are continuous functions where $N * N$ has the$N \times N$ subspace topology . A topological groupoid is denoted by TG.

Definition (12):[3]
A morphism of TG is morphism of groupoids $(f_1, f_2): (N, M) \rightarrow (N_1, M_1)$ such that f_1 and f_2 are continuous.

3. SC-groupoid , SSC-groupoid , $SC\ \mathcal{T}$ -space , $SSC\ \mathcal{T}$ -space

Definition (1):[1]
TG known as the source proper groupoid ((SC-groupoid)) if:
(1) The base space M is a Hausdorff.
(2) The map $\alpha : N \rightarrow M$ is a proper.

Proposition (2):[1]
If (N, M) be an SC-groupoid then the functions $w: M \rightarrow N$, $\beta: N \rightarrow M$ and $w: M \rightarrow N$ are proper.

Definition (3):
If (N, M) be an SC-groupoid then the function $\xi_x: N_x \times N_x \times N_x \rightarrow N$, is defined as $\xi_x(n_1, n_2, n_3) = \lambda(n_1, \delta(n_2 . n_3))$ is proper map, for every $x \in M$.

Definition (4):[1]
A $\mathcal{T}$-space $\mathcal{S}$ is referred to as source proper group space (($SC\mathcal{T}$ -space)) if:
(1) $\mathcal{S}$ is free $\mathcal{T}$-space
(2) The action groupoid $(\mathcal{S} \times \mathcal{T}, \mathcal{S})$ is SC-groupoid.

Definition (5):[4]
Let $(f_1, \grave{\hat{f}}_1): (N_1, M_1) \rightarrow (\grave{N}_1, \grave{M}_1)$ and $(\xi_2, \grave{\hat{\xi}}_2): (N_2, M_2) \rightarrow (\grave{N}_2, \grave{M}_2)$ each are proper functions, implies a direct sum$(\xi_1 \oplus \xi_2, \grave{\hat{\xi}}_1 \oplus \grave{\hat{\xi}}_2): (N_1 \oplus N_2, M_1 \oplus M_2) \rightarrow (\grave{N}_1 \oplus \grave{N}_2, \grave{M}_1 \oplus \grave{M}_2)$ be proper functions.

Proposition (6):[2]
Let $\mathcal{T}(\mathcal{S}, \varphi, M)$ is the cartan principal bundle implies $\mathcal{S} \times \mathcal{S} / \mathcal{T}$ be the TG of base M. A pair $(\mathcal{S} \times \mathcal{S} / \mathcal{T}, M)$ is known as the Ehresmann groupoid.

Proposition (7):[4]
The function $\xi^*: \mathcal{S} \underset{M}{\times} \mathcal{S} \rightarrow \mathcal{T}$ is continuous if $\mathcal{S}$ be $SC\ \mathcal{T}$ -space.

Proposition (8):[4]

If (N, M) be an SC-groupoid then the α-fiber space N_x is $SC_x N_x$-space, for all $x \in M$.

Definition (9) :

A transitive SC-groupoid (N, M) is known as submersive groupoid (SSC -groupoid), when the function $\beta_x : N_x \times N_x \to M$ is submersion for every $x \in M$.

Example (10) :

Every compact transitive TG on discrete space M is SSC-groupoid. Since for all $n \in N_x$ then $U = \{ \beta_x (n_1, n_2) \}$ is a neighborhood that is open in M. $\beta_x (n_1, n_2)$ and the constant map $\upsilon : U \to N_x$, $\upsilon(\beta_x(n_1, n_2)) = n_1 n_2$ is continuous right inverse to $\beta_x : N_x \times N_x \to M$

Definition (11):[1]

An $SC\mathcal{T}$-space $\mathcal{S}$ is called submersive group space "$SSC\mathcal{T}$-space" if the map $\varphi : \mathcal{S} \to \mathcal{S}/\mathcal{T}$ is submersion.

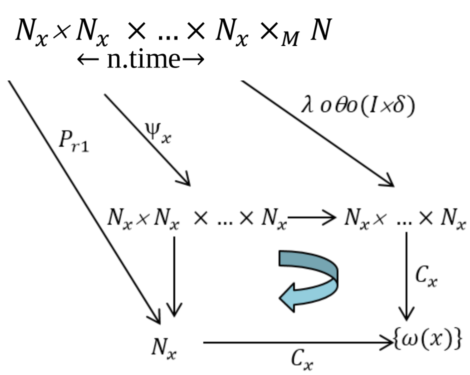
4. The results of SC-groupoid and SSC-groupoid

Proposition (1):

If (N, M) be an SC-groupoid then the function $\xi_x : \underbrace{N_x \times N_x \times \dots \times N_x}_{\leftarrow n.time \rightarrow} \to N$, defined by $(n_1, n_2, \dots, n_n) = \lambda(n_1, \delta(n_2 \cdot \dots \cdot n_n))$ is proper map, for every $x \in M$.

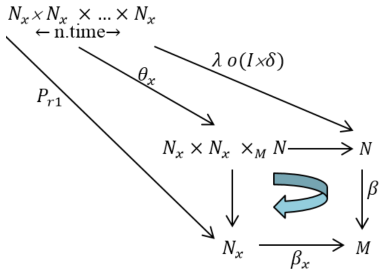
Proof:

Consider the following diagram:



In which $C_x o P_{r1}(n, h) = C_x o \lambda \, o \theta o (I \times \delta)(n, h)$ where C_x is the constant function, θ be the permutation function and, $\omega(x)$ be identity element in ${}_x N_x$ and $N_x \times N_x \times \dots \times N_x \times_M N$ is the fiber product of β_x and β over M. Hence there exists a unique morphism

$\psi_x : \underbrace{N_x \times N_x \times \dots \times N_x}_{\leftarrow n.time \rightarrow} \times_M N \to \underbrace{N_x \times N_x \times \dots \times N_x}_{\leftarrow n.time \rightarrow}$ is given by $\psi_x(n_1, n_2, \dots, n_n) = (\lambda(\delta(n_1). n_2), \dots n_n)$ by the universal property of fiber product making $\boldsymbol{T}$ commutative over the whole diagram. Now, consider the following diagram:



$(\alpha_1 \oplus \alpha_2 \oplus \dots . \oplus \alpha_n)$

In which $\beta o \lambda o(I \times \delta)(n_1, n_2, \dots, n_n) = \beta x o Pr_1(n_1 . n_2 . \dots . n_n)$, since $\beta(\lambda(n_1, \delta(n_1 . n_2 . \dots . n_n))) = \beta(n_1)$. Hence there exist a unique morphism $\theta_x : N_x \times N_x \times \dots \times N_x \rightarrow N_x \times N_x \times \dots \times N_x \underset{M}{\times} N$, given by $\theta_x(n_1 . n_2 . \dots . n_n) = \big(\lambda\big(n_1, \delta(n_1 . n_2 . \dots . n_n)\big), n_1\big)$ by the universal property of fiber product making the whole diagram commutative in $\boldsymbol{T}$. $\xrightarrow{C_x}$

Clearly $\theta_x o \psi_x = I$ and $\psi_x o \theta_x = I$. Hence θ_x is homeomorphism an then $\xi_x : N_x \times \dots \times N_x$, $\xi_x(n_1 . n_2 . \dots . n_n) =$then $\xi_x : N_x \times N_x \times \dots \times N_x$, $\xi_x(n_1 . n_2 . \dots . n_n) = \lambda\big(n_1, \delta(n_2 . \dots . n_n)\big)$, $\forall (n_1, n_2, \dots, n_n) \in N_x \times N_x \times \dots \times N_x$,is proper map (Propositions(2,4)), since $N_x \times N_x \times \dots \times N_x \underset{M}{\times} N = ((\beta \times \dots \times \beta)_x \times \beta)^{-1}(\Delta M)$ is closed subspace of $N_x \times N_x \times \dots \times N$.

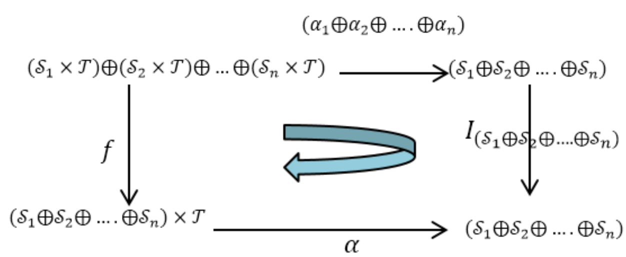
Proposition (2):

Let $\mathcal{S}_i$ be $SC\mathcal{T}$ -space, $i = 1,2, \dots, n$ then $\oplus_{i=1}^{n} \mathcal{S}_i$ is $SC\mathcal{T}$ -space.

Proof:

Define $\psi : \oplus_{i=1}^{n} \mathcal{S}_i \times \mathcal{T} \rightarrow \oplus_{i=1}^{n} \mathcal{S}_i$ by $\psi((u_1 \oplus u_2 \oplus \dots \oplus u_n), r) = (\pi_1(u_1, r) \oplus \pi_2(u_2, r) \oplus \dots \oplus \pi_n(u, r))$ for every $(u_1 \oplus u_2 \oplus \dots \oplus u_n) \in \oplus_{i=1}^{n} \mathcal{S}_i$ *and* $r \in \mathcal{T}$, which is continuous . Where π_i is a law of action of $\mathcal{T}$ on $\mathcal{S}_i, i = 1,2, \dots, n$.

Now if $\psi((u_1 \oplus u_2 \oplus \dots \oplus u_n), r) = (u_1 \oplus u_2 \oplus \dots \oplus u_n)$ then $r = e$ since $\mathcal{S}_i$ is free $\mathcal{T}$ -space, $i = 1,2, \dots, n$. Hence $\oplus_{i=1}^{n} \mathcal{S}_i$ is free $\mathcal{T}$ -space and the action groupoid $((\oplus_{i=1}^{n} \mathcal{S}_i) \times \mathcal{T}, \oplus_{i=1}^{n} \mathcal{S}_i)$ is SC-groupoid since $\oplus_{i=1}^{n} \mathcal{S}_i$ is a Hausdorff ($\mathcal{S}_1$, $\mathcal{S}_{2,\dots,}$ $\mathcal{S}_n$ are Hausdorff) and the source map:α: $(\oplus_{i=1}^{n} \mathcal{S}_i) \times \mathcal{T} \rightarrow \oplus_{i=1}^{n} \mathcal{S}_i$; $\alpha((u_1 \oplus u_2 \oplus \dots \oplus u_n), r) = (u_1 \oplus u_2 \oplus \dots \oplus u_n) =$
$(\alpha_1 (u_1, r) \oplus \alpha_2 (u_2, r) \oplus \dots \oplus \alpha_n(u_n, r))$ be proper by using a next commutative diagram into $\boldsymbol{T}$:



The map defined by: f:$f((u_1, r_1) \oplus (u_2, r_2) \oplus \dots, (u_n, r_n))$= $((u_1 \oplus u_2 \oplus \dots \oplus u_n), r_n)$ which is

$(\mathcal{S}_1 \oplus \mathcal{S}_2 \oplus \dots . \oplus \mathcal{S}_n)$

surjective continuous since

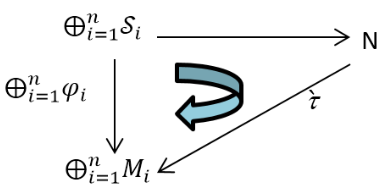
$f=(\mathcal{S}_1\times\mathcal{T})\oplus(\mathcal{S}_2\times\mathcal{T})\oplus\ldots\oplus(\mathcal{S}_n\times\mathcal{T})\overset{\cong}{\rightarrow}(\mathcal{S}_1\oplus\mathcal{S}_2\oplus\ldots.\oplus\mathcal{S}_n)\times\mathcal{T}\xrightarrow{p_{r_{1,2,3,\ldots,n}}}(\mathcal{S}_1\oplus\mathcal{S}_2\oplus\ldots.\oplus\mathcal{S}_n)\times\mathcal{T}$

$((u_1,r_1)\oplus(u_2,r_2)\oplus\ldots\oplus(u_n,r_n))\longrightarrow((u_1\oplus u_2\oplus\ldots.\oplus u_2),(r_1,r_2\ldots,r_n))\rightarrow$

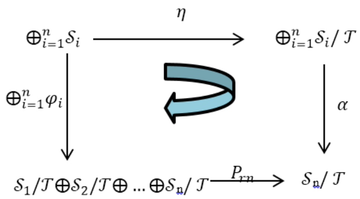
$((u_1\oplus u_2\oplus\ldots.\oplus u_2),r_n)$Therefore $(\mathcal{S}_1\oplus\mathcal{S}_2\oplus\ldots.\oplus\mathcal{S}_n)$ is $SC\mathcal{T}$-space.

Proposition (3):
Let $\mathcal{S}_i$ be $SC\mathcal{T}$ -space, $i=1,2,\ldots,n$ then $\oplus_{i=1}^{n}\mathcal{S}_i$ is $SC\mathcal{T}$ -space and the collection of every orbits $N=\oplus_{i=1}^{n}\mathcal{S}_i/\mathcal{T}$ is SC -groupoid of base $M=\mathcal{S}_i/\mathcal{T}, i=1,2,\ldots,n$ with identification topology associated to the morphismhh η: $\oplus_{i=1}^{n}\mathcal{S}_i\rightarrow\oplus_{i=1}^{n}\mathcal{S}_i/\mathcal{T}$, $\eta(u_1,u_2,\ldots,u_n)=[(u_1,u_2,\ldots,u_n)]$.

Proof:
Since$\oplus_{i=1}^{n}\mathcal{S}_i$ is $SC\mathcal{T}$ -space (Proposition(4,2)).By ψ: $(\oplus_{i=1}^{n}\mathcal{S}_i)\times\mathcal{T}\rightarrow\oplus_{i=1}^{n}\mathcal{S}_i$,defined by $\psi((u_1\oplus u_2\oplus\ldots.\oplus u_n),r)=(\varphi(u_n,r))$ where π_i is the law of action of $\mathcal{T}$ on $\mathcal{S}_i$, $i=1,2,\ldots,n$.To show that $(N=\oplus_{i=1}^{n}\mathcal{S}_i/\mathcal{T}, M=\mathcal{S}_i/\mathcal{T}, i=1,2,\ldots,n)$ is an SC-groupoid.(1)$(N=\oplus_{i=1}^{n}\mathcal{S}_i/\mathcal{T}, M=\mathcal{S}_i/\mathcal{T}, i=1,2,\ldots,n)$ is TG since the functions w and δ are continuous functions and λ is continuous and then α is continuous since the function $\oplus_{i=1}^{n}\varphi_i$: $\oplus_{i=1}^{n}\mathcal{S}_i\rightarrow\mathcal{S}_1/\mathcal{T}\oplus\mathcal{S}_2/\mathcal{T}\oplus\ldots\oplus\mathcal{S}_n/\mathcal{T}$ is constant on the identification function's fibers, η. Consequently, unique morphism exists $\grave{\tau}=\tau: N\rightarrow\oplus_{i=1}^{n}M_i$ in ***T*** If There is a commutative relationship in ***T*** in the following diagram:



by way of the identification function 's universal property, so α: $N=\oplus_{i=1}^{n}\mathcal{S}_i/\mathcal{T}\overset{\grave{\tau}}{\rightarrow}\oplus_{i=1}^{n}M_i\overset{P_1}{\rightarrow}M_1$ is continuous and β is continuous, because $\beta=\alpha o\delta$. (2) The base space $M=\mathcal{S}_i/\mathcal{T}$, $i=1,2,\ldots,n$ is a Hausdorff (Theorem (2,10)).(3)To prove the source function α: $\oplus_{i=1}^{n}\mathcal{S}_i/\mathcal{T}\rightarrow\mathcal{S}_i/\mathcal{T}$,$i=1,2,\ldots,n$, $\alpha([(u_1\oplus u_2\oplus\ldots\oplus u_n)])=\varphi_n(u_n)$ is proper. The map $\mathcal{T}\overset{\cong}{\rightarrow}\{(u_1\oplus u_2\oplus\ldots\oplus u_n,u_o)\}\times\mathcal{T}\overset{\Psi^*}{\rightarrow}\psi((u_1\oplus u_2\oplus\ldots\oplus u_n,u_o),\mathcal{T})$ is continuous and then all orbit $\psi((u_{1\oplus}^{\circ}\cdots_{\oplus}u_n^{\circ}),\mathcal{T})$ is compact ($\mathcal{T}$ is compact) s.t $\psi^*=\psi|\{(u_1\oplus u_2\oplus\ldots\oplus u_n,u_o)\times\mathcal{T}\}$. But α-fiber space, however $N_{\varphi(u_o)}=\alpha^{-1}(\varphi(u_o))$ is closed subspace of η $\psi((u_1\oplus u_2\oplus\ldots\oplus u_n,u_o),\mathcal{T})$ since $\mathcal{S}_i/\mathcal{T}$, $i=1,2,\ldots,n$ is Hausdorff (Theorem (2,10). Hence, α-fiber space $N_{\varphi(u_o)}$is compact for all $u_o\in\mathcal{S}_i$, $i=1,2,\ldots,n$. Hence, a fibers of α be compact. To prove that the function α be closed, the function f^{u_o}: $\mathcal{S}_i\rightarrow N^{\varphi(u_o)}$, $i=1,2,\ldots,n$ defined by $f_{u_o}(u_1\oplus u_2\oplus\ldots\oplus u_n)=[(u_o,u_1\oplus u_2\oplus\ldots\oplus u_n)]$ is homeomorphism. Thus $\mathcal{S}_1,\mathcal{S}_{2,\ldots,}\mathcal{S}_n$ are compact ($N_{\varphi(u_o)}$are compact) and then $\mathcal{S}_i/\mathcal{T}$ is compact, $i=1,2,..,n$. Consider the commutative diagram that follows in ***T*:**



where the η , $\oplus_{i=1}^{n}\varphi_i$ will be closed (Proposition (2,4)) P_{rn} is closed. Thus, the function α is closed. Thus, the function α is proper and then $(\oplus_{i=1}^{n}\mathcal{S}_i/\mathcal{T}, \mathcal{S}_n/\mathcal{T})$ is an SC-groupoid.

Proposition (4):
If (N, M) be an SC-groupoid then the α-fiber spaces $\oplus_{i=1}^{n} N_{x_i}$ and $\oplus_{j=1}^{n} N_{y_j}$ are isomorphic group spaces for any any $n \in {}_y N_x$.

Proof:
$\oplus_{i=1}^{n} N_{x_i}$ is $SC_x N_x$-space and $\oplus_{j=1}^{n} N_{y_j}$ is $SC_y N_y$-space $\oplus_{i=1}^{n} N_{x_i}$ (Proposition (3,8)).

Homeomorphic to $\oplus_{j=1}^{n} N_{y_j}$ by $R_{\delta(n_1,n_2\dots,n_n)}: N_{x1}\oplus N_{x2}\oplus \dots\oplus N_{x3} \to$

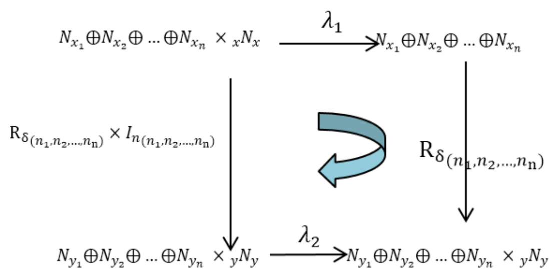
$N_{y1}\oplus N_{y2}\oplus\dots\oplus N_{yn}$

$R_{\delta(n_1,n_2\dots,n_n)}(h_1\oplus h_2\oplus\dots\oplus h_n) = \lambda(n_1,n_2,\dots,n_n, \lambda(h_1\oplus h_2\oplus\dots\oplus h_n, \delta(n_1,n_2,\dots,$

$n_n)))$, vertex group ${}_x N_x$ isomorphic to the vertex group ${}_y N_y$ using inner automorphism

$I_{n(n_1,n_2\dots,n_n)}(h_1\oplus h_2\oplus\dots\oplus h_n) = \lambda(n_1, n_2 \dots, n_n, \lambda(h_1\oplus h_2\oplus\dots\oplus h_n, \delta(n_1,n_2,\dots, n_n)))$ and the

next diagram be commutative into ***T***:



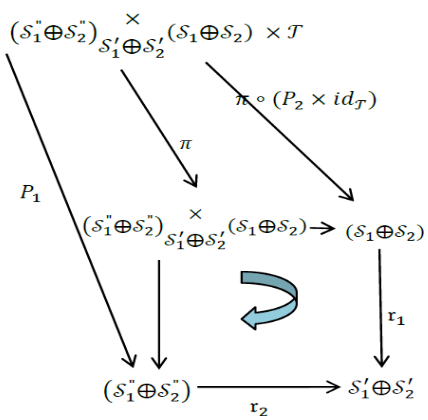
$N_{y_1}\oplus N_{y_2}\oplus\dots\oplus N_{y_n}$

Where $\lambda_1=\lambda|_{N_{x_1}\oplus N_{x_2}\oplus\dots\oplus N_{x_n}\times {}_xN_x}$
and$\lambda_2 = \lambda|_{N_{y_1}\oplus N_{y_2}\oplus\dots\oplus N_{y_n}\times {}_yN_y}$.
Hence the pair
$(R_{\delta(n_1,n_2,\dots,n_n)}, I_{n(n_1,n_2,\dots,n_n)})$ represent an isomorphism of group spaces.

Proposition (5):
Suppose $\mathcal{S}_1\oplus\mathcal{S}_2$ is the $SC\mathcal{T}$ -space , $\acute{\mathcal{S}}_1\oplus\acute{\mathcal{S}}_2$ is the equivarient space for $\mathcal{S}_1\oplus\mathcal{S}_2$, let $\mathcal{S}_1''\oplus\mathcal{S}_2''$ is a Hausdorff space and $\mathfrak{r}_2: \mathcal{S}_1''\oplus\mathcal{S}_2'' \to \mathcal{S}_1'\oplus\mathcal{S}_2'$ is continuous function then $(\mathcal{S}_1''\oplus\mathcal{S}_2'') \underset{\mathcal{S}_1'\oplus\mathcal{S}_2'}{\times} (\mathcal{S}_1\oplus\mathcal{S}_2)$ is the fiber product of equivarient function $\mathfrak{r}_1$ and $\mathfrak{r}_2$ over $\mathcal{S}_1'\oplus\mathcal{S}_2'$.

Proof:

Let π' ☹$(\mathcal{S}_1^{"}\oplus\mathcal{S}_2^{"})\ \underset{\acute{\mathcal{S}}_1\oplus\acute{\mathcal{S}}_2}{\times}(\mathcal{S}_1\oplus\mathcal{S}_2))\times\mathcal{T}\rightarrow$
$(\mathcal{S}_1^{"}\oplus\mathcal{S}_2^{"})\underset{\acute{\mathcal{S}}_1\oplus\acute{\mathcal{S}}_2}{\times}(\mathcal{S}_1\oplus\mathcal{S}_2$)by
$\pi'((u_1^{"},u_2^{"}),(u_1,u_2),a)=((u_1^{"},u_2^{"}),\pi\left((u_1,u_2),a\right))$ where π be a law action for $\mathcal{T}$ onto $\mathcal{S}_1\oplus\mathcal{S}_2$. π'be the continuous action for $\mathcal{T}$ onto $(\mathcal{S}_1^{"}\oplus\mathcal{S}_2^{"})\underset{\mathcal{S}_1'\oplus\mathcal{S}_2'}{\times}(\mathcal{S}_1\oplus\mathcal{S}_2)$,
(1)π'(($u_1^{"}\oplus u_2^{"}$),($u_1\oplus u_2$),e)=($u_1^{"}\oplus u_2^{"}$,
π($u_1\oplus u_2$,e))=(($u_1^{"}\oplus u_2^{"}$),($u_1\oplus u_2$)),since $\pi\left((u_1\oplus u_2),e\right)=(u_1\oplus u_2)$ for every $\left((u_1^{"}\oplus u_2^{"}),(u_1\oplus u_2)\right)\in(u_1^{"}\oplus u_2^{"})\underset{\mathcal{S}_1'\times\mathcal{S}_2'}{\times}(u_1\oplus u_2)$. (2) π'
$((u_1^{"}\oplus u_2^{"}),(u_1\oplus u_2))$,μ($r_1,r_1$))=(($u_1^{"}\oplus u_2^{"}$),$\pi$(($u_1\oplus u_2$),$\lambda(r_1,r_1)$)=
(($u_1^{"}\oplus u_2^{"}$) , π (π (($u_1\oplus u_2$) ,r_1)$,r_2$)= π' (π' ((($u_1^{"}\oplus u_2^{"}$),($u_1\oplus u_2$) ,r_1)$,r_2$), where λ is law of composition.(3) If π' ((($u_1^{"}\oplus u_2^{"}$),($u_1\oplus u_2$)),r_1)=(($u_1^{"}\oplus u_2^{"}$),($u_1\oplus u_2$)) ⇒π (($u_1\oplus u_2$),r)= ($u_1\oplus u_2$) ⇒ r=e, since $\mathcal{S}_1\oplus\mathcal{S}_2$ is free $\mathcal{T}$-space. (4) Consider the following diagram:



In which $r_1\circ\pi\circ(P_2\times id_{\mathcal{T}})((u_1^{"}\oplus u_2^{"}),(u_1\oplus u_2),r)$
$=r_2\circ P_1((u_1^{"}\oplus u_2^{"}),(u_1\oplus u_2),r)$. Hence there exist a unique morphism $\theta=\pi'$:
$((\mathcal{S}_1^{"}\oplus\mathcal{S}_2^{"})\underset{\mathcal{S}_1'\oplus\mathcal{S}_2'}{\times}(\mathcal{S}_1\oplus\mathcal{S}_2))\times\mathcal{T}\rightarrow(\mathcal{S}_1^{"}\oplus\mathcal{S}_2^{"})\underset{\mathcal{S}_1'\oplus\mathcal{S}_2'}{\times}(\mathcal{S}_1\oplus\mathcal{S}_2)$ given by $\pi'((u_1^{"}\oplus u_2^{"}),(u_1\oplus u_2),r)=$
$((u_1^{"}\oplus u_2^{"})$,
$\pi(\ (\mathcal{S}_1\oplus\mathcal{S}_2),r))$ making a whole diagram commutative into ***T*** by an universal property for fiber product. Hence $(\mathcal{S}_1^{"}\oplus\mathcal{S}_2^{"})\underset{\mathcal{S}_1'\oplus\mathcal{S}_2'}{\times}(\mathcal{S}_1\oplus\mathcal{S}_2)$ is free $\mathcal{T}$-space. To show that action groupoid $(((\mathcal{S}_1^{"}\oplus\mathcal{S}_2^{"})\underset{\mathcal{S}_1'\oplus\mathcal{S}_2'}{\times}(\mathcal{S}_1\oplus\mathcal{S}_2)\times\mathcal{T},(\mathcal{S}_1^{"}\oplus\mathcal{S}_2^{"})\underset{\mathcal{S}_1'\oplus\mathcal{S}_2'}{\times}(\mathcal{S}_1\oplus\mathcal{S}_2)\ )$ is SC-groupoid. $(\mathcal{S}_1^{"}\oplus\mathcal{S}_2^{"})$ is Hausdorff space and $(\mathcal{S}_1\oplus\mathcal{S}_2)$ is Hausdorff space (Definition (3,4)). Hence $(\mathcal{S}_1^{"}\oplus\mathcal{S}_2^{"})\underset{\mathcal{S}_1'\oplus\mathcal{S}_2'}{\times}(\mathcal{S}_1\oplus\mathcal{S}_2)$ is Hausdorff

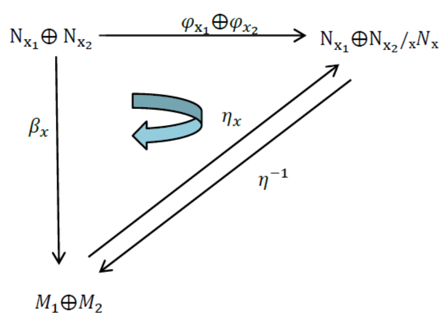
space (subspace of Hausdorff $\left(\mathcal{S}_1''\oplus\mathcal{S}_2''\right) \underset{\mathcal{S}_1'\oplus\mathcal{S}_2'}{\times} (\mathcal{S}_1\oplus\mathcal{S}_2)$.The fibers $\alpha^{-1}(u_1\oplus u_2) = \{\,(u_1\oplus u_2)\}\times\mathcal{T}$ of a source function for the action groupoid $((\mathcal{S}_1\oplus\mathcal{S}_2)\ \times\mathcal{T}(\mathcal{S}_1\oplus\mathcal{S}_2))$ are compact since $(\mathcal{S}_1\oplus\mathcal{S}_2)$ is $SC\mathcal{T}$-space, but $\{(u_1\oplus u_2)\}\times\mathcal{T}\cong\mathcal{T}$ hence a source function for an action groupoid $\left(\mathcal{S}_1''\oplus\mathcal{S}_2''\right) \underset{\mathcal{S}_1'\oplus\mathcal{S}_2'}{\times} (\mathcal{S}_1\oplus\mathcal{S}_2)\times\mathcal{T},\left(\mathcal{S}_1''\oplus\mathcal{S}_2''\right) \underset{\mathcal{S}_1'\oplus\mathcal{S}_2'}{\times} (\mathcal{S}_1\oplus\mathcal{S}_2))$ is proper.(Proposition (2,4)).

Proposition (6):

Let (N,M) be an SSC-groupoid then $N_{x_1}\oplus N_{x_2}$ is SSC_xN_x -space, for all $x\in M$.

Proof:

N_{x_1} and N_{x_2} are SC_xN_x-space, (Proposition (3,8)).
$G_{x_1}\oplus G_{x_2}$ is SC_xN_x-space, (Proposition (4,2)).
To show that the function $\varphi_x : N_{x_1}\oplus N_{x_2}\to N_{x_1}\oplus N_{x_2}/{}_xN_x$ is submersion, for every $x\in M$.The maps $\varphi_{x_1} * \varphi_{x_2}: N_{x_1}\oplus N_{x_2}\to N_{x_1}\oplus N_{x_2}/{}_xN_x$ and $\beta_x : N_{x_1}\oplus N_{x_2}\to M_1\oplus M_2$are both identification function (β_x is surjective proper function,(Proposition (3,2))and Proposition (2,4))) and constant on each other's fibres. The dotted arrows in the figure below:



existing, are unique into ***T*** by universal property for identification , a function η_x be provided by $\eta_x(\varphi_{x_1}\oplus\varphi_{x_2}(n_1,n_2)) = \beta_x\,(n_1,n_2)$. Now, to show that the function $\varphi_{x_1}\oplus\varphi_{x_2}: N_{x_1}\oplus N_{x_2}\to N_{x_1}\oplus N_{x_2}/{}_xN_x$ is submersion. unique in ***T***.

Let$(n_1\oplus n_2)\in N_{x_1}\oplus N_{x_2}$, $\varphi_{x_1}\oplus\varphi_{x_2}\,(n_1\oplus n_2)\in N_{x_1}\oplus N_{x_2}/{}_xN_x$, $\eta_x\left(\varphi_{x_1}\oplus\varphi_{x_2}(n_1\oplus\ n_2)\right) =$ $\beta_x(n_1,n_2)\in M_1\oplus M_2$ then there is an open neighborhood $U_{(n_1,n_2)}$ of $\beta_x(n_1,n_2)$ in $M_1\oplus M_2$ and continuous right inverse $v: U\to N_{x_1}\oplus N_{x_2}$ to $\beta_x: N_{x_1}\oplus N_{x_2}\to M_1\oplus M_2$ such that $vo\beta_x\,(n_1\oplus n_2) =$ $(\,n_1\oplus n_2)$, (β_x is submersion (N,M) is SSC-groupiod).

Now define $v'_{(n_1\oplus n_2)}: {\eta_x}^{-1}(U_{(n_1\oplus n_2)}) \overset{\eta_x}{\to} U_{(n_1\oplus n_2)} \overset{v}{\to} N_{x_1}\oplus N_{x_2}$ by $v'_{(n_1\oplus n_2)}(a) = vo\,\eta_x(a)$ where

${\eta_x}^{-1}(\mathrm{U}_{(n_1 \oplus n_2)})$ is open neighborhood of $\varphi_{\mathrm{x}_1} \oplus \varphi_{x_2}\ (n_1 \oplus n_2)$ in $\mathrm{N}_{\mathrm{x}_1} \oplus \mathrm{N}_{\mathrm{x}_2}/_{\mathrm{x}}\mathrm{N}_{\mathrm{x}}$. $v'_{(n_1 \oplus n_2)}$ is continuous and $(\varphi_{\mathrm{x}_1} \oplus \varphi_{x_2}) o v'_{(n_1 \oplus n_2)})(a) = (\varphi_{\mathrm{x}_1} \oplus \varphi_{x_2}) o v o \eta_x(a) = \eta^{-1} o \eta_x o((\varphi_{\mathrm{x}_1} \oplus \varphi_{x_2})\ o\, v o \eta_x(a) = \eta_x^{-1} o \beta_x o\, v o(\varphi_{\mathrm{x}_1} \oplus \varphi_{x_2})\ (a) = \eta_x^{-1} o \eta_x(a) = a$

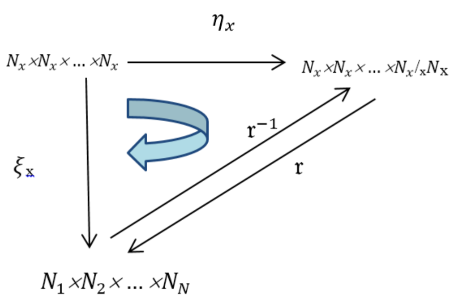
And $v'_{(n_1 \oplus n_2)}(\varphi_{\mathrm{x}_1} \oplus \varphi_{x_2})(n_1 \oplus n_2)) = v o \eta_x(\varphi_{x_1} \oplus \varphi_{x_2}(n_1 \oplus n_2)) = v o \beta_x(n_1 \oplus n_2) = n.$

Proposition (7):

Let $(N_1 \times N_2 \times \ldots \times N_n, M_1 \times M_2 \times \ldots \times M_n)$ be transitive SC-groupoid then Ehressmann groupoid $(\underbrace{N_x \times N_x \times \ldots \times N_x}_{\leftarrow n.\text{time} \rightarrow}/_{\mathrm{x}}\mathrm{N}_{\mathrm{x}}, \underbrace{N_x}_{\leftarrow n.\text{time} \rightarrow}/_{\mathrm{x}}\mathrm{N}_{\mathrm{x}})$ isomorphic to $(N_1 \times N_2 \times \ldots \times N_n, M_1 \times M_2 \times \ldots \times M_n)$ in $\boldsymbol{TG}$.

Proof:

The function ξ_{x}: $N_x \times N_x \times \ldots \times N_x \rightarrow N_1 \times N_2 \times \ldots \times N_n$ is surjective proper (Definition (3,3)). Next, the functions ξ_{x}: $\underbrace{N_x \times N_x \times \ldots \times N_x}_{\leftarrow n.\text{time} \rightarrow} \rightarrow N_1 \times N_2 \times \ldots \times N_n$ and η_x: $N_x \times N_x \times \ldots \times N_x \rightarrow N_x \times N_x \times \ldots \times N_x/_{\mathrm{x}}N_{\mathrm{x}}$ are both constants on the fiber of each other and identification functions. Hence, in the following figure, the dotted arrows:



exist single into $\boldsymbol{T}$ by a universal property for identification function ,a function $\mathfrak{r}$ be given by $\mathfrak{r}([(n_1, n_2 \ldots, , n_{\mathrm{n}})])$=$\lambda(n_1, \delta(n_2 \ldots, n_{\mathrm{n}}))$ has to become homeomorphism. $(\mathfrak{r}, \eta_x)$ is the isomorphism for TG where η_x is the function presented by $\eta_x(\varphi_{\mathrm{x}}(n_1, n_2 \ldots, , n_{\mathrm{n}}))$=$\beta_x(n_1, n_2 \ldots, , n_{\mathrm{n}})$ where φ_{x}:$N_{\mathrm{x}} \times N_{\mathrm{x}} \times \ldots \times N_{\mathrm{x}} \rightarrow\ N_{\mathrm{x}} \times N_X \times \ldots \times N_X/_{\mathrm{x}}N_{\mathrm{x}}$. (Proposition (4,6)).

Proposition (8):

Let $\mathcal{S}_1 \oplus \mathcal{S}_2$ be an $SSC\mathcal{T}$-space then Ehresmann groupoid $((\mathcal{S}_1 \oplus \mathcal{S}_2) \times (\mathcal{S}_1 \oplus \mathcal{S}_2)/\mathcal{T}, (\mathcal{S}_1 \oplus \mathcal{S}_2)/\mathcal{T})$ is submersive groupoid.

Proof:

Let N =$((\mathcal{S}_1 \oplus \mathcal{S}_2) \times (\mathcal{S}_1 \oplus \mathcal{S}_2))/\mathcal{T}$, M=$(\mathcal{S}_1 \oplus \mathcal{S}_2)/\mathcal{T}$, (N, M) be an transitive SC-groupoid (Proposition

(4,3)) into prove that the function $\beta_x : N_\chi \oplus N_\chi \to M$ be a submersion. If h =[($u \oplus \grave{u}$), ($\grave{\grave{u}} \oplus \grave{\grave{\grave{u}}}$)]$\in N_\chi \oplus N_\chi$, then

$\beta_x\ (h)$ = φ ($u \oplus u'$) and there exists an open neighborhood V of φ (u$\oplus$u') in M as well as a continuous right inverse. $\nu *: V \to \mathcal{S}_1 \oplus \mathcal{S}_2$ to φ s.t $\nu * \circ\, \varphi$ (u$\oplus$u'))=($u \oplus u'$) ($\varphi : \mathcal{S}_1 \oplus \mathcal{S}_2 \to M$ = ($\mathcal{S}_1 \oplus \mathcal{S}_2$)/$\mathcal{T}$ is submersion, (Definition (3,11)). Define $\nu **: V \to N_x$ by: $\nu **\ (y)$ = [($\nu *(y)$, ($\grave{\grave{u}} \oplus \grave{\grave{\grave{u}}}$)] for every $y \in V$. $\beta_x \text{o}\, \nu **\ (y) = \beta_x$ ([($\nu *(y)$, ($u \oplus \grave{u}$)]) = $\varphi\ o\, \nu *(y)$ = $y = I_V$ for every $y \in V$ and $\nu **\ (y) \text{o}\, \beta_x\ (h)$ = $\nu **$ (β[($u \oplus u'$), ($\grave{\grave{u}} \oplus \grave{\grave{\grave{u}}}$)]) = $\nu **$ (φ(u$\oplus$ u')) = [($\nu *$(φ ($u \oplus u'$), ($\grave{\grave{u}} \oplus \grave{\grave{\grave{u}}}$))] = [($u \oplus u'$), ($\grave{\grave{u}} \oplus \grave{\grave{\grave{u}}}$)] = h

Proposition (9):

Let (N, M) be SSC-groupoid then Ehresmann groupoid (($N_x \times N_x \times \ldots \times N_x\ /_x N_x$, $N_x\ /_x N_x$) is SSC-groupoid for every $x \in M$.

Proof:

If $N = N_x \times N_x \times \ldots \times N_x /_x N_x$, $M = N_x /_x N_x$, (N, M) be an transitive SC-groupoid (Proposition (4,3)) to display that a function $\beta_X : N_x \to M$ be submersion. If h=[($u_1, u_2, \ldots, u_n$)]$\in N_x$, then β_X (h)= $\varphi(u_1)$, there exists the open neighborhood V of φ (u_1) in M and continuous right inverse $\nu *: V \to N_x$ into φ s.t $\nu *\ o\ \varphi\ (u)$ =u (φ:$N_\chi \to M$ = $N_\chi /_x N_\chi$ is submersion, (Definition (2,8)). Define $\nu *'$:U$\to N_x$ by: $\nu *'$ (y)=[(ν^*(y),u`)] for every y$\in$U .Define ν^{**} :V$\to N_\chi$ by: $\nu **\ (y)$=[($\nu *(y)$, ($u_2, \ldots, u_n$))] for every $y \in V$. $\beta_x \text{o}\, \nu **\ (y) = \beta_X$ ([($\nu *(y), u_1$))]) = $\varphi\ \text{o}\, \nu *(y)$ = $y = I_V$ for every $y \in V$ and $\nu **\ (y) o \beta_X\ (h)$= $\nu **$ (β[($u_1, u_2, \ldots, u_n$)])= $\nu **$ (φ (u_1))=[($\nu *$ (φ (u_1), $u_2, \ldots, u_n$))]=[($u_1, u_2, \ldots, u_n$)]= h .

5. Conclusion:

We have studied topological groupoid. we also studied privately type of topological groupoid which is SC-groupoid, SSC-groupoid, SC $\mathcal{T}$-space and SSC $\mathcal{T}$ -space and the relationships among them written as proposition.

6. Acknowledgement

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