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SKOLEM DIFFERENCE MEAN LABELING OF STAR RELATED GRAPHS

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ABSTRACT

The graph $G = (V(G), E(G)$ with p vertices and q edges is called Skolem difference mean labeling graph if $f: V(G) \longrightarrow \{1,2 \dots p+q\}$ is an injective mapping such that induced bijective edge labeling $f*: E(G) \longrightarrow \{1,2,\dots,q\}$ defined by $f*(uv) = \frac{|f(u)-f(v)|}{2}$, if $|f(u) - f(v)|$ is even otherwise $f*(uv) = |f(u) - f(v)| + \frac{1}{2}$, if $|f(u) - f(v)|$ is odd.

MSC Classification:05C76

INTRODUCTION:

Graph labelling is an assignment of labels to edges, vertices or both. Labelling of a graph G is an assignment of integers either to the vertices or edges or both subject to certain conditions. A dynamic survey on graph labelling is regularly up dated by Gallian [2]. Graph labelling is an important area of research in graph theory. The Concept of Skolem mean labelling was introduced by V. Balaji, D.S.T. Ramesh & A. Subramanian. Skolem difference mean labelling was introduced by K. Murugesan and A. Subramanian.

BASIC DEFINITIONS

Definition 2.1: The graph $G = (V(G), E(G)$ with p vertices and q edges is called Skolem difference mean labeling graph if $f: V(G) \longrightarrow \{1,2 \dots p+q\}$ is an injective mapping such that induced bijective edge labeling $f*: E(G) \longrightarrow \{1,2,\dots,q\}$ defined by $f*(uv) = \frac{|f(u)-f(v)|}{2}$, if $|f(u) - f(v)|$ is even otherwise $f*(uv) = |f(u) - f(v)| + \frac{1}{2}$, if $|f(u) - f(v)|$ is od

Definition 2.2: Double star $K_{1,n,n}$ is a tree obtained from the star $K_{1,n}$ by adding a new pendant edge to each of the existing n pendant vertices.

Definition 2.3: Bistar is a graph obtained from a path P_2 by joining the root of stars S_m and S_n at the terminal vertices of P_2. It is denoted by $B_{m,n}$.

Definition 2.4: A Subdivision of a graph G is a graph that can be obtained from G by a Sequence of edge Subdivisions.

Definition 2.5: A graph is a line graph of a tree if and only if it is a connected claw-free block graph or equivalently a connected block graph in which each cut vertex belongs to exactly two blocks.

MAIN RESULT

Theorem:1

The subdivision bistar $G = B_{n,n}S_n$ is Skolem Difference mean graph.

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Proof:

Let $[B_{n,n}S_n]$ graph. The order of the graph is $p = 3n + 2$ and the size is $q = 3n + 1$. By the definition of $B_nS_{2,n}$, the vertex set $V = \{u_1, u_2, \dots u_n, v_1, v_2, \dots v_n, w_1, w_2, \dots w_n, u, v\}$. Let $u_1, u_2, \dots u_n$ be the pendent vertices attaching by u and $v_1, v_2, \dots v_n$ be the pendent vertices attaching by v. $w_1, w_2, \dots w_n$ attaching by v_i. Let u,v be the central vertices of $B_{n,n}$. The edge set $E(G) = \{e, e_i, e_{1i}, e_{2i}\}$ Where $e_i = (u, u_i), e_{1i} = (v, v_j), e_{2k} = (v_j, w_k), e = (u, v)$

Now let us define the function $f: v \to \{1,2, \dots . p + q\}$ as follows

$$V(G) = \{1,2, \dots . .6n + 3\}$$

$$f(u) = 1,$$

$$f(u_i) = 2i + 1; i = 1,2, \dots . n$$

$$f(v) = 6n + 3$$

$$f(v_j) = 2(n + j) + 1; j = 1,2, \dots . n$$

$$f(w_k) = 2(n + 2k); k = 1,2, \dots . n$$

Then the edge labels are

$$f(e) = 3n + 1$$

$$f(e_i) = 3n - i + 1; i = 1,2, \dots . n$$

$$f(e_{1j}) = n + j; j = 1,2, \dots . n$$

$$f(e_{2k}) = 2k - j; k = 1,2, \dots n; \mathrm{j} = 1,2,,, \mathrm{n}$$

Then the above defined function f admits Skolem Difference mean labelling. Hence the graph $[B_{n,n}S_n]$ is Skolem Difference mean graph.

Example:

The graph $[B_{4,4}S_4]$ is shown in fig 1. In Sub division bistar graph, the order 14 and size 13. The vertex labels are $f(u_i) = 2i + 1; i = 1,2,3, \dots n$

$$f(u_1) = 3$$

$$f(u_2) = 5$$

$$f(u_3) = 7$$

$$f(u_4) = 9$$

$$f(u) = 6n + 3; f(v) = 1$$

$$f(u) = 27; f(v) = 1$$

$$f(v_j) = 2(n + j) + 1; j = 1,2, \dots . n$$

$$f(v_1) = 11$$

$$f(v_2) = 13$$

$$f(v_3) = 15$$

$$f(v_4) = 17$$

$$f(w_k) = 2(n + 2k); k = 1,2, \dots . n$$

$$f(w_1) = 12$$

$$f(w_2) = 16$$

$$f(w_3) = 20$$

$$f(w_4) = 24$$

The edge labels are

$$f(e_i) = 3n - i + 1; i = 1,2, \dots . n$$
$$f(e_1) = 12$$
$$f(e_2) = 11$$
$$f(e_3) = 10$$
$$f(e_4) = 9$$
$$f(e) = 3n + 1$$
$$f(e) = 13$$
$$f\left(e_{1j}\right) = n + j; j = 1,2, \dots . n$$
$$f(e_{11}) = 5$$
$$f(e_{12}) = 6$$
$$f(e_{13}) = 7$$
$$f(e_{14}) = 8$$
$$f(e_{2k}) = 2k - j; j = 1,2, \dots . n; k = 1,2, \dots n$$
$$f(e_{21}) = 1$$
$$f(e_{22}) = 2$$
$$f(e_{23}) = 3$$
$$f(e_{24}) = 4$$

The labels are satisfying skolem difference mean labelling
Hence the graph $B_{4,4}S_4$ is skolem difference mean graph.

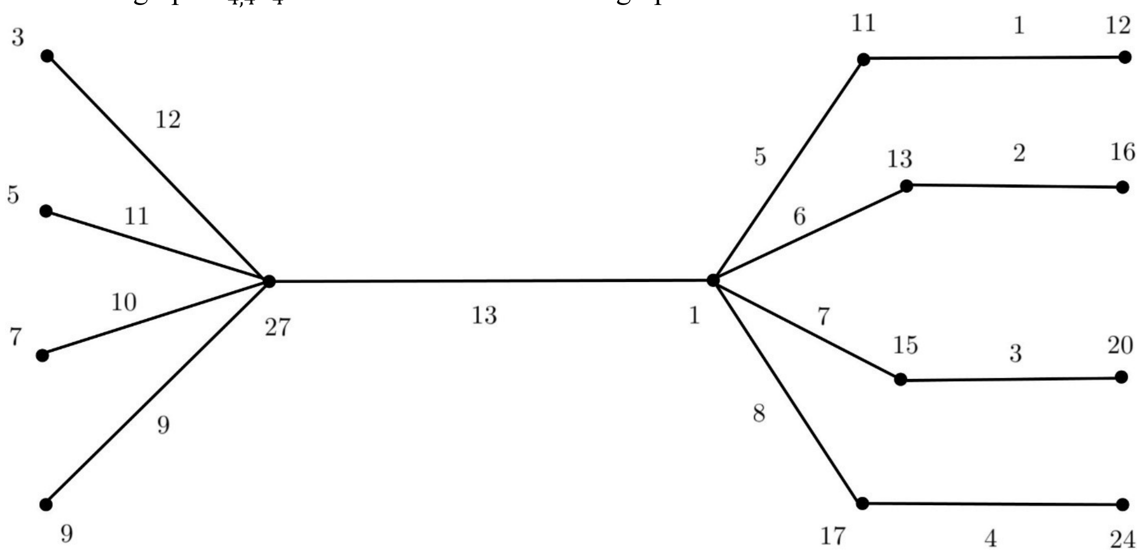

Fig 1: SDML of $B_{4,4}S_4$

Theorem: 2

The graph $C_5 \oplus K_{1,n}$ for $n \geq 2$, The order of the graph $p = n + 5$, and size of the graph $q = n + 5$. By the definition of the graph $C_5 \oplus K_{1,n}$ be the vertex set

$$V = \{u, u_1, \dots u_n, v_1, v_2, \dots v_n\}$$

Where $\{u_1, u_2, u_3, u_4, u_5\}$ be the vertex set C_5, $\{v_1, v_2, \dots v_n\}$ attaching the vertex u. The edge set $\{e_1, e_2, \dots e_5, e_{11}, e_{12}, \dots e_{1j}\}$ Where $e_i = \{u, u_i\}$ for $i \leq 5$, $e_{1j} = \{u_{1j}, u\}$

Now let us define the function $f: v \to \{1,2, \ldots p+q\}$ as follows,

$$f: V(G) = \{1,2, \ldots 2n+10\}, p = n+5, q = n+5$$

Vertex label

$$f(u) = 1$$
$$f(u_i) = i+1; i = 1,3$$
$$f(u_2) = 8,$$
$$f(u_4) = 11$$
$$f(v_j) = 2(n+1+j); j = 1,2, \ldots n$$

Edge label

$$f(e_{1j}) = n+1+j; j = 1,2, \ldots n$$
$$f(e_1) = 1,$$
$$f(e_2) = 3,$$
$$f(e_3) = 2,$$
$$f(e_4) = 4,$$
$$f(e_5) = 5,$$

Then the above defined the function of f admits skolem difference mean labelling. Hence the graph $C_5 \oplus K_{1,n}$ is skolem difference mean graph.

Example:

The graph $C_5 \oplus K_{1,4}$ is shown in fig 2. In the order 9, and size 9.

The vertex labels are

$$f(u) = 1$$
$$f(u_i) = i+1, i = 1,3$$
$$f(u_1) = 2, f(u_3) = 4$$
$$f(u_2) = 8, f(u_4) = 11$$
$$f(v_j) = 2(n+1+j); j = 1,2,3,4$$
$$f(v_1) = 12, f(v_3) = 14$$
$$f(v_2) = 16, f(v_4) = 18$$

The Edge labels

$$f(e_1) = 1, f(e_2) = 3, f(e_3) = 2,$$
$$f(e_4) = 4, f(e_5) = 5,$$
$$f(e_1') = n+1+j; j = 1,2,3,4$$
$$f(e_1') = 6, f(e_2') = 7, f(e_3') = 8, f(e_4') = 9$$

The labels are satisfying skolem difference mean labelling

Hence the graph $C_5 \oplus K_{1,4}$ is skolem difference mean graph.

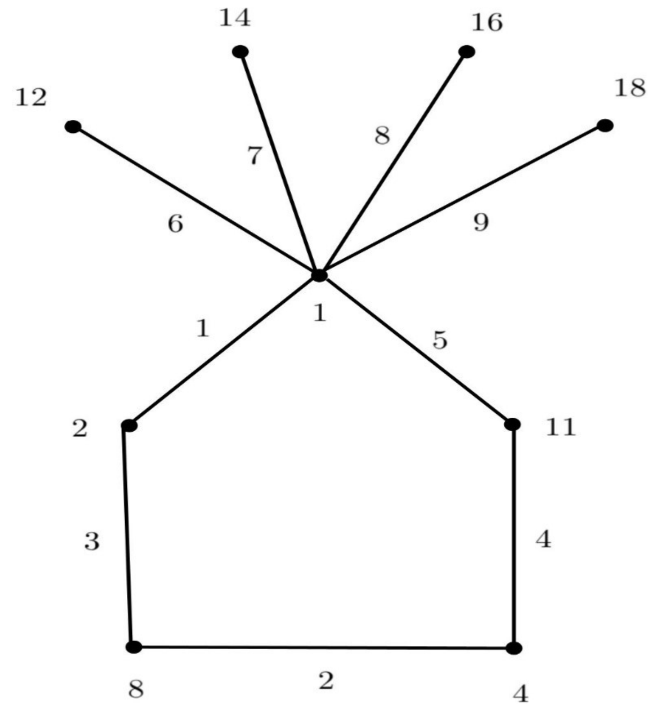

Fig 2: SDML of $C_5 \oplus K_{1,4}$

Theorem 3:
The graph $S_n \oplus P_2$ is skolem difference mean labelling, $n \geq 2$.
Proof:
Let the graph $S_n \oplus P_2$ for $n \geq 2$, The order of the graph $P = 3n + 1$, and size of the graph $q = 3n$. By definition of the given graph $S_n + P_2$, be the vertex set $V(G) = \{ u_{n+1}, u_1, \dots u_n, v_1, v_2, \dots v_n\}$
Where $\{u, u_1, \dots u_n\}$ are the vertices attaching the vertex u_{n+1} and $\{v_1, v_2, \dots v_n\}$ are the vertices attaching by the vertices $v_1, v_2, \dots v_n$.
The Edge set $E(G) = \{e_1, e_2, \dots e_n, e'_1, e'_2, \dots e'_n, \}$. Where $e_i = (u_{n+1}, u_i), e'_{ij} = (u_i, v_i)$ for $0 \leq i \leq n$. Define the function $f: v \to \{1,2, \dots p + q\}$ vertex labeled as follows:

$$f(u_{n+1}) = 6n + 1$$

$$f(u_i) = 2n + 1 - 2i; i = 1,2, \dots n$$

$$f(v_i) = \bigcup_{i=1; i \equiv 1(mod 2)}^{n} (5n + 4m - 3j)$$

$$f(v_i) = \bigcup_{i=1; i \equiv 0(mod 2)}^{n} (5n + 4m + 1 - 3j)$$

Edge labelled as follows:

$$f(e_i) = 2n + i; i = 1,2, \dots n$$

$$f(e_{ij}) = \bigcup_{i=1}^{n}[\bigcup_{j=1;j\equiv 1(mod2)}^{n}[3n+4m-3j+2i-1]]/2$$

$$f(e_{ij}) = \bigcup_{i=1}^{n}[\bigcup_{j=1;j\equiv 0(mod2)}^{n}[3n+4m-3j+2i]]/2$$

Then the above defined the function of f admits Skolem difference mean labelling. Hence the graph $S_n \oplus P_2$, is skolem difference mean graph.

Example:

The graph $S_6 \oplus P_2$ is shown in fig 3. In the order 19, and size 18.

The vertex labels are,

$$f(u_{n+!}) = 6n+1;$$

$$f(u_7) = 37,$$

$$f(u_i) = 2n+1-2i, i = 1,2,\dots 6; n = 6$$

$$f(u_1) = 11, f(u_4) = 5$$

$$f(u_2) = 9, f(u_5) = 3$$

$$f(u_3) = 7, f(u_6) = 1$$

$$f(v_i) = \bigcup_{i=1;i\equiv 1(mod2)}^{n}(5n+4m-3i)\,; m = 2$$

$$f(v_1) = 35, f(v_4) = 17$$

$$f(v_2) = 29, f(v_5) = 11$$

$$f(v_3) = 23, f(v_6) = 5$$

$$f(v_i) = \bigcup_{i=1;i\equiv 0(mod2)}^{n}(5n+4m+1-3i)\,; m = 2$$

$$f(v_2) = 33, f(v_7) = 15$$

$$f(v_4) = 27, f(v_9) = 9$$

$$f(v_6) = 21, f(v_{11}) = 3$$

Edge labels are

$$f(e_i) = 2n+i; i = 1,2,\dots 6$$

$$f(e_1) = 13, f(e_2) = 14,$$

$$f(e_3) = 15, f(e_4) = 16,$$

$$f(e_5) = 17, f(e_6) = 18$$

$$f(e_{ij}) = \frac{\bigcup_{i=1}^{n}[\bigcup_{j=1;j\equiv 1(mod2)}^{n}[3n+4m-3j+2i-1]]}{2}; n = 2, m = 2$$

$$f(e_{ij}) = \frac{\bigcup_{i=1}^{n}[\bigcup_{j=1;j\equiv 0(mod2)}^{n}[3n+4m-3j+2i]]}{2}; n = 6, m = 2$$

$$f(e_{11}) = 1, f(e_{13}) = 3, f(e_{15}) = 5, f(e_{17}) = 7,$$

$$f(e_{12}) = 2, f(e_{14}) = 4, f(e_{16}) = 6, f(e_{18}) = 8,$$

$$f(e_{19}) = 9, f(e_{21}) = 11,$$

$$f(e_{20}) = 10, f(e_{22}) = 12$$

The labels are satisfying skolem difference mean labelling.
Hence the graph $S_6 \oplus P_2$is skolem difference mean graph.

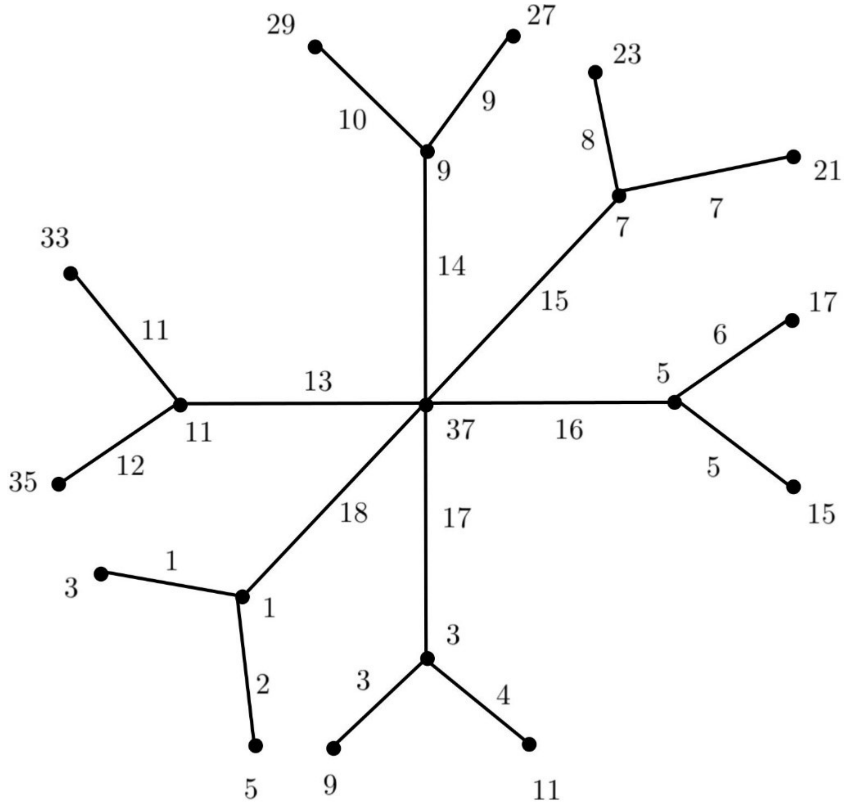

Fig 3: SDML of $S_6 \oplus P_2$

Theorem: 4

The star $SS_{1,n}$ is a skolem difference mean labelling, $n \geq 2$.

Proof:

Let $SS_{1,n}$ be the star graph. By the definition of $SS_{1,n}$ the order and size are $P = 2n + 1, Q = 2n$.

The vertex set is $V(G) = \{ u_{n+1}, u_1, \dots u_n, v_1, v_2, \dots v_n\}$

Where $u, u_1, \dots u_n$ are adjacent vertices of u_{n+1} and $u, u_1, \dots u_n$ are adjacent vertices of $v_1, v_2, \dots v_n$.

The edge set $E(G) = \{ e_1, e_2, \dots e_n, e_{11}, , \dots e_{1n}\}$.

$$e_i = \{u_{n+1}, u_i\}, e_{ij} = \{u_i, v_i\}$$

Define the function $f: V(G) \to \{1,2, \dots, p + q\}$. Vertex labels are:

$$f(u_{n+1}) = 4n + 1,$$
$$f(u_i) = 2n + 1 - 2i; i = 1,2, \dots n$$
$$f(v_i) = 4n - 4j + 2; i = 1,2, \dots n$$

Edge labels are:

$$f(e_i) = n + i; i = 1,2, \dots n$$

$$f(e_{ij}) = n + i - 2j + 1; i = 1,2, \dots n; j = 1,2 \dots n$$

Then the above function f admits Skolem difference mean labelling. Hence the graph $SS_{1,n}$ accepts skolem difference mean graph.

Example:

The graph $SS_{1,6}$ is shown in fig.4.

The order and size are 13 and 12.

The vertex labels are

$$f(u_{n+1}) = 4n + 1$$
$$f(u_7) = 25$$
$$f(u_i) = 2n + 1 - 2i; i = 1,2, \dots 6$$
$$f(u_1) = 11, f(u_3) = 7, f(u_5) = 3,$$
$$f(u_2) = 9, (u_4) = 5, f(u_6) = 7.$$
$$f(v_i) = 4n - 4j + 2; i = 1,2, \dots 6$$
$$f(v_1) = 22, f(v_3) = 14, f(v_5) = 6$$
$$f(v_2) = 18, (v_4) = 10, f(v_6) = 2.$$

Edge labels are

$$f(e_i) = n + i; i = 1,2, \dots 6$$
$$f(e_1) = 7, f(e_3) = 9, f(e_5) = 11$$
$$f(e_2) = 8, (e_4) = 10, f(e_6) = 12.$$
$$f(e_{ij}) = n + i - 2j + 1; i = 1,2, \dots 6; j = 1,2, \dots 6$$
$$f(e_{11}) = 6, f(e_{33}) = 4, f(e_{55}) = 2$$
$$f(e_{22}) = 5, (e_{44}) = 3, f(e_{66}) = 1$$

The labels are satisfying skolem difference mean labelling.

Hence the $SS_{1,n}$ is skolem difference mean graph.

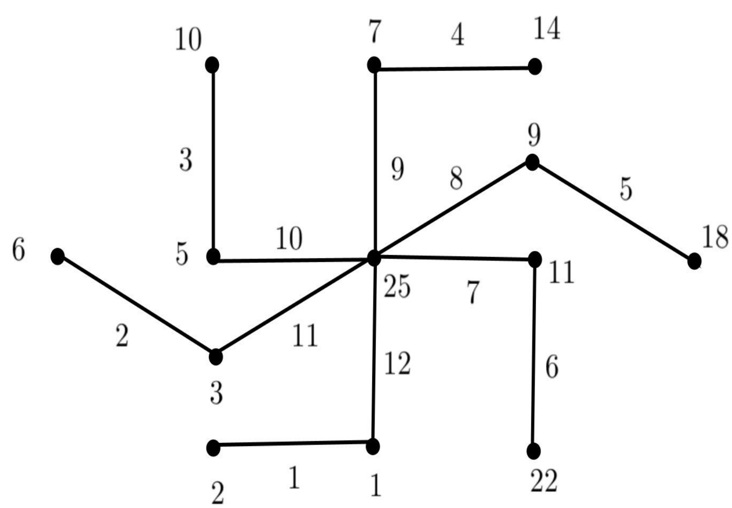

Fig 4: SDML of $SS_{1,n}$

CONCLUSION:

In this article the concept of Skolem Difference Mean Labelling Of Star Related Graphs are explained. In future different concept of labelling can also be developed.

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