SOME PROPERTIES OF ζ −CCOMPACTNESS IN ζ-TOPOLOGICAL SPACES

Nadia Abdulrahman Hamed [1], **Taha Hamed Jassem** [2]

[1] Tikrit university, College of Education for women, Department of Mathematics, Tikrit, Iraq.

[2] Tikrit university, College of Computer Science and Mathematics, Department of Mathematics, Tikrit, Iraq.

Corresponding Author: Taha Hamed Jassem, tahahameed91@gmail.com

Abstract:

In this article, we introduced the new concepts of mappings such as ζ −irresolute, $\zeta -$ Contra continuous, $\zeta -$ contra irresolute and $\zeta -$ homeomorphism and some properties with relationship among them are given, with more results to explain this topic. We investigation some topological properties of ζ −irresolute, $\zeta -$ Contra continuous, $\zeta -$ contra irresolute and $\zeta -$ homeomorphism. Also, we have introduce a new class such as $\zeta - T_{\frac{1}{2}}$ compact, $\zeta - T_{\frac{1}{2}}^{*}$ compact, $\zeta - T_c$, compact $\zeta - T_{ab}$ compact and $\zeta - T_{ac}$ compact with relationship among them.

Keywords: $\boldsymbol{\zeta}$ −Ccompactness; $\boldsymbol{\zeta}$ −contra; $\boldsymbol{\zeta}$ −irresolute; $\boldsymbol{\zeta} -$ homeomorphism.

INTRODUCTION

Compactness is a generalization of the concept of a closed and limited subset of real lines to spaces of topology. Compactness is an essential and fundamental idea in generic topology as well as other high-level mathematical disciplines. Numerous academics [1,2,3,4,5,6,7,8,9,10,11,12,13] have looked into the fundamental characteristics of compactness and connectivity. Scientists were inspired to generalize these ideas because of how productive and productive these ideas of compact. These efforts have resulted in the introduction and investigation of numerous stronger and weaker versions of compactness.

Das and Mahanta [14], Girija *et al.* [15], Jung and Nam [16], and Basumatary *et al.,*[17] studied more of the subjects of topological spaces.

Further than just continuity features in topological spaces are outlined in Mohammadi & Rashidi [18] and Farhan & Yang [19]. Also, Ozel *et al.,* [20] created the ζb-open continuous function in 2021. Zhang presented continuity and production topology in 2021 [21]. Finally, Al Ghour [22] presented the concept of soft complete continuity and soft strong continuity in soft topological spaces.

Submitted: 22/06/2023 **Accepted**: 25/07/2023

In this study, we give a new class of ζ −Ccompactness in ζ-Topological Spaces with some properties related it. Section 2 is devoted to ζ −irresolute, ζ −contra and the concept of ζ − homeomorphism was defined and some properties were introduced, with more examples to explain this subject. Section 3 −compact with some properties is presented. Furthermore, ζ −compact and generalization of the ζ −type of it are examined in this part. Through the use of a few instances, also the relationship between the compact and ζ −compact is demonstrated in this section.

PRELIMINARIES

In this part, we will give the most important definitions and theorems with related for our work.

Definition 1. [23] Suppose that $(\boldsymbol{X},\boldsymbol{\tau})$ be topological space and $\boldsymbol{Q} \subseteq \boldsymbol{X}$. If $\boldsymbol{Q} \cap \overline{\boldsymbol{G}} \neq \emptyset$ for every $\boldsymbol{q} \in \boldsymbol{Q}$ there exists open set $\boldsymbol{G}$ content $\boldsymbol{q}$ s.t $\emptyset \neq \boldsymbol{Q} \neq \boldsymbol{X}$ and $\boldsymbol{Q} \cap \overline{\boldsymbol{G}} = \emptyset$ if $\boldsymbol{Q} = \emptyset$ and $\boldsymbol{Q} \cap \overline{\boldsymbol{G}} = \boldsymbol{X}$ if $\boldsymbol{Q} = \boldsymbol{X} = \boldsymbol{G}$. Then $\boldsymbol{Q}$ is $\boldsymbol{\zeta}$-open set and set of all $\boldsymbol{\zeta}$-open is denoted by $\boldsymbol{\zeta}$–$\mathbf{O}(\mathbf{X})$.

Definition 2. [24] Let $\mathbf{h}: (\boldsymbol{X},\boldsymbol{\tau}) \rightarrow (\boldsymbol{Y},\boldsymbol{\sigma})$ be a mapping. Then:

1. If $\mathbf{h}^{-1}(\boldsymbol{W}) \in \boldsymbol{X}$ for every open set in $\boldsymbol{Y}$, then the mapping h is considered to be continuous.
2. if $\mathbf{h}(\boldsymbol{W})$ is an open set in $\boldsymbol{Y}$, $\boldsymbol{and}\ \boldsymbol{W} \in \boldsymbol{\tau}$ then the mapping h is considered to be an open.
3. if $\mathbf{h}(\boldsymbol{W})$ is a closed set in $\boldsymbol{Y}$, $\boldsymbol{and}\ \boldsymbol{W}^{\boldsymbol{c}} \in \boldsymbol{\tau}$ then the mapping h is considered to be closed.

Definition 3. A map $\mathrm{h}: (X,\tau) \rightarrow (Y,\sigma)$ is :
(i) Contra-continuous **[25]** if $h^{-1}(M)$ is closed in (X,τ) for every open set M of (Y,σ);
(ii) Strongly contra-continuous **[26]**, if h is a contra continuous mapping such that $h^{-1}(M)$ has an interior point whenever M is open set of (Y,σ).
(iii) irresolute **[27]** $h^{-1}(M) \in SO(X,\tau)$ for every $M \in SO(Y,\sigma)$.

Definition 4. [28] A map $h: (\mathrm{X},\tau) \rightarrow (\mathrm{Y},\sigma)$is considered to be homeomorphism if h is a continuous, bijective and open map.

Definition 5. [29] A collection $\{\Psi_\iota : \iota \in \xi\}$ of open sets in (X,τ) is named a ζ-open cover of a subset Ω in (X,τ) if $\Omega \subset \cup\{\Psi_\iota : \iota \in \xi\}$ satisfied.

Definition 6.[29] A topological space $(X,\tau)$is compact if every open cover of (X,τ) has a finite subcover.

Some Properties of $\boldsymbol{\zeta}$ −irresolute Map, $\boldsymbol{\zeta}$ − Contra continuous, $\boldsymbol{\zeta}$ − contra irresolute and $\boldsymbol{\zeta}$ − homeomorphism

In this section, we have introduced the new mappings in ζ − topological spaces such as, ζ − irresolute map, ζ − Contra continuous map, ζ − contra irresolute map and ζ −homeomorphism map in ζ −topological spaces with some properties and relationship among them.

Definition 7. Let X and Y be topological spaces. A map $h:(X,\tau_\zeta)\to(Y,\sigma_\zeta)$ is said to be $\zeta-$ irresolute map if the inverse image of every $\zeta-$open set in Y is $\zeta - semi$ open in X.

Theorem 8. Let X,Y,Z be topological spaces and let $h:\left(X,\tau_\zeta\right)\to\left(Y,\sigma_\zeta\right)$, $k:(Y,\sigma_\zeta)\to(Z,\rho_\zeta)$ be two maps. Their composition $koh:(X,\tau_\zeta)\to(Z,\rho_\zeta)$ is $\zeta-$continuous if h is $\zeta-$irresolute and k is $\zeta-$continuous.
Proof: Let V be an $\zeta-$open set in Z. Then $(k\mathrm{oh})^{-1}(\mathrm{V})=\mathrm{h}^{-1}(k^{-1}(\mathrm{V}))=\mathrm{h}^{-1}(\mathrm{V})$, where $\mathrm{V}=k^{-1}(\mathrm{V})$ is $\zeta-$open in Y as k is $\zeta-$continuous. Since h is $\zeta-$irresolute $\mathrm{h}^{-1}(\mathrm{V})$ is $\zeta-$open in X. Thus, koh is $\zeta-$continuous.

Theorem 9. Let X,Y,Z be $\zeta-$topological spaces. Let $h:\left(X,\tau_\zeta\right)\to\left(Y,\sigma_\zeta\right)$, $k:(Y,\sigma_\zeta)\to(Z,\rho_\zeta)$ be two $\zeta-$irresolute maps. Then their composition $koh:(X,\tau_\zeta)\to(Z,\rho_\zeta)$ is a $\zeta-$irresolute map.
Proof: Let V be $\zeta-$open in Z. Consider $(k\mathrm{oh})^{-1}(\mathrm{V})=\mathrm{h}^{-1}(\mathrm{V})$, where $\mathrm{V}=k^{-1}(\mathrm{V})$ is $\zeta-$open in Y, as k is $\zeta-$irresolute. Since h is $\zeta-$irresolute $h^{-1}(\mathrm{V})$ is $\zeta-$open in X. Thus, koh is $\zeta-$ irresolute maps.

Theorem 10. Let X,Y,Z be topological spaces. Let $h:\left(X,\tau_\zeta\right)\to\left(Y,\sigma_\zeta\right)$, $k:(Y,\sigma_\zeta)\to(Z,\rho_\zeta)$ be two maps. Then their composition $koh:(X,\tau_\zeta)\to(Z,\rho_\zeta)$ is $\zeta-$closed map. The following implications are valid:

(i) h is $\zeta-$continuous and surjective then k is $\zeta-$closed.

(ii) if k $\zeta-$irresolute and injective then h is $\zeta-$closed.

Proof:(i) Let H be a $\zeta-$closed set of Y. Since $h^{-1}(H)$ is $\zeta-$closed in X. So, $(k\circ h)(h^{-1})(H)$ is $\zeta-$closed in Y and hence $k(H)$ is $\zeta-$closed in Z. Thus, k is $\zeta-$closed.
(ii) Let F be $\zeta-$closed set of X. Then $(k\circ h)(F)$ is $\zeta-$closed in and $k^{-1}(k\circ h)(F)$ is $\zeta-$closed in Y. Since k is injective $h(F)=\ ^1(k\circ h)(F)$ is $\zeta-$closed in Y. Therefore h is $\zeta-$closed.

Theorem 11. If a map $h:\left(X,\tau_\zeta\right)\to\left(Y,\sigma_\zeta\right)$ is $\zeta-$continuous then it is $\zeta-$irresolute.
Proof: Let F be any $\zeta-$open set in Y.Then F is $\zeta-$semi open in Y. As h is $\zeta-$irresolute $h^{-1}(F)$ is $\zeta-$ semi open in X. Therefore, h is $\zeta-$irresolute.

Remark 12. The converse of the above theorem need not be true as seen from the following example.

Example 13. Let $\mathrm{X}=\mathrm{Y}=\{\mathrm{a},\mathrm{b},\mathrm{c}\}$ with $\tau_\zeta=\{\phi,\mathrm{X},\{\mathrm{a}\},\{\mathrm{c}\},\{\mathrm{a},\mathrm{c}\}\}$ and $\sigma_\zeta=\{\phi,\mathrm{Y},\{\mathrm{a}\}\}$ and a map $h:\left(X,\tau_\zeta\right)\to\left(Y,\sigma_\zeta\right)$ be denoted by $h(a)=a=h(c)$ and $h(b)=b$ then h is $\zeta-$irresolute but not $\zeta-$continuou as the inverse image of $\zeta-$open {b} in X is not $\zeta-$open in X.

Theorem 14. If a bijection map $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ is ζ −open and ζ −continuous then h is ζ −irresolute.

Proof: Let M be a ζ −open set in Y such that $h^{-1}(\mathrm{M}) \subseteq \mathrm{O}$ where O is ζ −open set in X. Therefore $\mathrm{M} \subseteq \mathrm{h(O)}$ holds Since $h(O)$ is open and M is ζ −open in Y. So, cl (int (M)) $\subseteq h(O)$ holds and hence $h^{-1}(\mathrm{cl\,(int\,(M))}) \subseteq O$. Since h is $\zeta -$ continuous and cl (int (M)) is ζ −closed in Y. $\mathrm{cl\,(int\,(h^{-1}\ (cl\,(int\,(M)))))} \subseteq \mathrm{O}$ and so $\mathrm{cl}\left(\mathrm{int\,(h^{-1}(\,M))}\right) \subseteq \mathrm{O}$. Therefore $\mathrm{h^{-1}(\,M)}$ is ζ −semi closed. So, $\mathrm{h^{-1}(\,M^c)}$ is ζ −semi open. Hence h is ζ −irresolute.

Remark 15. The following example show that the reverse of above theorem not true.

Example 16. Let $\mathrm{X} = \mathrm{Y} = \{\mathrm{a, b, c}\}$ with $\tau_\zeta = \{\phi, \mathrm{X}, \{\mathrm{a}\}, \{\mathrm{c}\}, \{\mathrm{a, c}\}\}$, $\sigma_\zeta = \{\phi, \mathrm{Y}, \{\mathrm{a}\}, \{\mathrm{a, b}\}\}$.Let $h: (X, \zeta_\tau) \to (Y, \sigma_\zeta)$ be denoted by $h(a) = a = h(c), h(b) = b$ then h is ζ −irresolute and ζ −open but not bijective and so h is not ζ −continuous.

Theorem 17. Let X, Y and Z be any ζ −topological spaces. For any $\zeta -$ irresolute map $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ and any ζ −continuous map $k: (Y, \sigma_\zeta) \to (Z, \rho_\zeta)$, the composition $koh: (X, \tau_\zeta) \to (Z, \rho_\zeta)$ is ζ −continuous.

Proof: Let F be any ζ −closed set in Z. Since k is ζ −continuous, $k^{-1}(F)$ is ζ −closed set in Y. Since h is ζ −irresolute, $h^{-1}(k^{-1}(F))$is ζ −closed in X. But $h^{-1}(k^{-1}(F)) = (koh)^{-1}(F)$. Therefore, $koh: (X, \tau_\zeta) \to (Z, \rho_\zeta)$ is ζ −continuous.

Definition 18. The mapping is considered to have the form $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$:
(i) ζ −contra-continuous if $h^{-1}(M)$ is ζ −closed in (X, τ_ζ) for every ζ −open set M of (Y, σ_ζ);
(iii) $\zeta -$ contra-irresolute if $h^{-1}(M)$ is ζ −semi closed in (X, τ_ζ) for every $M \in SO(Y, \sigma_\zeta)$.

Remark 19. The ζ −contra continuous maps and ζ −irresolute maps are independent of each other as seen from the following example.

Example 2.1.14. Let $\mathrm{X} = \mathrm{Y} = \{\mathrm{a, b, c}\}$ with $\tau_\zeta = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma_\zeta = \rho_\zeta = \{\phi, Y, \{a\}, \{b, c\}\}$. Let $h: (X, \zeta_\tau) \to (Y, \sigma_\zeta)$ be identity map. Then the map h is irresolute but it is not ζ −contra continuous, since $G = \{c\}$ is ζ −closed in (Z, ρ_ζ), where $h^{-1}(G) = \{c\}$ is not ζ −closed in (X, τ_ζ).

Recall Example 16 we see that the map h is ζ −contra continuous but it is not $\zeta -$ irresolute, since $G = \{b\}$ is ζ −closed in (Z, ρ_ζ), where $h^{-1}(G) = \{b\}$ is not ζ −closed in (X, τ_ζ).

Theorem 20. If a bijection map $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ is ζ −continuous then h is $\zeta -$ contra-irresolute .

Proof: Let M be a ζ −semi closed set in Y such that $h^{-1}(\mathrm{M}) \subseteq \mathrm{O}$ where O is ζ −semi open set in X. Therefore $\mathrm{M} \subseteq \mathrm{h(O)}$ holds. Since $h(O)$ is ζ −semi open and M is $\zeta -$ semi closed in

Y. So, cl (int (M)) $\subseteq h(O)$ holds and hence h^{-1}(cl (int (M))) $\subseteq O$. Since h is $\zeta -$ continuous and cl (int (M)) is ζ −closed in Y. cl (int (h^{-1} (cl (int (M))))) $\subseteq$ O and so cl (int (h^{-1}(M))) $\subseteq$ O. Therefore h^{-1}(M) is ζ −closed. Hence h is $\zeta -$ contra-irresolute.

Remark 21. The following example show that the reverse of above theorem not true.

Recall Example 16 We see that h is $\zeta -$ contra-irresolute but not h is not ζ −continuous.

Remark 22. ζ −contra irresolute and ζ −irresolute are actually separate concepts. as shown in example 13 , the opposite is demonstrated in the following example.

Example 23. ζ −contra irresolution is not necessary for an ζ −irresolute map.

Then irresolute map is the identity map over the topological space (X, τ_ζ) where $\tau = \{0, \{b\}, X\}$ and $X = \{a, b\}$, but not ζ −contra irresolute.

Theorem 24. If a map $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ is ζ −continuous then it is ζ −contra continuous.
Proof: Let F be any ζ −open set in Y.Then F^c is ζ −closed in Y. As h is $\zeta -$ continuous $h^{-1}(F^c)$ is ζ −closed in X. Therefore, h is $\zeta -$ contra continuous.

Remark 25. The converse of the above theorem need not be true as seen from the following example.

Example 26. Let $\mathrm{X} = \mathrm{Y} = \{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$ with $\tau_\zeta = \{\phi, \mathrm{X}, \{\mathrm{a}\}, \{\mathrm{a}, \mathrm{c}, \mathrm{d}\}\}$ and $\sigma_\zeta = \{\phi, \mathrm{Y}, \{\mathrm{a}, \mathrm{b}\}\}$ and a map $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ be denoted by $h(a) = b, h(c) = \mathrm{a}$ and $h(b) = c = h(d)$ then h is $\zeta -$ contra continuous but not $\zeta -$ continuous as the inverse image of $\zeta -$ open {c} in X is not ζ −open in X.

Remark 27. $\zeta -$ irresolute and ζ −contra continuous are actually separate concepts. as shown in Example 13, then h is $\zeta -$ contra continuous but not ζ −irresolute as the inverse image of $\zeta -$ open {c} in X is not ζ −open in X.

Example 28. Let $\mathrm{X} = \{a, \mathrm{b}, \mathrm{c}\}, \mathrm{Y} = \{a, \mathrm{b}\}$ with $\tau_\zeta = \{\phi, \mathrm{X}, \{a\}, \{\mathrm{b}, \mathrm{c}\}\}$ and $\sigma_\zeta = \{\phi, \mathrm{Y}, \{a\}, \{\mathrm{b}\}\}$ and a map $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ be denoted by $h(a) = a \; and \; h(c) = \mathrm{b}$, then h is ζ −irresolute but not $\zeta -$ contra continuous as the inverse image of $\zeta -$ closed $\{a\}$ in X is not ζ −open in X.

Theorem 29. If a map $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ is ζ −contra continuous then it is ζ −contra irresolute.
Proof: Let F be any ζ −semi open set in Y.Then F^c is ζ −semi closed in Y. As h is $\zeta -$ contra continuous $h^{-1}(F^c)$ is ζ −semi closed in X. Therefore, h is $\zeta -$ contra irresolute.

Remark 30. The converse of the above theorem need not be true as seen from the following example.

Recall Example 28. We see that h is ζ −contra irresolute but not ζ −contra continuous. Because, the inverse image of $\zeta -$ open $\{a\}$ in X is not ζ −closed in X.

Theorem 31. Let X, Y, Z be topological spaces. Let $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta), k: (Y, \sigma_\zeta) \to (Z, \rho_\zeta)$ be two $\zeta -$ contra irresolute maps. Then their composition $koh: (X, \tau_\zeta) \to (Z, \rho_\zeta)$ is $\zeta -$ contra irresolute map.

Proof: Let V be ζ −semi open in Z. Consider $(k\mathrm{oh})^{-1}(\mathrm{V}) = \mathrm{h}^{-1}(\mathrm{V})$, where $\mathrm{V} = k^{-1}(\mathrm{V})$ is ζ −semi open in Y, as k is ζ −irresolute. Since h is $\zeta -$ contra irresolute $h^{-1}(\mathrm{V})$ is ζ −semi open in X. Thus, koh is $\zeta -$ contra irresolute map.

Theorem 32. Let X, Y, Z be topological spaces and let $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta), k: (Y, \sigma_\zeta) \to (Z, \rho_\zeta)$ be two maps. Their composition $koh: (X, \tau_\zeta) \to (Z, \rho_\zeta)$ is $\zeta -$ contra irresolute if h is $\zeta -$ contra irresolute and k is $\zeta -$ contra irresolute.

Proof: Let V be an ζ −semi open set in Z. Then$(k\mathrm{oh})^{-1}(\mathrm{V}) = \mathrm{h}^{-1}(k^{-1}(\mathrm{V})) = \mathrm{h}^{-1}(\mathrm{V})$, where $V^c = k^{-1}(\mathrm{V})$ is ζ −semi closed in Y as k is $\zeta -$ contra irresolute. Since h is $\zeta -$ contra irresolute $\mathrm{h}^{-1}(\mathrm{V})$ is ζ −semi closed in X. Thus, koh is $\zeta -$ contra irresolute.

Theorem 33. If a map $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ is $\zeta -$ irresolute then it is ζ −contra irresolute.

Proof: Let F be any ζ −semi open set in Y.Then F^c is ζ −semi closed in Y. As h is $\zeta -$ irresolute $h^{-1}(F^c)$ is ζ −semi closed in X. Therefore, h is $\zeta -$ contra irresolute.

Remark 34. The converse of the above theorem need not be true as seen from the following example.

Example 35. Let $\mathrm{X} = \mathrm{Y} = \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ with $\tau_\zeta = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma_\zeta = \{\phi, Y, \{b\}\}$. Let $h: (X, \zeta_\tau) \to (Y, \sigma_\zeta)$be denoted by $h(a) = b, h(b) = c, h(c) = a$, then h is ζ −contra irresolute and so h is not ζ −irresolute.

Definition 36. Let X and Y be two topological spaces. A bijection map $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ from a ζ −topological space X into ζ −topological space Y is called $\zeta -$ homeomorphism if h and h^{-1} are ζ −continuous.

Theorem 37. Every homeomorphism is ζ −homeomorphism.

Proof: Let $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ be homeomorphisms from the ζ −topological space X to Y then h and h^{-1} are continuous. As every continuous function is ζ −continuous. We have h and h^{-1} are ζ −continuous. Thus h is ζ −homeomorphism.

Remark 38. The converse of the above theorem need not be true from the following example.

Example 39. Let $X = Y = \{a, b, c\}$ with $\tau_\zeta = \{\phi, X, \{a\}, \{a, c\}\}$ and $\sigma_\zeta = \{\phi, Y, \{a, b\}\}, h: (X, \zeta_\tau) \to (Y, \sigma_\zeta)$ is defined by $\mathrm{h(a) = a, h(b) = c, h(c) = b}$ then h is ζ −homeomorphism but not homeomorphism as the inverse image of the ζ −open set $\{a\}$ in X is $\{a\}$ is not ζ −open in Y.

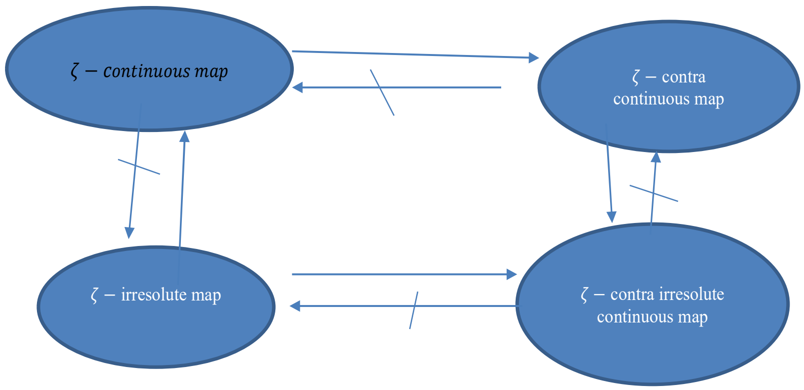
Theorem 40. A bijection map $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ is ζ −homeomorphism then it is ζ −homeomorphism.
Proof: Since h is homeomorphism both h and h^{-1} are ζ −continuous.As every continuous functions are ζ −continuous h and h^{-1} are ζ −continuous. Thus h is ζ −homeomorphism.

Remark 41. The converse of the above theorem need not be true as seen from the following example.

Example 42. Let $\mathrm{X = Y = \{a, b, c\}}$ with $\tau_\zeta = \{\phi, \mathrm{X}, \{\mathrm{a}\}\}$ and $\sigma_\zeta = \{\phi, Y, \{a, b\}\}$ and $h: (X, \tau_\zeta) \to (Y, \sigma_\zeta)$ is defined as $h(a) = b, h(b) = c, h(c) = a$. Then h is ζ −homeomorphism but not ζ −homeomorphism as $h(\{a\}) = \{b\}$ is not ζ −open in Y.

Theorem 43. Assume that (X, τ_ζ) and (Y, σ_ζ) are topological spaces and let h be a bijective mapping from X onto Y. Then the following conditions are equivalent.
(a). h is $\zeta - open$ and ζ −continuous.
(b). h is ζ −homeomorphism.
(c). h is ζ −closed and ζ −continuous.
Proof:
(i) To prove (a) ⇒ (b)
Let h be bijective ζ −open and ζ −continuous.Let G be ζ −open set in X.Then G is ζ −open as h is ζ −open map, $h(G)$ is ζ −open in Y. i.e. $(h^{-1})^{-1}(G) = h(G)$ is ζ −open in Y. Thus h^{-1} is ζ −continuous. Thus (a) ⇒ (b).
(ii). To prove (b) ⇒ (a)
Let h be ζ −homeomorphism and $\mathrm{h}^{-1} = k$ then $k^{-1} = h$. Since h is bijctive g is also bijective. If G is ζ −open set $k^{-1}(G)$ is a ζ −open set for k is ζ −continuous. i.e. h(G) is ζ −open i.e. Thus h is ζ −open. Therefore (b) ⇒ (a). Hence (a), ⇔ (b)
(iii) To prove (b) ⇒ (c)
Assume that h is ζ −homeomorphism. Let F be a ζ −closed set in X. Then $(X - F)$ is open and $h^{-1} = k$ is ζ −continuous, $k^{-1}(\mathrm{X - F})$ is ζ −open i.e $k^{-1}(X - F) = Y - k^{-1}(F)$ is ζ −open. Thus $k^{-1}(F)$ is ζ −closed. i.e. $h(F)$ is ζ −closed. Hence h is ζ −closed map.
(iv) To prove (c) ⇒ (b)
If h is ζ −closed and ζ −continuous then we have to prove h^{-1} is also ζ −cotenuous. Let G be ζ −open set. Then $\mathrm{X - G}$ is ζ −closed. Since f is ζ −closed, $h(X - G)$ is ζ −closed. i.e. $k^{-1}(X - G) = Y - k^{-1}(G)$ is ζ −closed, implies $k^{-1}(G)$ is ζ −open. Thus, inverse image under k of every open set is ζ −open. i.e. $k = \mathrm{h}^{-1}$ is ζ −continuous. Thus h is ζ −homeomorphism. Hence (c) ⇒ (b). Thus, (c) ⇔ (b)

Here we have proved that a ζ −closed and ζ −continuous mapping is ζ −homeomorphism in (a). We have proved that a ζ −homeomorphism is ζ −open and ζ −continuous. Thus ζ −closed and ζ −continuous mapping is also ζ −open and conversely.
Now, the relationships among mappings, ζ − continuous map, ζ − contra continuous map, ζ − irresolute map and ζ − contra irresolute map are shown in the figure below.



Properties of ζ-Compact Space

As a generalization of the concepts of ζ-topological space, we introduce the concepts of ζ-compact space. we give many characterizations and many properties of this concept. Firstly, we began with the following definition:

Definition 44. a collection $\{\Psi_\iota : \iota \in \xi\}$ of ζ-open sets in (X, ρ_ζ) is named a ζ-open cover of a subset Ω in (X, ρ_ζ) if $\Omega \subset \cup\{\Psi_\iota : \iota \in \xi\}$ satisfied.

Definition 45. A ζ-topological space $(X, \rho_\zeta)$is ζ-compact if every ζ -open cover of (X, ρ_ζ)has a finite subcover.

Definition 46. A subset Ω of a topological space (X, ρ_ζ) is ζ-compact relative to (X, ρ_ζ) if, there exists a finite subset ξ_0 of ξ such that $\Omega \subseteq \cup\{\Psi_\iota : \iota \in \xi_0\}$ for every collection $\{\Psi_\iota : \iota \in \xi\}$ of ζ -open subsets of (X, ρ_ζ) such that $\Omega \subset \cup\{\Psi_\iota : \iota \in \xi\}$.

Theorem 47. every ζ -closed subset of a ζ -compact space (X, ρ_ζ) is ζ -compact relative to (X, ρ_ζ).

proof. assume that Ψ be ζ -closed subset of ζ -compact space (X, ρ_ζ). so, Ψ^c is ζ -open in (X, ρ_ζ). let $H = \{\phi_\lambda : \lambda \in \xi\}$ be a cover of Ψ by ζ -open sets in (X, ρ_ζ). thus, $H^* = H \cup \Psi^c$ is a ζ -open cover of (X, ρ_ζ). because (X, ρ_ζ) is ζ -compact, H^* is reducible to a finite subcover of (X, ρ_ζ). set $X = \phi_{\lambda 1} \cup \phi_{\lambda 2} \cup \ldots \cup \phi_{\lambda n} \cup \Psi^c, \phi_{\lambda n} \in H$. but, Ψ and Ψ^c are disjoint hence $\Psi \subset \phi_{\lambda 1} \cup \phi_{\lambda 2} \cup \ldots \cup \phi_\lambda, \phi_\lambda \in H$, and so any ζ-open cover H of Ψ contains a finite subcover. thus, $\boldsymbol{\Psi}$ is $\boldsymbol{\zeta}$-compact relative to $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$. therefore, every $\boldsymbol{\zeta}$ -closed subset of $\boldsymbol{\zeta}$ -compact space $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ is $\boldsymbol{\zeta}$ -compact.

Theorem 48. every compact space is $\boldsymbol{\zeta}$-compact.

proof. let $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ be a compact space. Suppose that $\{\boldsymbol{\Psi_\iota} : \boldsymbol{\iota} \in \boldsymbol{\xi}\}$ be $\boldsymbol{\zeta}$-open cover of $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$. thus, $\{\boldsymbol{\Psi_\iota} : \boldsymbol{\iota} \in \boldsymbol{\xi}\}$ is a $\boldsymbol{\zeta}$-open cover of $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ as every open set is $\boldsymbol{\zeta}$-open set. since $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ is compact, the $\boldsymbol{\zeta}$ -open cover $\{\boldsymbol{\Psi_\iota} : \boldsymbol{\iota} \in \boldsymbol{\xi}\}$ of $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ has a finite subcover $\{\boldsymbol{\Psi_\iota} : \boldsymbol{i} = \mathbf{1},\ldots,\boldsymbol{n}\}$ for $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$. therefore, $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ is $\boldsymbol{\zeta}$-compact.

Remark 49. the converse of Theorem 48 is not true in general.

Example 50. consider the set of real numbers with the topology $\boldsymbol{\tau}$, such that $\boldsymbol{\tau} = \{\boldsymbol{G} \subseteq \mathbb{R} \mid \mathbf{0} \in \boldsymbol{G}$ or $\boldsymbol{G} = \emptyset\}$, we see that every open cover of $\mathbb{R}$ must contain $\mathbb{R}$. we take $\{\mathbb{R}, \boldsymbol{M}\}$ as $\boldsymbol{\zeta}$- open cover of $\mathbb{R}$ with $\mathbb{R} \cap \mathbf{M}$ $\boldsymbol{\zeta}$-compact and $\mathbb{R} \cap \boldsymbol{M} = \boldsymbol{M}$, thus $\boldsymbol{M}$ is $\boldsymbol{\zeta}$-compact. hence $(\mathbb{R}, \boldsymbol{\tau})$ is $\boldsymbol{\zeta}$-compact. however, since $\boldsymbol{W} = \{(-\mathbf{m}, \mathbf{m}); \mathbf{m} \in \mathbb{N}\}$ is a collection to open subsets of $\mathbb{R}$ and does not have a terminating open subcover $\mathbb{R}$, the resulting space cannot be compact.

Definition 51. let $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ be a topological space. then, $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ is considered to be $\boldsymbol{g\zeta}$-compact if every $\boldsymbol{\zeta}$-open cover of $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ has a finite subcover.

Theorem 52. every $\boldsymbol{g\zeta}$ -compact space is $\boldsymbol{\zeta}$-compact.

proof: let $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ be a $\boldsymbol{g\zeta}$-compact space. let $\{\boldsymbol{\Psi_\iota} : \boldsymbol{\iota} \in \boldsymbol{\xi}\}$ be a $\boldsymbol{\zeta}$ -open cover of $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ by $\boldsymbol{\zeta}$ -open set in $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$. because each $\boldsymbol{g\zeta}$ -open set is $\boldsymbol{\zeta}$ -open, hence, $\{\boldsymbol{\Psi_\iota} : \boldsymbol{\iota} \in \boldsymbol{\xi}\}$ is $\boldsymbol{g\zeta}$ cover of $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$. since $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ is $\boldsymbol{g\zeta}$ -compact, the $\boldsymbol{\zeta}$-open cover $\{\boldsymbol{\Psi_\iota} : \boldsymbol{\iota} \in \boldsymbol{\xi}\}$ of $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ has a finite subcover $\{\boldsymbol{\Psi_\iota} : \boldsymbol{i} = \mathbf{1},\ldots,\boldsymbol{n}\}$ of $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$.therefore, $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ is $\boldsymbol{\zeta}$ -compact.

Remark 53. the converse of Theorem 53 is not true in general.

recall example 50, we see that $(\mathbb{R}, \boldsymbol{\tau})$ is $\boldsymbol{\zeta} -$ space. but $(\mathbb{R}, \boldsymbol{\tau})$ is not $\boldsymbol{g\zeta}$ -compact since $\boldsymbol{W} = \{(-\mathbf{m}, \mathbf{m}); \mathbf{m} \in \mathbb{N}\}$ is a collection to open subsets of $\mathbb{R}$ and does not have a terminating $\boldsymbol{g\zeta}$ open subcover $\mathbb{R}$, the resulting space cannot be $\boldsymbol{g\zeta}$ compact.

Theorem 54. every $\boldsymbol{\zeta}$-compact space is $\boldsymbol{\zeta}$-countably compact.
proof: suppose that $(\boldsymbol{X},\ \boldsymbol{\rho_\zeta})$ be $\boldsymbol{\zeta}$-compact space. let $\{\boldsymbol{M_j}: \boldsymbol{j} \in \boldsymbol{\Lambda}\}$ be a $\boldsymbol{\zeta}$-countable

open cover of $\boldsymbol{X}$ containing $\boldsymbol{\zeta}$ $-$open sets. thus, $\{\boldsymbol{M_j}: \boldsymbol{j} \in \boldsymbol{\Lambda}\}$ is $\boldsymbol{\zeta}$-open cover $\{\boldsymbol{M_j}: \boldsymbol{j} \in \boldsymbol{\Lambda}\}$ of $\boldsymbol{X}$ that has a finite subcover say $\{\boldsymbol{M_j}: \mathbf{j} = \mathbf{1} \ldots \mathbf{n}\}$. therefore, $\mathbf{X}$ is $\boldsymbol{\zeta}$-countably compact.

Remark 55. the converse of Theorem 54 is not true in general.

Example 56. taken as a closed cover on with that is countably compact, the usual topological space $\mathbb{R}$ and $\{\mathbb{R}^+, \mathbb{R}^-\}$is what we will be working with $\mathbb{R}^+ \cap \mathbb{R}^- = \{\mathbf{0}\}$. due to the fact that we are aware that is not zero space, Theorem 52 cannot be true.

Theorem 57. if a map $\boldsymbol{h}: (\boldsymbol{X}, \boldsymbol{\tau_\zeta}) \rightarrow (\boldsymbol{Y}, \boldsymbol{\mathfrak{I}_\zeta})$ is $\boldsymbol{\zeta}$-continuous from a $\boldsymbol{\zeta}$- compact space **X** onto a topological space **Y**, then **Y** is $\boldsymbol{\zeta}$- compact.

proof. assume that $\boldsymbol{h}: (\boldsymbol{X}, \boldsymbol{\tau_\zeta}) \rightarrow (\boldsymbol{Y}, \boldsymbol{\mathfrak{I}_\zeta})$ be $\boldsymbol{\zeta}$-continuous map. set $\{\boldsymbol{M_j}: \boldsymbol{j} \in \boldsymbol{\Lambda}\}$ be $\boldsymbol{\zeta}$-compact be an open cover of $\boldsymbol{Y}$. thus, $\{\mathbf{h^{-1}}(\boldsymbol{M_j}): \boldsymbol{j} \in \boldsymbol{\Lambda}\}$is a $\boldsymbol{\zeta}$ -open cover of $\boldsymbol{X}$. because $(\boldsymbol{X}, \boldsymbol{\tau_\zeta})$ is $\boldsymbol{\zeta}$-compact it has a finite sub-cover like $\{\mathbf{h^{-1}}(\boldsymbol{M_1}), \mathbf{h^{-1}}(\boldsymbol{M_2}), \cdots, \mathbf{h^{-1}}(\boldsymbol{M_n})\}$. therefore, $\{\boldsymbol{M_1}, \boldsymbol{M_2}, \cdots, \boldsymbol{M_n}\}$ is finite cover of $\boldsymbol{Y}$ because $\boldsymbol{h}$ is onto. hence, $\boldsymbol{Y}$ is $\boldsymbol{\zeta}$-compact.

Theorem 58. if a map $\boldsymbol{h}: (\boldsymbol{X}, \boldsymbol{\tau_\zeta}) \rightarrow (\boldsymbol{Y}, \boldsymbol{\mathfrak{I}_\zeta})$ is $\boldsymbol{\zeta}$-contra continuous from a $\boldsymbol{\zeta}$- compact space **X** onto a topological space **Y**, then **Y** is $\boldsymbol{\zeta}$- compact.

proof. assume that $\boldsymbol{h}: (\boldsymbol{X}, \boldsymbol{\tau_\zeta}) \rightarrow (\boldsymbol{Y}, \boldsymbol{\mathfrak{I}_\zeta})$ be $\boldsymbol{\zeta}$-contra continuous map. set $\{\boldsymbol{M_j}: \boldsymbol{j} \in \boldsymbol{\Lambda}\}$ be $\boldsymbol{\zeta}$-compact be an open cover of $\boldsymbol{Y}$. thus, $\{\mathbf{h^{-1}}(\boldsymbol{M_j}): \boldsymbol{j} \in \boldsymbol{\Lambda}\}$is a $\boldsymbol{\zeta}$ -open cover of $\boldsymbol{X}$, because $(\boldsymbol{X}, \boldsymbol{\tau_\zeta})$ is $\boldsymbol{\zeta}$-compact it has a finite sub-cover like

$\{\mathbf{h^{-1}}(\boldsymbol{M_1}), \mathbf{h^{-1}}(\boldsymbol{M_2}), \cdots, \mathbf{h^{-1}}(\boldsymbol{M_n})\}$. Therefore, $\{\boldsymbol{M_1}, \boldsymbol{M_2}, \cdots, \boldsymbol{M_n}\}$ is finite cover of $\boldsymbol{Y}$ because $\boldsymbol{h}$ is onto. hence, $\boldsymbol{Y}$ is $\boldsymbol{\zeta}$-compact.

Theorem 59. if a map $\boldsymbol{h}: (\boldsymbol{X}, \boldsymbol{\tau_\zeta}) \rightarrow (\boldsymbol{Y}, \boldsymbol{\mathfrak{I}_\zeta})$ is $\boldsymbol{\zeta}$-irresolute from a $\boldsymbol{\zeta}$- compact space **X** onto a topological space **Y**, then **Y** is $\boldsymbol{\zeta}$- compact.

proof. assume that $\boldsymbol{h}: (\boldsymbol{X}, \boldsymbol{\tau_\zeta}) \rightarrow (\boldsymbol{Y}, \boldsymbol{\mathfrak{I}_\zeta})$ be $\boldsymbol{\zeta}$- irresolute map. set $\{\boldsymbol{M_j}: \boldsymbol{j} \in \boldsymbol{\Lambda}\}$ be $\boldsymbol{\zeta}$-compact be an open cover of $\boldsymbol{Y}$. thus, $\{\mathbf{h^{-1}}(\boldsymbol{M_j}): \boldsymbol{j} \in \boldsymbol{\Lambda}\}$is a $\boldsymbol{\zeta}$ -open cover of $\boldsymbol{X}$. because $(\boldsymbol{X}, \boldsymbol{\tau_\zeta})$ is $\boldsymbol{\zeta}$-compact it has a finite sub-cover like $\{\mathbf{h^{-1}}(\boldsymbol{M_1}), \mathbf{h^{-1}}(\boldsymbol{M_2}), \cdots, \mathbf{h^{-1}}(\boldsymbol{M_n})\}$. therefore, $\{\boldsymbol{M_1}, \boldsymbol{M_2}, \cdots, \boldsymbol{M_n}\}$ is finite cover of $\boldsymbol{Y}$ because $\boldsymbol{h}$ is onto. hence, $\boldsymbol{Y}$ is $\boldsymbol{\zeta}$-compact.

Theorem 60. if a map $\boldsymbol{h}: (\boldsymbol{X}, \boldsymbol{\tau_\zeta}) \rightarrow (\boldsymbol{Y}, \boldsymbol{\mathfrak{I}_\zeta})$ is $\boldsymbol{\zeta}$-contra irresolute from a $\boldsymbol{\zeta}$- compact space **X** onto a topological space **Y**, then **Y** is $\boldsymbol{\zeta}$- compact.

proof. assume that $\boldsymbol{h}: (\boldsymbol{X}, \boldsymbol{\tau_\zeta}) \rightarrow (\boldsymbol{Y}, \boldsymbol{\mathfrak{I}_\zeta})$ be $\boldsymbol{\zeta}$-contra irresolute map. set $\{\boldsymbol{M_j}: \boldsymbol{j} \in \boldsymbol{\Lambda}\}$ be $\boldsymbol{\zeta}$-compact be an open cover of $\boldsymbol{Y}$. thus, $\{\mathbf{h^{-1}}(\boldsymbol{M_j}): \boldsymbol{j} \in \boldsymbol{\Lambda}\}$is a $\boldsymbol{\zeta}$ -open cover of $\boldsymbol{X}$. because $(\boldsymbol{X}, \boldsymbol{\tau_\zeta})$ is $\boldsymbol{\zeta}$-compact it has a finite sub-cover like $\{\mathbf{h^{-1}}(\boldsymbol{M_1}), \mathbf{h^{-1}}(\boldsymbol{M_2}), \cdots, \mathbf{h^{-1}}(\boldsymbol{M_n})\}$. therefore, $\{\boldsymbol{M_1}, \boldsymbol{M_2}, \cdots, \boldsymbol{M_n}\}$ is finite cover of $\boldsymbol{Y}$ because $\boldsymbol{h}$ is onto. hence, $\boldsymbol{Y}$ is $\boldsymbol{\zeta}$-compact.

Theorem 61. if a map $\boldsymbol{h}: (\boldsymbol{X}, \boldsymbol{\tau_\zeta}) \rightarrow (\boldsymbol{Y}, \boldsymbol{\mathfrak{I}_\zeta})$ is $\boldsymbol{\zeta}$-continuous from a $\boldsymbol{\zeta}$-countably compact space **X** onto a topological space **Y**, then **Y** is $\boldsymbol{\zeta}$-countably compact.

proof: assume $\{M_j: j \in \Lambda\}$ be a ζ-countable open cover of Y. because h is ζ-continuous, then $\{h^{-1}(M_j): j \in \Lambda\}$ is ζ-countable open cover of X. now, since X is ζ-countably compact, then the ζ-countable open cover $\{h^{-1}(M_j): j \in \Lambda\}$ of X has a finite subcover like $\{h^{-1}(M_j): j = 1 \ldots n\}$. thus, $X = \bigcup_{i=1}^{n} h^{-1}(M_i)$ and so $h(X) = \bigcup_{i=1}^{n} M_j$. thus, $Y = \bigcup_{i=1}^{n} M_j$. therefore $\{M_1, M_2, \ldots, M_n\}$ is a finite subcover of $\{M_j: j \in \Lambda\}$ for Y. therefore, Y is ζ-countably compact.

Theorem 62. if a map $h: (X, \tau_\zeta) \to (Y, \mathfrak{I}_\zeta)$ is ζ - irresolute and a subset N of X is ζ - compact relative to X, then the image $h(N)$ is ζ - compact relative to Y.

proof. assume a collection $\{M_j: j \in \Phi\}$ of ζ - open subsets of Y such that $h(N) \subset \bigcup\{M_j: j \in \Phi\} \subset$. thus, $N \subset \bigcup\{h^{-1}(M_j): j \in \Phi\}$. because N is - compact relative to X, there exists a finite subset $M_0 \in \Phi$ such that $N \subset \bigcup\{h^{-1}(M_j): h \in \Phi_0\}$. hence, $h(N) \bigcup \subset \{(M_j): \alpha \in \Phi_0\}$. therefore, $h(N)$ is ζ -compact relative to Y.

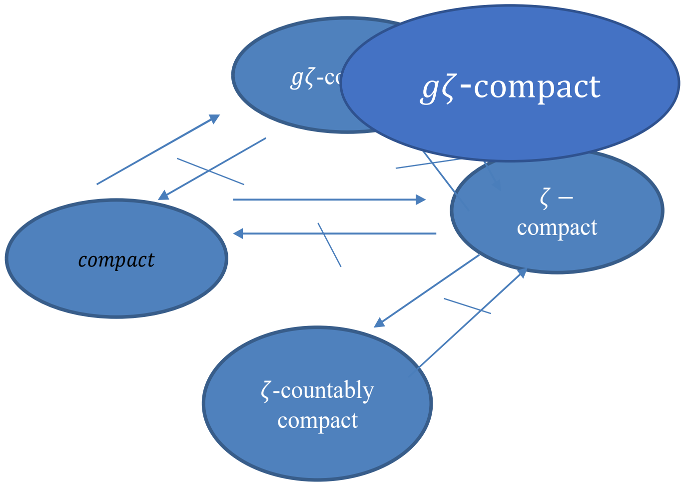
Theorem 63. if a map $h: (X, \tau_\zeta) \to (Y, \mathfrak{I}_\zeta)$ is ζ – contra irresolute and a subset N of X is ζ - compact relative to X, then the image $h(N)$ is ζ - compact relative to Y.

proof. assume a collection $\{M_j: j \in \Phi\}$ of ζ – semi open subsets of Y such that $h(N) \subset \bigcup\{M_j: j \in \Phi\} \subset$. thus, $N \subset \bigcup\{h^{-1}(M_j): j \in \Phi\}$. because N is - compact relative to X, there exists a finite subset $M_0 \in \Phi$ such that $N \subset \bigcup\{h^{-1}(M_j): h \in \Phi_0\}$. hence, $h(N) \bigcup \subset \{(M_j): \alpha \in \Phi_0\}$. therefore, $h(N)$ is ζ -compact relative to Y.

Theorem 64. if a map $h: (X, \tau_\zeta) \to (Y, \mathfrak{I}_\zeta)$ is ζ – contra continuous and a subset N of X is ζ - compact relative to X, then the image $h(N)$ is ζ - compact relative to Y.

proof. assume a collection $\{M_j: j \in \Phi\}$ of ζ - closed subsets of Y such that $h(N) \subset \bigcup\{M_j: j \in \Phi\} \subset$. thus, $N \subset \bigcup\{h^{-1}(M_j): j \in \Phi\}$. because N is - compact relative to X, there exists a finite subset $M_0 \in \Phi$ such that $N \subset \bigcup\{h^{-1}(M_j): h \in \Phi_0\}$. hence, $h(N) \bigcup \subset \{(M_j): \alpha \in \Phi_0\}$. therefore, $h(N)$ is ζ -compact relative to Y.

now, the relationships among compact, $\zeta -$ compact, $\zeta -$ compact, and $\zeta -$ contra irresolute are shown in the figure below.



CONCLUSION

We obtain several results as follows:

1- ζ −continuous map is ζ − contra-continuous map but not converse.
2- ζ − irresolute map is ζ −continuous map but not converse.
3- ζ − irresolute map is ζ −contra irresolute continuous map but not converse.
4- ζ − contra continuous map is ζ −contra irresolute continuous map but not converse.
5- ζ − irresolute map and ζ −contra continuous map independent notion.
6- $g\zeta$-compact is $\zeta - \mathbf{compact}$ but not converse.

Acknowledgments

The authors would like to express they're thanks to the AIP Editorial Board for their valuable suggestions and comments for the acceptance of this paper.

References

[1] Balachandran, K. (1991). On generalized continuous maps in topological spaces. Mem. Fac. Sci. Kochi Univ. Ser. A Math., 12, 5-13.
[2] Di Majo, G. (1987). On s-closed spaces. Indian Journal. Pure applied Mathematic., 18(3), 226-233.
[3] Devi, R. (1994). Studies on generalizations of closed maps and homeomorphisms in topological spaces.
[4] Gnanambal, Y., & Balachandran, K. (1999). On gpr-continuous functions in topological spaces. Indian Journal of Pure and Applied Mathematics, 30, 581-594.
[5] Pushpalatha, A. (2000). Studies on generalizations of mappings in topological spaces.
[6] Sheik, M. J. (2002). A Study on Generalizations of Closed Sets and Continuous Maps in Topological Spaces (Doctoral dissertation, Ph. D. Thesis, Bharathiar Univ., Coimbatore).

[7] Song, Y. K. (2008). On σ-starcompact spaces. Applied General Topology, 9(2), 293-299.

[8] Žitković, G. (2010). Convex compactness and its applications. Mathematics and Financial Economics, 3, 1-12.

[9] Gnanachandra, P. (2015). Remarks on rg-compact, gpr-compact and gpr-connected spaces. Malaya Journal of Matematik, 3(02), 207-210.

[10] Sekar, S., & Brindha, R. (2018). On pgb-Connectedness and pgb-Compactness in Topological Spaces. School of Mathematics Northwest University Xi'an, Shaanxi, China, 13(1), 12-18.

[11] Parthasarathy, K. (2022). Locally Compact Spaces. In Topology: An Invitation (pp. 151-161). Singapore: Springer Nature Singapore.

[12] Al Ghour, S. (2023). Soft C-Continuity and Soft Almost C-Continuity Between Soft Topological Spaces. Heliyon.

[13] Bagaria, J., & da Silva, S. G. (2023). ω1-strongly compact cardinals and normality. Topology and its Applications, 323, 108276.

[14] Das, D., & Mahanta, J. (2016). Boundary and exterior of a multiset topology. Journal of new theory, (12), 75-84.

[15] Girija, S., Selvanayaki, S., & Ilango, G. (2018, December). Study on frontier, border and exterior sets in inuitionistic topological spaces. In Journal of Physics: Conference Series (Vol. 1139, No. 1, p. 012043). IOP Publishing.

[16] Jung, S. M., & Nam, D. (2019). Some properties of interior and closure in general topology. Mathematics, 7(7), 624.

[17] Basumatary, B., Wary, N., Mwchahary, D. D., Brahma, A. K., Moshahary, J., Basumatary, U. R., & Basumatary, J. (2021). A study on some properties of neutrosophic multi topological group. Symmetry, 13(9), 1689.

[18] Mohammadi, K., & Rashidi, A. (2019). A new method for compactification with the help of order topology and limit point. arXiv preprint arXiv:1908.09366.

[19] Farhan, A. M., & Yang, X. S. (2015). New Type of Strongly Continuous Functions in Topological Spaces via Î´-Î²-Open Sets. European journal of pure and applied mathematics, 8(2), 185-200.

[20] Ozel, C., Al, S., Kaymakci, A. K., Park, C., & Shin, D. Y. (2021). On \delta b-open continuous functions. AIMS Mathematics, 6(3), 2947-2955.

[21] Zhang, J. H. (2021). Topological Properties and Characterizations. arXiv preprint arXiv:2108.07061.

[22] Al Ghour, S. (2023). Soft Complete Continuity and Soft Strong Continuity in Soft Topological Spaces. Axioms, 12(1), 78.

[23] Singh, T. B. (2019). Continuity and the Product Topology. Introduction to Topology, 29-50.

[24] Padlewska, B., & Darmochwał, A. (1990). Topological spaces and continuous functions. Formalized Mathematics, 1(1), 223-230.

[25] Dontchev, J. (1996). Contra-continuous functions and strongly S-closed spaces. International journal of mathematics and mathematical sciences, 19(2), 303-310.

[26] El-Magbrabi, A. I. (2010). Some properties of contra-continuous mappings. Int. J. General Topol, 3(1-2), 55-64.

[27] Crossley, S., & Hildebrand, S. (1972). Semi-topological properties. Fundamenta Mathematicae, 74(3), 233-254.

[28] Vidhya, D., & Parimelazhagan, R. (2012). g* b-Homeomorphisms and Contra-g* b-continuous Maps in Topological Spaces. International Journal of Computer Applications, 58(14).

[29] Munkres, J. R. (1974). Topology; a first course [by] james r. munkres. Prentice-Hall.