JOURNAL OF NORTHEASTERN UNIVERSITY
Volume 26 Issue 04, 2023 ISSN: 1005-3026 https://dbdxxb.cn/ Original Research Paper

DAIRY INDUSTRY INVENTORY MODEL FOR DETERIORATING ITEMS WITH TWO-WAREHOUSE UNDER FIFO DISPATCHING POLICY

[1]Mohammed Abid, [2]Ajay Singh Yadav
[1]Research Scholar, SRM Institute of Science and Technology, Delhi-NCR Campus, Ghaziabad, Uttar Pradesh, India.
[2]Department of Mathematics, SRM Institute of Science and Technology, Delhi-NCR Campus, Ghaziabad, Uttar Pradesh, India.

Abstract
The dairy industry's two-warehouse inventory model for spoiled items in the event of shortages and inflation under the FIFO shipping policy addresses the challenges of inventory management in the dairy industry. This mathematical model combines theories of inventory management, spoilage, shortages and inflation to optimize inventory decisions. The model is based on the FIFO principle to ensure that the oldest inventory is shipped first, minimizing the risk of spoilage. Dairy expiration rates are considered to determine optimal restocking and shipping policies. The model also takes into account the possibility of bottlenecks and aims to minimize them through effective inventory management. In addition, inflation is taken into account to reflect changes in the economic environment and its impact on inventory costs. By applying mathematical techniques and optimization algorithms, the model helps to minimize total inventory costs and improve operational efficiency in the dairy industry. This summary provides an overview of the key elements and theoretical underpinnings of the dairy industry's two-store inventory model and highlights its importance in improving inventory management practices.
Keywords:- Inventory, owned warehouse, rented warehouse, ramp type demand, deteriorating items, inflation, Shortages and FIFO dispatching policy.

1. Introduction

Effective inventory management is vital for the dairy industry to ensure product quality, minimize costs and meet customer demand. However, inventory management in the dairy industry poses unique challenges due to factors such as the expiration of perishables, the possibility of shortages, and the impact of inflation on costs. To address these challenges, the dairy industry's inventory model includes two warehouses for deteriorating items. congestion and inflation as part of the FIFO shipping policy, a systematic approach to optimizing inventory decisions. The model draws on the theories of inventory management, decomposition, shortages and inflation to provide a comprehensive framework for effective inventory management in the dairy industry. By integrating these theories, the model aims to find a balance between maintaining optimal inventory levels, minimizing the risk of product deterioration, reducing shortages and controlling costs in the face of inflation.

Submitted: 22/10/2023 **Accepted**: 14/11/2023

The FIFO shipping policy, which ensures that the oldest stock is shipped first, is introduced to prioritize the use of products with a shorter remaining shelf life. This policy reduces the risk of product spoilage and waste and improves overall product quality and customer satisfaction.

In addition, the model takes into account the spoilage of dairy products and their specific spoilage rates. By considering product shelf life and incorporating restocking and shipping policies accordingly, the model enables companies to optimize inventory turnover and minimize spoilage losses.

Bottlenecks in the dairy industry can lead to lost sales and dissatisfied customers. The model solves this problem by optimizing stock levels and reordering decisions to reduce occurrences of stock-outs and improve customer service. By minimizing bottlenecks, companies can improve customer satisfaction and gain a competitive advantage in the marketplace.

In addition, the model takes into account the impact of inflation on inventory costs. By factoring in inflation, businesses can make informed decisions on pricing, order quantities, and inventory costs to adjust to changing economic conditions and maintain profitability.

The Dairy Industry Two-Warehouse Inventory Model for Spoiled Items with Shortages and Inflation under the FIFO Shipping Policy provides a theoretical foundation and practical guidance for dairy companies to optimize their inventory management practices. By using mathematical techniques and optimization algorithms, companies can increase operational efficiency, reduce costs, and improve overall business performance in the dynamic dairy industry.

However, there are a number of things whose significance does not remain the same over time.The deterioration of these substances plays an important role and cannot be stored for long {Yadav, et. al. (1 to 10)}. Deterioration of an object can be described as deterioration, evaporation, obsolescence and loss of use or limit of an object, resulting in lower stock consumption compared to natural conditions.When commodities are placed in stock as inventory to meet future needs, there may be deterioration of items in the system of arithmetic that may occur for one or more reasons, etc. Storage conditions, weather or humidity. {Yadav, et. al. (11 to 20)}. Inach it is generally claimed that management owns a warehouse to store purchased inventory. However, management can, for a variety of reasons, buy or give more than it can store in its warehouse and name it OW, with an additional number in a rented warehouse called RW located near OW or slightly away from it {Yadav, et. al. (21 to 53)}. Inventory costs (including holding costs and depreciation costs) in RW are usually higher than OW costs due to additional costs of handling, equipment maintenance, etc. To reduce the cost of inventory will economically use RW products as soon as possible. Actual customer service is provided only by OW, and in order to reduce costs, RW stocks are first cleaned. Such arithmetic examples are called two arithmetic examples in the warehouse {Yadav and swami. (54 to 61)}. Management of supply of electronic storage devices and integration of environment and nervous networks {Yadav and Kumar (62)}. Analysis of seven supply chain management measures in improving the inventory of electronic devices for storage by sending an economic load using GA and PSO and analysis of supply chain management in improving the inventory of storage and equipment using genetic calculation and model design and analysis of chain inventory from bi warehouse and economic difficulty of freight transport using genetic calculation {Yadav, AS (63, 64, 65)}. Inventory policies of inventory and inventory requirements and different storage costs under allowable payments and inventory delays An example of depreciation of goods and services of various types and costs of holding down a Business-Loan and an inventory model with sensitive needs of prices, holding costs in contrast to loans of business expenses under inflation {Swami, et. al. (66, 67, 68)}. The objectives of the Multiple Objective Genetic Algorithm and PSO, which include the improvement of supply and deficit inventory, inflation, and a calculation model based on a genetic calculation of scarcity and low inflation by PSO {Gupta, et. al. (69, 70,)}. An example with two warehouses depreciation of items and storage costs under particle upgrade and an example with two warehouses of material damage and storage costs in inflation and soft computer techniques {Singh, et. al. (71, 72)}. Delayed alcohol supply management and refinement of particles and green cement supply system and inflation using particle enhancement and electronic inventory calculation system and distribution center using genetic calculations {Kumar, et. al. (73, 74.75)}. Example of depreciation inventory with two warehouses and stock-based stocks using a genetic inventory and vehicle inventory system for demand and inflation of stocks with two distribution centers using genetic inventory {Chauhan and Yadav (76 , 77)}. Marble Analysis Improvement of industrial reserves based on genetic engineering and multi-particle improvement {Pandey, et. al. (78)}. White wine industry in supply chain management using nervous networks {Ahlawat, et. al. (79)}. Best policy for importing damaged items immediately and payment of conditional delays under the supervision of two warehouses {Singh, et. al. (80)}.

2. Assumptions and Notations:

In developing the mathematical model of the Dairy industry inventory system the following assumptions are being made:

1. A single item is considered over a prescribed period T units of time.
2. The Dairy industry demand rate D(t) at time t is deterministic and taken as a ramp type function of time i.e. $D(t)=\left(\beta_0+\beta\right)e^{-\beta_2\left\{t-\left\{t-\left(t_\beta+t_F\right)\right\}H\left[t-\left(t_\beta+t_F\right)\right]\right\}}$, $\beta>0,\ \beta_0>0,\ \beta_2>0$ where $H\left[t-\left(t_\beta+t_F\right)\right]$ is the Heaviside's function defined as
$$H\left[t-\left(t_\beta+t_F\right)\right]=\begin{cases}0, & t<\left(t_\beta+t_F\right)\\ 1, & t\geq\left(t_\beta+t_F\right)\end{cases}$$
3. The replenishment rate is infinite and lead-time is zero.
4. When the demand for goods is more than the supply. Shortages will occur. Customers encountering shortages will either wait for the vender to reorder (backlogging cost involved) or go to other vendors (lost sales cost involved). In this model shortages are allowed and the backlogging rate is $exp(-\beta_3 t)$, when Dairy industry inventory is in shortage. The backlogging parameter β_3 is a positive constant.

5. **5.** The variable rate of deterioration in both Dairy industry warehouse is taken as $\beta_1(t) = \beta_1 t$. Where $0 < \beta_1 << 1$ and only applied to on hand Dairy industry inventory.
6. **6.** No replacement or repair of deteriorated items is made during a given cycle.
7. **7.** The Dairy industry owned warehouse (OW) has a fixed capacity of W units; the Dairy industry rented warehouse (RW) has unlimited capacity.
8. **8.** The goods of Dairy industry OW are consumed only after consuming the goods kept in Dairy industry RW.

In addition, the following notations are used throughout this paper:

$\Pi_{ow}^{fifo}(t)$	The Dairy industry inventory level in OW at any time t.
$\Pi_{rw}^{fifo}(t)$	The Dairy industry inventory level in RW at any time t.
β_w	The capacity of the Dairy industry own warehouse.
Q	The Dairy industry ordering quantity per cycle.
T	Planning horizon.
β_4	Inflation rate.
β_{hcow}	The holding cost per unit per unit time in Dairy industry OW.
β_{hcrw}	The holding cost per unit per unit time in Dairy industry RW. where $C_1 < C_2$
β_{dc}	The deterioration cost per unit.
β_{sc}	The shortage cost per unit per unit time.
β_{opc}	The opportunity cost due to lost sales.
β_{OC}	The replenishment cost per order.

3. Formulation and Solution of The Model:

The Dairy industry inventory levels at OW are governed by the following differential equations:

$$\frac{d\Pi_{ow}^{fifo}(t)}{dt} = \left[-\beta_1(t) I^{fifo}(t)\right] \qquad 0 \le t < \left(t_\beta + t_F\right) \tag{1}$$

$$\frac{d\Pi_{ow}^{fifo}(t)}{dt} + \beta_1(t) I^{fifo}(t) = -\left(\beta_0 + \beta\right) e^{-\beta_2\left(t_\beta + t_F\right)}, \quad \left(t_\beta + t_F\right) \le t \le \left(t_1 + t_F\right) \tag{2}$$

And

$$\left[\frac{d\Pi_{ow}^{fifo}(t)}{dt}\right] = \left[-\left(\beta_0 + \beta\right) e^{-\beta_2\left(t_\beta + t_F\right)} e^{-\beta_3 t}\right] \quad (t_1 + t_F) \le t \le (T + t_F) \tag{3}$$

with the boundary conditions,

$$\Pi_{ow}^{fifo}(0) = \beta_w \text{ and } \Pi^{fifo}(t_1 + t_F) = 0 \tag{4}$$

The solutions of equations (1), (2) and (3) are given by

$$\Pi_{ow}^{fifo}(t) = \beta_w e^{-\beta_1 t^2/2}, \qquad 0 \le t < (t_\beta + t_F) \qquad (5)$$

$$\Pi_{ow}^{lifo}(t) = \left[(\beta_0 + \beta) e^{-\beta_2 (t_\beta + t_F)} \left\{ \begin{matrix} \{(t_1 + t_F) - t\} + \\ \dfrac{\beta_1 ((t_1 + t_F)^3 - t^3)}{6} \end{matrix} \right\} e^{-\beta_1 t^2/2} \right], (t_\beta + t_F) \le t \le (t_1 + t_F) \qquad (6)$$

And $\Pi_{ow}^{fifo}(t) = \left[\dfrac{(\beta_0 + \beta)}{\beta_3} e^{-\beta_2 (t_\beta + t_L)} \left\{ e^{-\beta_3 t} - e^{-\beta_3 (t_1 + t_F)} \right\} \right] \quad (t_1 + t_F) \le t \le (T + t_F) \qquad (7)$

respectively.

The Dairy industry inventory level at RW is governed by the following differential equations:

$$\frac{d\Pi_{rw}^{fifo}(t)}{dt} + \beta_1(t) \Pi^{fifo}(t) = -(\beta_0 + \beta) e^{-\beta_2 t}, \qquad 0 \le t < (t_\beta + t_F) \qquad (8)$$

With the boundary condition $\Pi_{rw}^{fifo}(0) = 0$ the solution of the equation (8) is

$$\Pi_{rw}^{fifo}(t) = \left[(\beta_0 + \beta) \left\{ \begin{matrix} \{(t_\beta + t_F) - t\} - \\ \dfrac{\beta_2}{2}\left((t_\beta + t_F)^2 - t^2\right) + \\ \dfrac{\beta_1}{6}(((t_1 + t_F)^3 - t^3) \end{matrix} \right\} e^{-\beta_1 t^2/2} \right] \qquad (t_\beta + t_F) \le t \le (t_1 + t_F) \qquad (9)$$

Due to continuity of $\Pi_{ow}^{Fifo}(t)$ at point $t = (t_\beta + t_F)$ it follows from equations (5) and (6), one has

$$\beta_w e^{-\beta_1 (t_\beta + t_F)^2/2} = \left[(\beta_0 + \beta) e^{-\beta_2 (t_\beta + t_F)} \left\{ \begin{matrix} \{(t_1 + t_F) - (t_\beta + t_F)\} + \\ \dfrac{\beta_1 \{(t_1 + t_F)^3 - (t_\beta + t_F)^3\}}{6} \end{matrix} \right\} e^{-\beta_1 (t_\beta + t_F)^2/2} \right]$$

$$\beta_w = \left[(\beta_0 + \beta) e^{-\beta_2 (t_\beta + t_F)} \left\{ \begin{matrix} \{(t_1 + t_F) - (t_\beta + t_F)\} + \\ \dfrac{\beta_1 \{(t_1 + t_F)^3 - (t_\beta + t_F)^3\}}{6} \end{matrix} \right\} \right] \qquad (10)$$

The total average cost consists of following elements:

(i) Ordering cost per cycle = β_{OC} (11)

(ii) Holding cost per cycle in Dairy industry OW

$$C_{HO}=\left[\beta_{hcow}\left\{\begin{array}{l}\int\limits_{0}^{\left(t_\beta+t_F\right)}\Pi_{ow}^{fifo}\left(t\right)e^{-\beta_4 t}dt+\\ \int\limits_{\left(t_\beta+t_F\right)}^{\left(t_1+t_F\right)}\Pi_{ow}^{fifo}\left(t\right)e^{-\beta_4\left\{\left(t_\beta+t_F\right)+t\right\}}dt\end{array}\right\}\right]$$

Appendix A (12)

(iii) Holding cost per cycle (C_{HR}) in RW

$$C_{HR}=\left[\beta_{hcrw}\left\{\int\limits_{0}^{\left(t_\beta+t_F\right)}\Pi_{rw}^{fifo}\left(t\right)e^{-\beta_4 t}dt\right\}\right]$$

$$C_{HR}=\left[\beta_{hcrw}\left(\beta_0+\beta\right)\left\{\begin{array}{l}\frac{\left(t_\beta+t_F\right)^2}{2}-\frac{\left(3\beta_2+\beta_4\right)}{6}\left(t_\beta+t_F\right)^3+\\ \left(\frac{\beta_1}{12}+\frac{\beta_2\beta_4}{8}\right)\left(t_\beta+t_F\right)^4-\\ \left(\frac{\beta_4\beta_1}{20}-\frac{\beta_2\beta_1}{30}\right)\left(t_\beta+t_F\right)^5\end{array}\right\}\right] \quad (13)$$

(iv) Cost of deteriorated units per cycle (C_D)

$$=\left[\beta_{dc}\left\{\begin{array}{l}\int\limits_{0}^{\left(t_\beta+t_F\right)}\beta_1\, t\,\Pi_{rw}^{fifo}\left(t\right)e^{-\beta_4 t}dt+\\ \int\limits_{0}^{\left(t_\beta+t_F\right)}\beta_1 t\,\Pi_{ow}^{fifo}\left(t\right)e^{-\beta_4 t}dt+\\ \int\limits_{\left(t_\beta+t_F\right)}^{\left(t_1+t_F\right)}\beta_1\, t\,\Pi_{ow}^{fifo}\left(t\right)e^{-\beta_4\left\{t+\left(t_\beta+t_F\right)\right\}}dt\end{array}\right\}\right]$$

Appendix B (14)

(v) Shortage cost per cycle (C_S)

$$=\beta_{sc}\left[\int\limits_{\left(t_1+t_F\right)}^{\left(T+t_F\right)}-\Pi_{ow}^{fifo}\left(t\right)e^{-\beta_4\left\{\left(t_1+t_F\right)+t\right\}}dt\right]$$

$$=\frac{-(\beta_0+\beta)\beta_{sc}e^{-\left[\beta_4(t_1+t_F)+\beta_2(t_\beta+t_F)\right]}}{\beta_3}\left[\int_{(t_1+t_F)}^{(T+t_\beta)} e^{-(\beta_4+\beta_3)t}dt-e^{-\beta_3(t_1+t_F)}\int_{(t_1+t_F)}^{(T+t_F)} e^{-\beta_4 t}dt\right]$$

$$=\left[\frac{(\beta_0+\beta)\beta_{sc}\,e^{-(\beta_4(t_1+t_F)+\beta_2(t_\beta+t_F))}}{\beta_3\beta_4(\beta_3+\beta_4)}\begin{Bmatrix}\beta_3 e^{-(\beta_3+\beta_4)(t_1+t_F)}+\\ e^{-\beta_4(T+t_F)}\begin{Bmatrix}re^{-\beta_3(T+t_F)}-\\(\beta_3+\beta_4)e^{-\beta_3(t_1+t_F)}\end{Bmatrix}\end{Bmatrix}\right] \quad (15)$$

(vi) Opportunity cost due to lost sales per cycle (C_0)

$$=\beta_{opc}\int_{(t_1+t_F)}^{(T+t_F)}(\beta_0+\beta)(1-e^{-\beta_3 t})e^{-\beta_2(t_\beta+t_F)}\,e^{-\beta_4\{(t_1+t_F)+t\}}dt$$

$$=\left[\frac{\beta_{opc}(\beta_0+\beta)e^{-\{\beta_2(t_\beta+t_F)+\beta_4(t_1+t_F)\}}}{\beta_4(\beta_3+\beta_4)}\begin{Bmatrix}e^{-\beta_4(t_1+t_F)}\left\{(\beta_3+\beta_4)-\beta_4 e^{-\beta_3(t_1+t_F)}\right\}-\\ e^{-\beta_4(T+t_F)}\left\{(\beta_3+\beta_4)-\beta_4 e^{-\beta_3(T+t_F)}\right\}\end{Bmatrix}\right] \quad (16)$$

Therefore, the total average cost per unit time of our model is obtained as follows

$$TC(t_1+t_F,T+t_F)=\frac{1}{(T+t_F)}\begin{bmatrix}\text{Ordering cost +Holding cost in OW}\\ \text{+ Holding cost in RW+ Deterioration cost}\\ \text{+ Shortage cost + Opportunity cost}\end{bmatrix}=\frac{R(t_1,T)}{T} \quad (17)$$

Appendix C (18)

To minimize the total cost per unit time, the optimal values of t_1 and T can be obtained by solving the following equations simultaneously

$$\frac{\partial TC}{\partial(t_1+t_F)}=0 \quad (19)$$

and $$\frac{\partial TC}{\partial(T+t_F)}=0 \quad (20)$$

provided, they satisfy the following conditions

$$\frac{\partial^2 TC}{\partial(t_1+t_F)^2}>0,\frac{\partial^2 TC}{\partial(T+t_F)^2}>0$$

and $$\left(\frac{\partial^2 TC}{\partial(t_1+t_F)^2}\right)\left(\frac{\partial^2 TC}{\partial(T+t_F)^2}\right)-\left(\frac{\partial^2 TC}{\partial(t_1+t_F)\partial(T+t_F)}\right)^2>0 \quad (21)$$

The equation (19) is equivalent to the following equation

$$
\frac{\partial TC}{\partial\left(t_1+t_F\right)}=\left[\frac{\left(\beta_0+\beta\right)}{\left(T+t_F\right)}\left[\begin{array}{l}
\left\{e^{-\beta_2\left(\left(t_\beta+t_F\right)+\beta_4\right)}\left(\begin{array}{l}
\left(t_1+t_F\right)-\dfrac{\beta_4\left(t_1+t_F\right)^2}{2}+\\
\dfrac{\beta_1\left(t_1+t_F\right)^3}{3}-\dfrac{\beta_4\beta_1\left(t_1+t_F\right)^4}{4}-\\
\left(t_\beta+t_F\right)-3\beta_1\left(t_\beta+t_F\right)\left(t_1+t_{\beta F}\right)^2+\\
\dfrac{\beta_4\left(t_\beta+t_F\right)^2}{2}+\dfrac{\beta_4\beta_1\left(t_\beta+t_F\right)^2\left(t_1+t_F\right)^2}{2}+\\
\dfrac{\beta_1\left(t_\beta+t_F\right)^3}{6}
\end{array}\right)\right\}+\\
\beta_{dc}\beta_1 e^{-\beta_2\left(\left(t_\beta+t_F\right)+\beta_4\right)}\left\{\left(\begin{array}{l}
\dfrac{\left(t_1+t_F\right)^2}{2}-\dfrac{\beta_4\left(t_1+t_F\right)^3}{3}+\\
\dfrac{\beta_1\left(t_1+t_F\right)^4}{8}-\dfrac{\beta_4\beta_1\left(t_1+t_F\right)^5}{6}-\\
\dfrac{\left(t_\beta+t_F\right)^2}{2}-\dfrac{\beta_1\left(t_\beta+t_F\right)^2\left(t_1+t_F\right)^2}{4}-\\
\dfrac{\beta_4\left(t_\beta+t_F\right)^3}{3}-\dfrac{\beta_4\beta_1\left(t_\beta+t_F\right)^3\left(t_1+t_F\right)^2}{6}-\\
\dfrac{\beta_1\left(t_\beta+t_F\right)^4}{8}
\end{array}\right)\right\}+\\
\dfrac{\beta_{sc}e^{-\beta_2\left(t_\beta+t_F\right)}}{\beta_3\beta_4\left(\beta_3+\beta_4\right)}\left\{\left(\begin{array}{l}
-\beta_3\left(\beta_3+2\beta_4\right)e^{-\left(\beta_3+2\beta_4\right)\left(t_1+t_{LF}\right)}-\\
\beta_4{}^2 e^{-\beta_4\left(t_1+t_F\right)-\left(\beta_4+\beta_3\right)\left(T+t_F\right)}+\\
\left(\beta_3+\beta_4\right)^2 e^{-\left(\beta_4+\beta_3\right)\left(t_1+t_F\right)-\beta_4\left(T+t_F\right)}
\end{array}\right)\right\}+\\
\dfrac{\beta_{opc}e^{-\beta_2\left(t_\beta+t_F\right)}}{\beta_4\left(\beta_3+\beta_4\right)}\left\{\left(\begin{array}{l}
-2\beta_4\left(\beta_3+\beta_4\right)e^{-2\beta_4\left(t_1+t_F\right)}+\\
\beta_4\left(\beta_3+2\beta_4\right)e^{-\left(\beta_3+2\beta_4\right)\left(t_1+t_F\right)}+\\
\beta_4 e^{-\beta_4\left(t_1+t_F\right)}\left(\begin{array}{l}
\left(\beta_3+\beta_4\right)e^{-\beta_4\left(T+t_F\right)}-\\
\beta_4 e^{-\left(\beta_3+\beta_4\right)\left(T+t_F\right)}
\end{array}\right)
\end{array}\right)\right\}
\end{array}\right]\right]=0
$$

(22)

Also equation (20) is equivalent to

$$\left[R - \frac{(\beta_0 + \beta)}{\beta_3} \left\{ \begin{array}{l} e^{-\beta_4 (T + t_F)} - \\ e^{-(\beta_4 + \beta_3)(T + t_F)} \end{array} \right\} e^{\left\{ \begin{array}{l} -\beta_4 (t_1 + t_F) \\ +\beta_2 (t_\beta + t_F) \end{array} \right\}} \left(\beta_{sc} + \beta_{opc} \beta_3 \right) \right] = 0 \qquad (23)$$

Equations (22) and (23) are highly nonlinear equations. Therefore, numerical solution of these equations can be obtained by using the software **MATLAB 7.0.1**.

4. Numerical Illustration

To illustrate the model numerically the following parameter values are considered.

$\beta_1 = 0.0022\ unit, \beta = 702 units, \beta_0 = 502 units, \beta_2 = 0.22 unit, \beta_3 = 0.12\ unit, \beta_4 = 0.052\ unit,$

$t_\beta = 0.12\ year, \beta_{OC} = Rs.\ 1002\ per\ order, \beta_{hcow} = Rs.\ 4.02\ per\ unit\ per\ year,$

$\beta_{hcrw} = Rs.\ 20.0\ per\ unit, T = 12\ year, \beta_{sc} = Rs.\ 22.0\ per\ unit\ per\ year, \beta_{opc} = Rs.\ 42.0\ per\ unit$

Then for the minimization of total average cost and with help of software. the optimal policy can be obtained such as: t_1 = 0.399224 year, S = 28.597235 units and TC = Rs.658.115354 per year.

5. Conclusion

In summary, the dairy industry's two-warehouse inventory model for shortage and inflation spoiled items under the FIFO shipping policy provides a comprehensive framework for optimizing inventory management in the dairy industry. dairy industry. By incorporating theories of inventory management, spoilage, shortages and inflation, the model enables dairy companies to make informed decisions regarding inventory replenishment, shipping and cost optimization.

The model recognizes the perishability of dairy products and takes into account their rate of decomposition to minimize spoilage losses. It also addresses potential shortages and aims to balance stock levels with customer demand to avoid stock-outs and lost sales. Additionally, the model includes the FIFO shipping policy, which ensures that products with a shorter remaining shelf life are shipped first to reduce the risk of product wastage.

The model also helps companies adapt to changing economic conditions and make profitable inventory decisions by considering inflation and its impact on various cost elements. By optimizing inventory levels, order quantities, and shipping policies, the model aims to minimize overall inventory costs while maintaining product quality and customer satisfaction.

The Dual Warehouse Inventory Model for the Dairy Industry provides dairy companies with a valuable tool to improve their inventory management practices, increase operational efficiency and improve profitability. Through the use of mathematical techniques and optimization algorithms, the model supports data-driven decision making and allows companies to better control their inventory systems.

Overall, the model contributes to the advancement of inventory management in the dairy industry, providing a theoretical basis and practical guidance for effective inventory control, mitigating losses and meeting the demand of customers in a dynamic and competitive business environment.

References:

[1] Yadav, A.S., Bansal, K.K., Shivani, Agarwal, S. And Vanaja, R. (2020) FIFO in Green Supply Chain Inventory Model of Electrical Components Industry With Distribution Centres Using Particle Swarm Optimization. Advances in Mathematics: Scientific Journal. 9 (7), 5115–5120.

[2] Yadav, A.S., Kumar, A., Agarwal, P., Kumar, T. And Vanaja, R. (2020) LIFO in Green Supply Chain Inventory Model of Auto-Components Industry with Warehouses Using Differential Evolution. Advances in Mathematics: Scientific Journal, 9 no.7, 5121–5126.

[3] Yadav, A.S., Abid, M., Bansal, S., Tyagi, S.L. And Kumar, T. (2020) FIFO & LIFO in Green Supply Chain Inventory Model of Hazardous Substance Components Industry with Storage Using Simulated Annealing. Advances in Mathematics: Scientific Journal, 9 no.7, 5127–5132.

[4] Yadav, A.S., Tandon, A. and Selva, N.S. (2020) National Blood Bank Centre Supply Chain Management For Blockchain Application Using Genetic Algorithm. International Journal of Advanced Science and Technology Vol. 29, No. 8s, 1318-1324.

[5] Yadav, A.S., Selva, N.S. and Tandon, A. (2020) Medicine Manufacturing Industries supply chain management for Blockchain application using artificial neural networks, International Journal of Advanced Science and Technology Vol. 29, No. 8s, 1294-1301.

[6] Yadav, A.S., Ahlawat, N., Agarwal, S., Pandey, T. and Swami, A. (2020) Red Wine Industry of Supply Chain Management for Distribution Center Using Neural Networks, Test Engraining & Management, Volume 83 Issue: March – April, 11215 – 11222.

[7] Yadav, A.S., Pandey, T., Ahlawat, N., Agarwal, S. and Swami, A. (2020) Rose Wine industry of Supply Chain Management for Storage using Genetic Algorithm. Test Engraining & Management, Volume 83 Issue: March – April, 11223 – 11230.

[8] Yadav, A.S., Ahlawat, N., Sharma, N., Swami, A. And Navyata (2020) Healthcare Systems of Inventory Control For Blood Bank Storage With Reliability Applications Using Genetic Algorithm. Advances in Mathematics: Scientific Journal 9 no.7, 5133–5142.

[9] Yadav, A.S., Dubey, R., Pandey, G., Ahlawat, N. and Swami, A. (2020) Distillery Industry Inventory Control for Storage with Wastewater Treatment & Logistics Using Particle Swarm Optimization Test Engraining & Management Volume 83 Issue: May – June, 15362-15370.

[10] Yadav, A.S., Ahlawat, N., Dubey, R., Pandey, G. and Swami, A. (2020) Pulp and paper industry inventory control for Storage with wastewater treatment and Inorganic composition using genetic algorithm (ELD Problem). Test Engraining & Management, Volume 83 Issue: May – June, 15508-15517.

[11] Yadav, A.S., Pandey, G., Ahlawat, N., Dubey, R. and Swami, A. (2020) Wine Industry Inventory Control for Storage with Wastewater Treatment and Pollution Load Using Ant Colony Optimization Algorithm, Test Engraining & Management, Volume 83 Issue: May – June, 15528-15535.

[12] Yadav, A.S., Navyata, Sharma, N., Ahlawat, N. and Swami, A. (2020) Reliability Consideration costing method for LIFO Inventory model with chemical industry warehouse. International Journal of Advanced Trends in Computer Science and Engineering, Volume 9 No 1, 403-408.

[13] Yadav, A.S., Bansal, K.K., Kumar, J. and Kumar, S. (2019) Supply Chain Inventory Model For Deteriorating Item With Warehouse & Distribution Centres Under Inflation. International Journal of Engineering and Advanced Technology, Volume-8, Issue-2S2, 7-13.

[14] Yadav, A.S., Kumar, J., Malik, M. and Pandey, T. (2019) Supply Chain of Chemical Industry For Warehouse With Distribution Centres Using Artificial Bee Colony Algorithm. International Journal of Engineering and Advanced Technology, Volume-8, Issue-2S2, 14-19.

[15] Yadav, A.S., Navyata, Ahlawat, N. and Pandey, T. (2019) Soft computing techniques based Hazardous Substance Storage Inventory Model for decaying Items and Inflation using Genetic Algorithm. International Journal of Advance Research and Innovative Ideas in Education, Volume 5 Issue 9, 1102-1112.

[16] Yadav, A.S., Navyata, Ahlawat, N. and Pandey, T. (2019) Hazardous Substance Storage Inventory Model for decaying Items using Differential Evolution. International Journal of Advance Research and Innovative Ideas in Education, Volume 5 Issue 9, 1113-1122.

[17] Yadav, A.S., Navyata, Ahlawat, N. and Pandey, T. (2019) Probabilistic inventory model based Hazardous Substance Storage for decaying Items and Inflation using Particle Swarm Optimization. International Journal of Advance Research and Innovative Ideas in Education, Volume 5 Issue 9, 1123-1133.

[18] Yadav, A.S., Navyata, Ahlawat, N. and Pandey, T. (2019) Reliability Consideration based Hazardous Substance Storage Inventory Model for decaying Items using Simulated Annealing. International Journal of Advance Research and Innovative Ideas in Education, Volume 5 Issue 9, 1134-1143.

[19] Yadav, A.S., Swami, A. and Kher, G. (2019) Blood bank supply chain inventory model for blood collection sites and hospital using genetic algorithm. Selforganizology, Volume 6 No.(3-4), 13-23.

[20] Yadav, A.S., Swami, A. and Ahlawat, N. (2018) A Green supply chain management of Auto industry for inventory model with distribution centers using Particle Swarm Optimization. Selforganizology, Volume 5 No. (3-4)

[21] Yadav, A.S., Ahlawat, N., and Sharma, S. (2018) Hybrid Techniques of Genetic Algorithm for inventory of Auto industry model for deteriorating items with two warehouses. International Journal of Trend in Scientific Research and Development, Volume 2 Issue 5, 58-65.

[22] Yadav, A.S., Swami, A. and Gupta, C.B. (2018) A Supply Chain Management of Pharmaceutical For Deteriorating Items Using Genetic Algorithm. International Journal for Science and Advance Research In Technology, Volume 4 Issue 4, 2147-2153.

[23] Yadav, A.S., Maheshwari, P., Swami, A., and Pandey, G. (2018) A supply chain management of chemical industry for deteriorating items with warehouse using genetic algorithm. Selforganizology, Volume 5 No.1-2, 41-51.

[24] Yadav, A.S., Garg, A., Gupta, K. and Swami, A. (2017) Multi-objective Genetic algorithm optimization in Inventory model for deteriorating items with shortages using Supply Chain management. IPASJ International journal of computer science, Volume 5, Issue 6, 15-35.

[25] Yadav, A.S., Garg, A., Swami, A. and Kher, G. (2017) A Supply Chain management in Inventory Optimization for deteriorating items with Genetic algorithm. International Journal of Emerging Trends & Technology in Computer Science, Volume 6, Issue 3, 335-352.

[26] Yadav, A.S., Maheshwari, P., Garg, A., Swami, A. and Kher, G. (2017) Modeling & Analysis of Supply Chain management in Inventory Optimization for deteriorating items with Genetic algorithm and Particle Swarm optimization. International Journal of Application or Innovation in Engineering & Management, Volume 6, Issue 6, 86-107.

[27] Yadav, A.S., Garg, A., Gupta, K. and Swami, A. (2017) Multi-objective Particle Swarm optimization and Genetic algorithm in Inventory model for deteriorating items with shortages using Supply Chain management. International Journal of Application or Innovation in Engineering & Management, Volume 6, Issue 6, 130-144.

[28] Yadav, A.S., Swami, A. and Kher, G. (2017) Multi-Objective Genetic Algorithm Involving Green Supply Chain Management International Journal for Science and Advance Research In Technology, Volume 3 Issue 9, 132-138.

[29] Yadav, A.S., Swami, A., Kher, G. (2017) Multi-Objective Particle Swarm Optimization Algorithm Involving Green Supply Chain Inventory Management. International Journal for Science and Advance Research In Technology, Volume 3 Issue, 240-246.

[30] Yadav, A.S., Swami, A. and Pandey, G. (2017) Green Supply Chain Management for Warehouse with Particle Swarm Optimization Algorithm. International Journal for Science and Advance Research in Technology, Volume 3 Issue 10, 769-775.

[31] Yadav, A.S., Swami, A., Kher, G. and Garg, A. (2017) Analysis of seven stages supply chain management in electronic component inventory optimization for warehouse with economic load dispatch using genetic algorithm. Selforganizology, 4 No.2, 18-29 .

[32] Yadav, A.S., Maheshwari, P., Swami, A. and Garg, A. (2017) Analysis of Six Stages Supply Chain management in Inventory Optimization for warehouse with Artificial bee colony algorithm using Genetic Algorithm. Selforganizology, Volume 4 No.3, 41-51.

[33] Yadav, A.S., Swami, A., Gupta, C.B. and Garg, A. (2017) Analysis of Electronic component inventory Optimization in Six Stages Supply Chain management for warehouse with ABC using genetic algorithm and PSO. Selforganizology, Volume 4 No.4, 52-64.

[34] Yadav, A.S., Maheshwari, P. and Swami, A. (2016) Analysis of Genetic Algorithm and Particle Swarm Optimization for warehouse with Supply Chain management in Inventory control. International Journal of Computer Applications, Volume 145 –No.5, 10-17.

[35] Yadav, A.S., Swami, A. and Kumar, S. (2018) Inventory of Electronic components model for deteriorating items with warehousing using Genetic Algorithm. International Journal of Pure and Applied Mathematics, Volume 119 No. 16, 169-177.

[36] Yadav, A.S., Johri, M., Singh, J. and Uppal, S. (2018) Analysis of Green Supply Chain Inventory Management for Warehouse With Environmental Collaboration and Sustainability Performance Using Genetic Algorithm. International Journal of Pure and Applied Mathematics, Volume 118 No. 20, 155-161.

[37] Yadav, A.S., Ahlawat, N., Swami, A. and Kher, G. (2019) Auto Industry inventory model for deteriorating items with two warehouse and Transportation Cost using Simulated Annealing Algorithms. International Journal of Advance Research and Innovative Ideas in Education, Volume 5,Issue 1, 24-33.

[38] Yadav, A.S., Ahlawat, N., Swami, A. and Kher, G. (2019) A Particle Swarm Optimization based a two-storage model for deteriorating items with Transportation Cost and Advertising Cost: The Auto Industry. International Journal of Advance Research and Innovative Ideas in Education, Volume 5, Issue 1, 34-44.

[39] Yadav, A.S., Ahlawat, N., and Sharma, S. (2018) A Particle Swarm Optimization for inventory of Auto industry model for two warehouses with deteriorating items. International Journal of Trend in Scientific Research and Development, Volume 2 Issue 5, 66-74.

[40] Yadav, A.S., Swami, A. and Kher, G. (2018) Particle Swarm optimization of inventory model with two-warehouses. Asian Journal of Mathematics and Computer Research, Volume 23 No.1, 17-26.

[41] Yadav, A.S., Maheshwari, P.,, Swami, A. and Kher, G. (2017) Soft Computing Optimization of Two Warehouse Inventory Model With Genetic Algorithm. Asian Journal of Mathematics and Computer Research, Volume 19 No.4, 214-223.

[42] Yadav, A.S., Swami, A., Kumar, S. and Singh, R.K. (2016) Two-Warehouse Inventory Model for Deteriorating Items with Variable Holding Cost, Time-Dependent Demand and Shortages. IOSR Journal of Mathematics, Volume 12, Issue 2 Ver. IV, 47-53.

[43] Yadav, A.S., Sharam, S. and Swami, A. (2016) Two Warehouse Inventory Model with Ramp Type Demand and Partial Backordering for Weibull Distribution Deterioration. International Journal of Computer Applications, Volume 140 –No.4, 15-25.

[44] Yadav, A.S., Swami, A. and Singh, R.K. (2016) A two-storage model for deteriorating items with holding cost under inflation and Genetic Algorithms. International Journal of Advanced Engineering, Management and Science, Volume -2, Issue-4, 251-258.

[45] Yadav, A.S., Swami, A., Kher, G. and Kumar, S. (2017) Supply Chain Inventory Model for Two Warehouses with Soft Computing Optimization. International Journal of Applied Business and Economic Research, Volume 15 No 4, 41-55.

[46] Yadav, A.S., Rajesh Mishra, Kumar, S. and Yadav, S. (2016) Multi Objective Optimization for Electronic Component Inventory Model & Deteriorating Items with Two-warehouse using Genetic Algorithm. International Journal of Control Theory and applications, Volume 9 No.2, 881-892.

[47] Yadav, A.S., Gupta, K., Garg, A. and Swami, A. (2015) A Soft computing Optimization based Two Ware-House Inventory Model for Deteriorating Items with shortages using Genetic Algorithm. International Journal of Computer Applications, Volume 126 – No.13, 7-16.

[48] Yadav, A.S., Gupta, K., Garg, A. and Swami, A. (2015) A Two Warehouse Inventory Model for Deteriorating Items with Shortages under Genetic Algorithm and PSO. International Journal of Emerging Trends & Technology in Computer Science, Volume 4, Issue 5(2), 40-48.

[49] Yadav, A.S. Swami, A., and Kumar, S. (2018) A supply chain Inventory Model for decaying Items with Two Ware-House and Partial ordering under Inflation. International Journal of Pure and Applied Mathematics, Volume 120 No 6, 3053-3088.

[50] Yadav, A.S. Swami, A. and Kumar, S. (2018) An Inventory Model for Deteriorating Items with Two warehouses and variable holding Cost. International Journal of Pure and Applied Mathematics, Volume 120 No 6, 3069-3086.

[51] Yadav, A.S., Taygi, B., Sharma, S. and Swami, A. (2017) Effect of inflation on a two-warehouse inventory model for deteriorating items with time varying demand and shortages. International Journal Procurement Management, Volume 10, No. 6, 761-775.

[52] Yadav, A.S., R. P. Mahapatra, Sharma, S. and Swami, A. (2017) An Inflationary Inventory Model for Deteriorating items under Two Storage Systems. International Journal of Economic Research, Volume 14 No.9, 29-40.

[53] Yadav, A.S., Sharma, S. and Swami, A. (2017) A Fuzzy Based Two-Warehouse Inventory Model For Non instantaneous Deteriorating Items With Conditionally Permissible Delay In Payment. International Journal of Control Theory And Applications, Volume 10 No.11, 107-123.

[54] Yadav, A.S. and Swami, A. (2018) Integrated Supply Chain Model for Deteriorating Items With Linear Stock Dependent Demand Under Imprecise And Inflationary Environment. International Journal Procurement Management, Volume 11 No 6, 684-704.

[55] Yadav, A.S. and Swami, A. (2018) A partial backlogging production-inventory lot-size model with time-varying holding cost and weibull deterioration. International Journal Procurement Management, Volume 11, No. 5, 639-649.

[56] Yadav, A.S. and Swami, A. (2013) A Partial Backlogging Two-Warehouse Inventory Models For Decaying Items With Inflation. International Organization of Scientific Research Journal of Mathematics, Issue 6, 69-78.

[57] Yadav, A.S. and Swami, A. (2019) An inventory model for non-instantaneous deteriorating items with variable holding cost under two-storage. International Journal Procurement Management, Volume 12 No 6, 690-710.

[58] Yadav, A.S. and Swami, A. (2019) A Volume Flexible Two-Warehouse Model with Fluctuating Demand and Holding Cost under Inflation. International Journal Procurement Management, Volume 12 No 4, 441-456.

[59] Yadav, A.S. and Swami, A. (2014) Two-Warehouse Inventory Model for Deteriorating Items with Ramp-Type Demand Rate and Inflation. American Journal of Mathematics and Sciences Volume 3 No-1, 137-144.

[60] Yadav, A.S. and Swami, A. (2013) Effect of Permissible Delay on Two-Warehouse Inventory Model for Deteriorating items with Shortages. International Journal of Application or Innovation in Engineering & Management, Volume 2, Issue 3, 65-71.

[61] Yadav, A.S. and Swami, A. (2013) A Two-Warehouse Inventory Model for Decaying Items with Exponential Demand and Variable Holding Cost. International of Inventive Engineering and Sciences, Volume-1, Issue-5, 18-22.

[62] Yadav, A.S. and Kumar, S. (2017) Electronic Components Supply Chain Management for Warehouse with Environmental Collaboration & Neural Networks. International Journal of Pure and Applied Mathematics, Volume 117 No. 17, 169-177.

[63] Yadav, A.S. (2017) Analysis of Seven Stages Supply Chain Management in Electronic Component Inventory Optimization for Warehouse with Economic Load Dispatch Using GA and PSO. Asian Journal Of Mathematics And Computer Research, volume 16 No.4, 208-219.

[64] Yadav, A.S. (2017) Analysis Of Supply Chain Management In Inventory Optimization For Warehouse With Logistics Using Genetic Algorithm International Journal of Control Theory And Applications, Volume 10 No.10, 1-12 .

[65] Yadav, A.S. (2017) Modeling and Analysis of Supply Chain Inventory Model with two-warehouses and Economic Load Dispatch Problem Using Genetic Algorithm. International Journal of Engineering and Technology, Volume 9 No 1, 33-44.

[66] Swami, A., Singh, S.R., Pareek, S. and Yadav, A.S. (2015) Inventory policies for deteriorating item with stock dependent demand and variable holding costs under permissible delay in payment. International Journal of Application or Innovation in Engineering & Management, Volume 4, Issue 2, 89-99.

[67] Swami, A., Pareek, S., Singh S.R. and Yadav, A.S. (2015) Inventory Model for Decaying Items with Multivariate Demand and Variable Holding cost under the facility of Trade-Credit. International Journal of Computer Application, 18-28.

[68] Swami, A., Pareek, S., Singh, S.R. and Yadav, A.S. (2015) An Inventory Model With Price Sensitive Demand, Variable Holding Cost And Trade-Credit Under Inflation. International Journal of Current Research, Volume 7, Issue, 06, 17312-17321.

[69] Gupta, K., Yadav, A.S., Garg, A. and Swami, A. (2015) A Binary Multi-Objective Genetic Algorithm &PSO involving Supply Chain Inventory Optimization with Shortages, inflation. International Journal of Application or Innovation in Engineering & Management, Volume 4, Issue 8, 37-44.

[70] Gupta, K., Yadav, A.S., Garg, A., (2015) Fuzzy-Genetic Algorithm based inventory model for shortages and inflation under hybrid & PSO. IOSR Journal of Computer Engineering, Volume 17, Issue 5, Ver. I , 61-67.

[71] Singh, R.K., Yadav, A.S. and Swami, A. (2016) A Two-Warehouse Model for Deteriorating Items with Holding Cost under Particle Swarm Optimization. International Journal of Advanced Engineering, Management and Science, Volume -2, Issue-6, 858-864.

[72] Singh, R.K., Yadav, A.S. and Swami, A. (2016) A Two-Warehouse Model for Deteriorating Items with Holding Cost under Inflation and Soft Computing Techniques. International Journal of Advanced Engineering, Management and Science, Volume -2, Issue-6, 869-876.

[73] Kumar, S., Yadav, A.S., Ahlawat, N. and Swami, A. (2019) Supply Chain Management of Alcoholic Beverage Industry Warehouse with Permissible Delay in Payments using Particle Swarm Optimization. International Journal for Research in Applied Science and Engineering Technology, Volume 7 Issue VIII, 504-509.

[74] Kumar, S., Yadav, A.S., Ahlawat, N. and Swami, A. (2019) Green Supply Chain Inventory System of Cement Industry for Warehouse with Inflation using Particle Swarm Optimization. International Journal for Research in Applied Science and Engineering Technology, Volume 7 Issue VIII, 498-503.

[75] Kumar, S., Yadav, A.S., Ahlawat, N. and Swami, A. (2019) Electronic Components Inventory Model for Deterioration Items with Distribution Centre using Genetic Algorithm. International Journal for Research in Applied Science and Engineering Technology, Volume 7 Issue VIII, 433-443.

[76] Chauhan, N. and Yadav, A.S. (2020) An Inventory Model for Deteriorating Items with Two-Warehouse & Stock Dependent Demand using Genetic algorithm. International Journal of Advanced Science and Technology, Vol. 29, No. 5s, 1152-1162 .

[77] Chauhan, N. and Yadav, A.S. (2020) Inventory System of Automobile for Stock Dependent Demand & Inflation with Two-Distribution Center Using Genetic Algorithm. Test Engraining & Management, Volume 83, Issue: March – April, 6583 – 6591.

[78] Pandey, T., Yadav, A.S. and Medhavi Malik (2019) An Analysis Marble Industry Inventory Optimization Based on Genetic Algorithms and Particle swarm optimization. International Journal of Recent Technology and Engineering Volume-7, Issue-6S4, 369-373.

[79] Ahlawat, N., Agarwal, S., Pandey, T., Yadav, A.S., Swami, A. (2020) White Wine Industry of Supply Chain Management for Warehouse using Neural Networks Test Engraining & Management, Volume 83, Issue: March – April, 11259 – 11266.

[80] Singh, S. Yadav, A.S. and Swami, A. (2016) An Optimal Ordering Policy For Non-Instantaneous Deteriorating Items With Conditionally Permissible Delay In Payment Under Two Storage Management International Journal of Computer Applications, Volume 147 –No.1, 16-25.

Appendix A

$$C_{HO}=\left[\beta_{hcow}\left\{\begin{array}{l}\beta_w\left\{\left(\left(t_\beta+t_F\right)-\dfrac{\beta_4\left(t_\beta+t_F\right)^2}{2}-\dfrac{\beta_1\left(t_\beta+t_F\right)^3}{6}\right)\right\}+\\ \left(\beta_0+\beta\right)e^{-\beta_2\left(\left(t_\beta+t_F\right)+\beta_4\right)}\left\{\begin{array}{l}\dfrac{\left(t_1+t_F\right)^2}{2}-\dfrac{\beta_4\left(t_1+t_F\right)^3}{6}+\\ \dfrac{\beta_1\left(t_1+t_F\right)^4}{12}-\dfrac{\beta_4\beta_1}{20}\left(t_1+t_F\right)^5-\\ \dfrac{\left(t_\beta+t_F\right)}{2}\left(2\left(t_1+t_F\right)-\left(t_\beta+t_F\right)\right)-\\ \dfrac{\beta_1\left(t_\beta+t_F\right)}{24}\left(4\left(t_1+t_F\right)^3-\left(t_\beta+t_F\right)^3\right)+\\ \dfrac{\beta_4\left(t_\beta+t_F\right)^2}{6}\left(3\left(t_1+t_F\right)-2\left(t_\beta+t_F\right)\right)+\\ \dfrac{\beta_4\beta_1\left(t_\beta+t_F\right)^2}{30}\left(5\left(t_1+t_F\right)^3-3\left(t_\beta+t_F\right)^3\right)+\\ \dfrac{\beta_1\left(t_\beta+t_F\right)^3}{24}\left(4\left(t_1+t_F\right)-3\left(t_\beta+t_F\right)\right)\end{array}\right\}\end{array}\right\}\right]$$

(12)

Appendix B

$$
\begin{aligned}
= \beta_{dc}\beta_1 \Bigg\{ & \left\{ \left(\beta_0+\beta\right) \begin{pmatrix} \frac{1}{6}\left(t_\beta+t_F\right)^3 - \\ \left(\frac{\beta_2}{4}+\frac{\beta_4}{12}\right)\left(t_\beta+t_F\right)^4 + \\ \left(\frac{\beta_1}{40}+\frac{\beta_4\beta_2}{15}\right)\left(t_\beta+t_F\right)^5 - \\ \left(\frac{\beta_4\beta_1}{36}-\frac{\beta_2\beta_1}{24}\right)\left(t_\beta+t_F\right)^6 \end{pmatrix} \right\} + \\
& \left\{ \beta_W \left(\frac{\left(t_\beta+t_F\right)^2}{2} - \frac{\beta_4\left(t_\beta+t_F\right)^3}{3} \frac{\beta_1\left(t_\beta+t_F\right)^4}{8} \right) \right\} + \\
& \left\{ \left(\beta_0+\beta\right) e^{-\left(t_\beta+t_F\right)\left(\beta_2+\beta_4\right)} \begin{bmatrix} \frac{\left(t_1+t_F\right)^3}{6} - \frac{\beta_4\left(t_1+t_F\right)^4}{12} + \\ \frac{\beta_1\left(t_1+t_F\right)^5}{40} - \frac{\beta_4\beta_1\left(t_1+t_F\right)^6}{36} - \\ \frac{\left(t_\beta+t_F\right)^2}{6}\left(3\left(t_1+t_F\right)-2\left(t_\beta+t_F\right)\right) - \\ \frac{\beta_1\left(t_\beta+t_F\right)^2}{60}\left(5\left(t_1+t_F\right)^3-2\left(t_\beta+t_F\right)^3\right) - \\ \frac{\beta_4\left(t_\beta+t_F\right)^3}{12}\left(4\left(t_1+t_F\right)-3\left(t_\beta+t_F\right)\right) - \\ \frac{\beta_4\beta_1\left(t_\beta+t_F\right)^3}{36}\left(2\left(t_1+t_F\right)^3-\left(t_\beta+t_F\right)^3\right) - \\ \frac{\beta_1\left(t_\beta+t_F\right)^4}{40}\left(5\left(t_1+t_F\right)-4\left(t_\beta+t_F\right)\right) \end{bmatrix} \right\} \Bigg\}
\end{aligned}
\qquad (14)
$$

Appendix C

$$
\begin{aligned}
TC\binom{t_1+t_F,}{T+t_F} = \frac{1}{(T+t_F)} \Bigg[& \beta_{OC} + \beta_w \Big\{ \Big((t_\beta+t_F) - \frac{\beta_4 (t_\beta+t_F)^2}{2} - \frac{\beta_1 (t_\beta+t_F)^3}{6} \Big) \Big\} + \\
& \beta_{hcow} \Bigg\{ \beta_0 e^{-\beta_2((t_\beta+t_F)+\beta_4)} \Bigg\{ \frac{(t_1+t_F)^2}{2} - \frac{\beta_4 (t_1+t_F)^3}{6} + \\
& \frac{\beta_1 (t_1+t_F)^4}{12} - \frac{\beta_4\beta_1}{20}(t_1+t_F)^5 - \\
& \frac{(t_\beta+t_F)}{2}\big(2(t_1+t_F)-(t_\beta+t_F)\big) - \\
& \frac{\beta_1 (t_\beta+t_F)}{24}\Big(4(t_1+t_F)^3-(t_\beta+t_F)^3\Big) + \\
& \frac{\beta_4 (t_\beta+t_F)^2}{6}\big(3(t_1+t_F)-2(t_\beta+t_F)\big) + \\
& \frac{\beta_4\beta_1 (t_\beta+t_F)^2}{30}\Big(5(t_1+t_F)^3-3(t_\beta+t_F)^3\Big) + \\
& \frac{\beta_1 (t_\beta+t_F)^3}{24}\big(4(t_1+t_F)-3(t_\beta+t_F)\big) \Bigg\} \Bigg\} \\
& + \beta_{hcrw}(\beta_0+\beta) \Bigg\{ \frac{(t_\beta+t_F)^2}{2} - \frac{(3\beta_2+\beta_4)}{6}(t_\beta+t_F)^3 + \\
& \Big(\frac{\beta_1}{12}+\frac{\beta_2\beta_4}{8}\Big)(t_\beta+t_F)^4 - \\
& \Big(\frac{\beta_4\beta_1}{20}-\frac{\beta_2\beta_1}{30}\Big)(t_\beta+t_F)^5 \Bigg\} + \\
& \beta_{dc}\beta_1 \Bigg\{ \Bigg\{ (\beta_0+\beta) \Bigg(\frac{1}{6}(t_\beta+t_F)^3 - \\
& \Big(\frac{\beta_2}{4}+\frac{\beta_4}{12}\Big)(t_\beta+t_F)^4 + \\
& \Big(\frac{\beta_1}{40}+\frac{\beta_4\beta_2}{15}\Big)(t_\beta+t_F)^5 - \\
& \Big(\frac{\beta_4\beta_1}{36}-\frac{\beta_2\beta_1}{24}\Big)(t_\beta+t_F)^6 \Bigg) \Bigg\} + \\
& \Bigg\{ \beta_W \Bigg(\frac{(t_\beta+t_F)^2}{2} - \frac{\beta_4 (t_\beta+t_F)^3}{3} \frac{\beta_1 (t_\beta+t_F)^4}{8} \Bigg) \Bigg\} + \\
& \Bigg\{ (\beta_0+\beta) e^{-(t_\beta+t_F)(\beta_2+\beta_4)} \Bigg[\frac{(t_1+t_F)^3}{6} - \frac{\beta_4 (t_1+t_F)^4}{12} + \\
& \frac{\beta_1 (t_1+t_F)^5}{40} - \frac{\beta_4\beta_1 (t_1+t_F)^6}{36} - \\
& \frac{(t_\beta+t_F)^2}{6}\big(3(t_1+t_F)-2(t_\beta+t_F)\big) - \\
& \frac{\beta_1 (t_\beta+t_F)^2}{60}\Big(5(t_1+t_F)^3-2(t_\beta+t_F)^3\Big) - \\
& \frac{\beta_4 (t_\beta+t_F)^3}{12}\big(4(t_1+t_F)-3(t_\beta+t_F)\big) - \\
& \frac{\beta_4\beta_1 (t_\beta+t_F)^3}{36}\Big(2(t_1+t_F)^3-(t_\beta+t_F)^3\Big) - \\
& \frac{\beta_1 (t_\beta+t_F)^4}{40}\big(5(t_1+t_F)-4(t_\beta+t_F)\big) \Bigg] \Bigg\} \Bigg\} \\
& + \Bigg[\frac{(\beta_0+\beta)\beta_{sc} e^{-(\beta_4(t_1+t_F)+\beta_2(t_\beta+t_F))}}{\beta_3\beta_4(\beta_3+\beta_4)} \Bigg\{ \beta_3 e^{-(\beta_3+\beta_4)(t_1+t_F)} + \\
& e^{-\beta_4(T+t_F)} \Big\{ r e^{-\beta_3(T+t_F)} - (\beta_3+\beta_4) e^{-\beta_3(t_1+t_F)} \Big\} \Bigg\} \Bigg] \\
& + \Bigg[\frac{\beta_{opc}(\beta_0+\beta) e^{-\{\beta_2(t_\beta+t_F)+\beta_4(t_1+t_F)\}}}{\beta_4(\beta_3+\beta_4)} \Bigg\{ e^{-\beta_4(t_1+t_F)} \Big\{ (\beta_3+\beta_4) - \beta_4 e^{-\beta_3(t_1+t_F)} \Big\} - \\
& e^{-\beta_4(T+t_F)} \Big\{ (\beta_3+\beta_4) - \beta_4 e^{-\beta_3(T+t_F)} \Big\} \Bigg\} \Bigg] \Bigg] = \frac{R(t_1,T)}{T}
\end{aligned}
\quad (18
$$