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SINGLE SERVER BULK SERVICE INTERDEPENDENT QUEUEING MODEL OF CONTROLLED ARRIVAL RATES WITH VACATION

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Abstract

Instead of individualized one to one assistance, here we describe systems that provide services for group of customers. This study introduces controllable arrival rates with vacation and interdependency of the system's service and arrival processes. A faster and slower arrival rates are meant to be controllable arrivals, with Poisson (each time Poisson occurrence has one arrival) being the default assumption. Service begins only when the count of customers in the queue approaches or surpasses a and the capacity b ($\geq a \geq 1$). A vacation period defines when a server goes for performing other uninterruptible work when the system is idle. Then, all the steady-state equations are derived to find the system's probabilities. We used M/M(a,b)/1 as the notation. For this model, steady-state solutions & characteristics are derived and explored. All the probabilities are expressed in terms of $P_{0,0}(0)$. The expected number of customers and waiting time depends on the interdependency, service rate, faster arrival rate, and slower arrival rate. According to each parameter, all the results are verified. There are works related to bulk service and vacation, but this is a new approach to give a bridge between bulk service and controllable arrival rates with vacation along with interdependency in the arrival and service process.

Keywords: M/M(a,b)/1 Queueing system, Bulk service, controllable arrival rates, steady states, interdependent model, stochastic processes, Vacation.

1. Introduction

Queueing models are used in many real-world contexts as a basis for the effective design and study of various technological systems and for the assessment of system behaviour, including client waiting times and estimated numbers of consumers. Bulk service was initially proposed by Bailey in 1954 [1]. The literature on bulk services has grown over time. These ideas can be applied in a number of situations, such as the transportation sector, where batch servers such as mass transit vehicles, lifts, and carriers are commonplace. Queuing problems can arise in many real-world systems, including those for production, voice or data transport, communications services, etc. Over computer communication networks, messages can be transmitted using any number of packets.

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Most queuing models start serving consumers as soon as they arrive. But with bulk service, the client count does not begin to count until number an is reached. However, service won't start until after a certain number of customers. Queue theorists are now investigating the performance characteristics of queuing systems when the service pattern is not constrained by a specific distribution, as a result of the single-server bulk service models. The body of research on bulk service queuing theory includes studies of the characteristics and actions of the system's servers as well as the conduct of customers in the line [1–7].

Numerous academics have significantly contributed to the bulk service queueing models due to their widespread applications. In queueing theory, several researchers deal with different types of services with other parameters and various arrivals in different models. Here it is a combined model as bulk service with control in arrivals. In previous studies, J. Medhi (2002) discussed bulk service systems [8]. Neuts M. F contributed to the idea of bulk queues in the literature in 1967[9]. A. Srinivasan and M. Thiagarajan (2006) researched the controllable arrival rates in various queueing models [10] in that study, discussing the concept of the speed of arrival rates in some queueing models. Various studies have been conducted on queueing systems to elucidate the concept of bulk service. These investigations provide a comprehensive understanding of queueing systems, such as k-stage bulk service, heterogeneous bulk service, group service for impatient customers, performance analysis of dependent bulk service queues with server breakdowns, multiple vacation transient behaviours of bulk service queueing systems with standby servers, and others [11-17]. Anyue Chen, Xiaohan Wu & Jing Zhang (2020) proposed "Markovian bulk-arrival and bulk-service queues with general state-dependent control" [18]. Additionally, K. H. Rahim and M. Thiagarajan proposed an innovative approach to the interdependent queueing model with controllable arrival rates in a single-server system [19], and it is an extension of that study.Finally, the present study deals with controllable arrivals with vacation by dividing the faster and slower rates in the bulk service queuing system, and it connects or makes a bridge between the bulk service to controlled arrivals. This system is using the idle time for internal uninterrupted work in the sense of vacation. So, all the probabilities can split according to the speed of arrivals with this concept. This interdependent model can apply to model the real-world situation to new queueing models. This model has controls for both service and arrival.

This study made an effort to examine the M/M(a,b)/1 interdependent queueing model with vacation and controllable arrival rates. We defined the model and steady-state equations and derived model properties. We produced numerical data for system performance metrics to conform to the analytical conclusions and facilitate sensitivity analysis. Following is a summary of the queueing model research. This study describes the model and then the steady-states, formulation, and notation. After that, it covers the properties of the models and then provides illustrative findings for system performance indicators to conform to the analytical conclusions and simplify the sensitivity analysis.

2. Model Description

This queuing system constitutes a single server and a limitless waiting space. The arrival and service completion process $\{X_1(t)\}$and $\{X_2(t)\}$ of the system follow a bivariate Poisson process and are correlated, given that,

$P(X_1(t) = x_1, X_2(t) = x_2)$

$$= e^{-(\lambda_j+\mu-2\epsilon)} \sum_{s=0}^{min\{x_1,x_2\}} (\varepsilon t)^s \left[\left(\lambda_j - \varepsilon\right)t\right]^{x_1-s} [(\mu-\varepsilon)t]^{x_2-s} \frac{1}{s!(x_1-s)!(x_2-s)!} \quad (2.1)$$

where x_i= 0, 1, 2,…(values of i is 1 and 2); $\lambda_j > 0$; j = 0,1; $\mu > 0$; $0 \leq \varepsilon < \min(\lambda_j, \mu)$; j = 0, 1.

The parameters $\mu, \varepsilon,\ \lambda_0, and\ \lambda_1$describe

μ = the mean service rate.

ε = the mean dependence rate (the covariance of $\{X_1(t)\}$ & $\{X_2(t)\}$).

λ_0 = the mean faster arrival rate.

λ_1 = the mean slower arrival rate.

FCFS is the queue discipline. Based on size [a, b], services are provided in batches. When the queue length reaches or surpasses a, and the capacity is b (≥a≥1), does service begin. A batch's service time distribution is considered exponential with parameter μ. The states of the system are denoted by (j, n), with n is the number of units in the queue, and j = 1 indicates the server in this system is busy serving a batch of size m (a≥ m ≥b), and j = 0 shows idle server. v denotes the probability of the vacation when the system is in idle state.

We consider the system's states.

Denote, $P_{j,n}(t) = Pr$[the state of the system (j, n) at t (time)].

$P_{j,n}(t)$ is non zero Only for j = 1, n ≥0, and j = 0; 0 ≤ n ≤a -1.

Now, the steady-state probabilities (SSP) are as follows.

Let $P_{0,n}(0)$, describe the SSP that there are queued n customers when idle server and the system is at a faster arrival rate.

Let $P_{0,n}(1)$, describe the SSP that there are queued n customers when idle server and the system is at a slower arrival rate.

Let $P_{1,n}(0)$, describe the SSP that there are queued n customers when the busy server and system are at a faster arrival rate.

Let $P_{1,n}(1)$, describe the SSP that there are queued n customers when the busy server and system are at a slower arrival rate.

Clearly, the process N(t); t ⩾ 0, where N(t) is the system size at time t, is a Markov chain with state space

{0,1, 2,…,f -1 f, f + 1,f+ 2,…,F -2,F -1,F,F+ 1,…} and F < a < b

3. Steady-State Equations

The steady state means the states of any queuing system at the probability of the count of the clients in any queueing system being independent of time t. In "Fundamentals of queueing theory," Donald Gross and Carl M. Harris explained steady-state equations and illustrated them in some models [11]. Here we can see that $P_{j,n}(0)$ exists only when n = 0,1, 2, f-1, f; $P_{j,n}(1)$ exists only when n = F, F + 1,…∞: both $P_{j,n}(0)$ & $P_{j,n}(1)$ exists elsewhere where j = 0, 1 .

The steady-state equations become

$$-(\lambda_0+\mu-2\varepsilon)P_{1,n}(0)+(\lambda_0-\varepsilon)P_{1,n-1}(0)=0$$
$$(n=1,2,3\ldots f-1) \quad (3.1)$$
$$-(\lambda o+\mu-2\varepsilon)P_{1,f}(0)+(\lambda_0-\varepsilon)P_{1,f-1}(0)+(\mu-\varepsilon)P_{1,f+b}(1)=0 \quad (3.2)$$
$$-(\lambda_0+\mu-2\varepsilon)P_{1,n}(0)+(\lambda_0-\varepsilon)P_{1,n-1}(0)=0$$
$$(n=f+1,f+2,\ldots,F-2) \quad (3.3)$$
$$-(\lambda_0+\mu-2\varepsilon)P_{1,F-1}(0)+(\lambda_0-\varepsilon)P_{1,F-2}(0)=0 \quad (3.4)$$
$$-(\lambda_1+\mu-2\varepsilon)P_{1,f+1}(1)+(\mu-\varepsilon)P_{1,f+1+b}(1)=0 \quad (3.5)$$
$$-(\lambda_1+\mu-2\varepsilon)P_{1,n}(1)+(\lambda_1-\varepsilon)P_{1,n-1}(1)+(\mu-\varepsilon)P_{1,n+b}(1)=0$$
$$(n=f+2,f+3,\ldots F-1) \quad (3.6)$$
$$-(\lambda_1+\mu-2\varepsilon)P_{1,F}(1)+(\lambda_1-\varepsilon)P_{1,F-1}(1)+(\lambda_0-\varepsilon)P_{1,F-1}(0)+(\mu-\varepsilon)P_{1,F+b}(1)=0 \quad (3.7)$$
$$-(\lambda_1-\mu+2\varepsilon)P_{1,n}(1)+(\lambda_1-\varepsilon)P_{1,n-1}(1)+(\mu-\varepsilon)P_{1,n+b}(1)=0$$
(n= F+1, F+2…) (3.8)
$$-(\lambda_0+\mu-2\varepsilon)P_{1,0}(0)+(\lambda_0-\varepsilon)P_{0,a-1}(0)=0 \quad (3.9)$$
$$-(\lambda_1+\mu-2\varepsilon)P_{1,0}(1)+(\lambda_1-\varepsilon)P_{0,a-1}(1)+(\mu-\varepsilon)\sum_{j=a}^{b}P_{1,j}(1)=0 \quad (3.10)$$
$$-(\lambda_0-\varepsilon)P_{0,0}(0)+(\mu-\varepsilon)P_{1,0}(0)=0 \quad (3.11)$$
$$-(\lambda_0-\varepsilon)P_{1,f+1}(0)+(\mu-\varepsilon)P_{1,f+1}(1)=0 \quad (3.12)$$
$$-(\lambda_0-v-2\varepsilon)P_{0,n}(0)+(\lambda_0-v-2\varepsilon)P_{0,n-1}(0)+(\mu-\varepsilon)P_{1,n}(0)=0$$
(n= 1,2,3,…,f-1) (3.13)
$$-(\lambda_0-v-2\varepsilon)P_{0,f}(0)+(\lambda_0-v-2\varepsilon)P_{0,f-1}(0)+(\mu-\varepsilon)P_{1,f}(0)+(\mu-\varepsilon)P_{1,f}(1)=0 \quad (3.14)$$
$$-(\lambda_0-v-2\varepsilon)P_{0,n}(0)+(\lambda_0-v-2\varepsilon)P_{0,n-1}(0)+(\mu-\varepsilon)P_{1,n}(0)=0$$
(n= f+1,f+2,…,F-2) (3.15)
$$-(\lambda_0-v-2\varepsilon)P_{0,F-1}(0)+(\lambda_0-v-2\varepsilon)P_{0,F-2}(0)=0 \quad (3.16)$$
$$-(\lambda_1-v-2\varepsilon)P_{0,f+1}(1)+(\mu-\varepsilon)P_{1,f+1}(0)=0 \quad (3.17)$$
$$-(\lambda_1-v-2\varepsilon)P_{0,n}(1)+(\lambda_1-v-2\varepsilon)P_{0,n-1}(1)+(\mu-\varepsilon)P_{1,n}(1)=0$$
$$(n=f+2,f+3,\ldots,F-1) \quad (3.18)$$
$$-(\lambda_1-v-2\varepsilon)P_{0,F}(1)+(\lambda_1-\varepsilon)P_{0,F-1}(1)+(\lambda_0-v-2\varepsilon)P_{0,F-1}(0)$$
$$+(\mu-\varepsilon)P_{1,F}(1)=0 \quad 3.19)$$
$$-(\lambda_1-v-2\varepsilon)P_{0,n}(1)+(\lambda_1-v-2\varepsilon)P_{0,n-1}(1)+(\mu-\epsilon)P_{1,n}(1)=0$$
(n=F+1, F+2,…,a-1) (3.20)

3.1 Computation Of Steady-State Solutions

Let $\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}=\rho_0$, $\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}=\rho_1$ and $v_\varepsilon=\frac{v-\varepsilon}{\mu-\varepsilon}$

Now, from equation (3.11)

$$P_{1,0}(0) = \rho_0\, v_\varepsilon P_{0,0}(0) \quad (3.2.1)$$

Recursively using equation (3.1.1) in equations (3.1),(3.2),(3.3), and (3.4), we get

$$P_{1,n}(0) = \frac{\rho_0\, {v_\varepsilon}^{n+1}}{\left(\rho_{0\, v_\varepsilon} + 1\right)^n} P_{0,0}(0)$$

(n= 1,2,3,…,F-1) (3.2.2)

From equation (3.6),(3.7) and (3.8) , also by using displacement operator.

$$-(\rho_1 + 1)EP_{1,n-1}(1) + \rho_1 P_{1,n-1}(1) + E^{b+1}P_{1,n-1}(1) = 0;$$

(n=f+1,f+2,f+3…)

Or $A\left(EP_{1,n}(1)\right) = 0$

With characteristic equation,

$$A(z) \equiv z^{b+1} - (\rho_1 + 1)z + \rho_1 = 0$$

Now, let $A_1(z) = -(\rho_1 + 1)z$ and $A_2(z) = z^{b+1} + \rho_1$

From the circle $|z| = 1 - \xi$ such that ξ is arbitrarily small. Then by Rouche's theorem, the roots are denoted by $\omega_1, \omega_2, \omega_3 \ldots \omega_b$ $|\omega_i| \geq 1$,

Thus $P_{1,n}(1) = \alpha\omega^n + \sum_{i=1}^{b} \alpha_i \omega_i^n$ for n= 0,1,2… where $\alpha_i's$ are constants.

Again $\sum_{n=0}^{\infty} P_{1,n}(1) < 1$ We must have $\alpha_i = 0 \forall i$ which implies $P_{1,n}(1) = \alpha\omega^n$

From equation (3.12)

$$P_{1,r+1}(1) = \frac{{\rho_0}^{f+3}}{(\rho_0+1)^{f+1}} P_{0,0}(0) \quad (3.2.3)$$

Now,

$$P_{1,n}(1) = \left(\frac{{\rho_0}^2}{\omega(\rho_0 + 1)}\right)^{f+1} \omega^n P_{0,0}(0)$$

(n= f+1,f+2,…) (3.2.4)

From equation (3.2.2) and (3.2.4)

$$P_{1,n} = P_{1,n}(0) + P_{1,n}(1)$$

$$\sum_{n=0}^{\infty} P_{1,n} = \sum_{n=1}^{F-1} P_{1,n}(0) + \sum_{n=f+1}^{\infty} P_{1,n}(1)$$

From equation (3.9), we have,

$$P_{0,a-1}(0) = (\rho_{01} + 1)P_{0,0}(0) \quad (3.2.5)$$

From equation (3.13) n= a-1,a-2,…,1 recursively using (3.2.5) we get,

$$P_{0,n}(0) = (\, v_\varepsilon\rho_0 + 1)\left\{1 - \left(\left(\frac{v_\varepsilon\rho_0}{(\, v_\varepsilon\rho_0 + 1)}\right)^F + \left(\frac{v_\varepsilon\rho_0}{(\, v_\varepsilon\rho_0 + 1)}\right)^{n+1}\right)\right\} P_{0,0}(0)$$

(n=1,2,3,…,F-1) (3.2.6)

Using equation (3.10) in (3.17), (3.18), and (3.19), we obtain,

$$P_{0,n}(1)=\frac{1}{v_\varepsilon\rho_1}\left(\frac{(v_\varepsilon\rho_0)^2}{\omega(\,v_\varepsilon\rho_0+1)}\right)^{f+1}\left((\,v_\varepsilon\rho_1+1)-\frac{\omega^{n+1}(1-\omega^{b-n})}{1-\omega}\right)P_{0,0}(0)$$
$$(n=f+1,f+2,\dots,a-1) \qquad (3.2.7)$$

Now, $P_{0,n}=P_{0,n}(0)+P_{0,n}(1)$

$$\sum_{n=1}^{a-1}P_{0,n}=\sum_{n=1}^{F-1}P_{0,n}(0)+\sum_{n=f+1}^{\infty}P_{0,n}(1)$$

We observed that every SSP of the system is defined by $P_{0,0}(0)$ values.

3.2 The Model's Characteristics

Here expected and analytical results are derived for the system.

Now, $P_{1,n}+P_{0,n}=1$

$$\left[\sum_{n=1}^{F-1}\frac{\rho_0\,{v_\varepsilon}^{n+1}}{(\rho_0\,v_\varepsilon+1)^n}+\sum_{n=f}^{\infty}\left(\frac{{\rho_0}^2}{\omega(\rho_0+1)}\right)^{f+1}\omega^n\right.$$
$$+\sum_{n=1}^{F-1}(\,v_\varepsilon\rho_0+1)\left\{1-\left(\left(\frac{v_\varepsilon\rho_0}{(\,v_\varepsilon\rho_0+1)}\right)^F+\left(\frac{v_\varepsilon\rho_0}{(\,v_\varepsilon\rho_0+1)}\right)^{n+1}\right)\right\}$$
$$\left.+\sum_{n=f+1}^{a-1}\frac{1}{v_\varepsilon\rho_1}\left(\frac{v_\varepsilon{\rho_0}^2}{\omega(\,v_\varepsilon\rho_0+1)}\right)^{f+1}\left((\,v_\varepsilon\rho_1+1)-\frac{\omega^{n+1}(1-\omega^{b-n})}{1-\omega}\right)\right]P_{0,0}(0)$$
$$=1$$
$$\Rightarrow MP_{0,0}(0)$$
$$=1 \qquad (3.3.1)$$

Where,

M =

$$\left[\sum_{n=1}^{F-1}\frac{\rho_{0\,v_\varepsilon}{}^{n+1}}{(\rho_0\,v_\varepsilon+1)^n}+\sum_{n=f}^{\infty}\left(\frac{\rho_0{}^2}{\omega(\rho_0+1)}\right)^{f+1}\omega^n\right.$$
$$+\sum_{n=1}^{F-1}(\,v_\varepsilon\rho_0+1)\left\{1-\left(\left(\frac{v_\varepsilon\rho_0}{(\,v_\varepsilon\rho_0+1)}\right)^F+\left(\frac{\rho_0}{(\,v_\varepsilon\rho_0+1)}\right)^{n+1}\right)\right\}$$
$$\left.+\sum_{n=f+1}^{a-1}\frac{1}{v_\varepsilon\rho_1}\left(\frac{(v_\varepsilon\rho_0)^2}{\omega(\,v_\varepsilon\rho_0+1)}\right)^{f+1}\left((\,v_\varepsilon\rho_1+1)-\frac{\omega^{n+1}(1-\omega^{b-n})}{1-\omega}\right)\right]$$

Hence,

$$P_{0,0}(0)=M^{-1}$$

$P_\lambda(0)$ represents the probability that this system will have a faster arrival rate. And it is given by

$$P_\lambda(0)=\sum_{n=0}^{F-1}\left(P_{0,n}(0)+P_{1,n}(0)\right)=$$
$$\left\{\sum_{n=1}^{F-1}(1+\,v_\varepsilon\rho_0)\left\{1-\left(\left(\frac{v_\varepsilon\rho_0}{(1+v_\varepsilon\rho_0)}\right)^F+\left(\frac{v_\varepsilon\rho_0}{(1+v_\varepsilon\rho_0)}\right)^{n+1}\right)\right\}+\sum_{n=1}^{F-1}\frac{\rho_{0\,v_\varepsilon}{}^{n+1}}{(\rho_0\,v_\varepsilon+1)^n}\right\}P_{0,0}(0) \quad (3.3.2)$$

$P_\lambda(1)$ represents the probability that this system will have a slower arrival rate. And it is given by

$$P_\lambda(1)=\sum_{n=f+1}^{\infty}\left(P_{0,n}(1)+P_{1,n}(1)\right)$$
$$=\left[\sum_{n=f+1}^{a-1}\frac{1}{v_\varepsilon\rho_1}\left(\frac{(v_\varepsilon\rho_0)^2}{\omega(\,v_\varepsilon\rho_0+1)}\right)^{f+1}\left((\,v_\varepsilon\rho_1+1)-\frac{\omega^{n+1}(1-\omega^{b-n})}{1-\omega}\right)\right.$$
$$\left.+\sum_{n=f}^{\infty}\left(\frac{\rho_0{}^2}{\omega(\rho_0+1)}\right)^{f+1}\omega^n\right]P_{0,0}(0) \quad (3.3.3)$$

Now, the probability that the count of units in the system between f and a - 1 can be expressed as

$$P(f\le n\le a-1)=\sum_{n=f}^{F-1}P_{1,n}(0)+\sum_{n=f+1}^{a-1}P_{1,n}(1)+\sum_{n=f}^{F-1}P_{0,n}(0)+\sum_{n=f+1}^{a-1}P_{0,n}(1)$$

$P(f \leq n \leq a-1) = TP_{0,0}(0)$ (3.3.4)

Where,

$$T = \left[\sum_{n=1}^{F-1} \frac{\rho_0\, v_\varepsilon{}^{n+1}}{\left(\rho_{0\, v_\varepsilon}+1\right)^n} + \sum_{n=f}^{a-1}\left(\frac{\rho_0{}^2}{\omega(\rho_0+1)}\right)^{f+1} \omega^n \right.$$
$$+ \sum_{n=1}^{F-1}(\, v_\varepsilon\rho_0+1)\left\{1-\left(\left(\frac{v_\varepsilon\rho_0}{(\, v_\varepsilon\rho_0+1)}\right)^F + \left(\frac{v_\varepsilon\rho_0}{(\, v_\varepsilon\rho_0+1)}\right)^{n+1}\right)\right\}$$
$$\left. + \sum_{n=f+1}^{a-1} \frac{1}{v_\varepsilon\rho_1}\left(\frac{(v_\varepsilon\rho_0)^2}{\omega(\, v_\varepsilon\rho_0+1)}\right)^{f+1}\left((\, v_\varepsilon\rho_1+1) - \frac{\omega^{n+1}(1-\omega^{b-n})}{1-\omega}\right)\right]$$

Now,

Conditional probability $P(0|f \leq n \leq a-1)$ that this system is in a faster arrival rate when the size of the system lies between f and a - 1 is given by

$P(0|f \leq n \leq a-1) = \frac{\sum_{n=f}^{a-1}\left(P_{0,n}(0)+p_{1,n}(0)\right)}{TP_{0,0}(0)}$ (3.3.5)

Now,

Conditional probability $P(1|f \leq n \leq a-1)$ that this system is in a slower arrival rate when the size of the system lies between f and a -1 is given by

$P(1|f \leq n \leq a-1) = \frac{\sum_{n=f}^{a-1}\left(P_{0,n}(1)+P_{1,n}(1)\right)}{TP_{0,0}(0)}$ (3.3.6)

The expected count of consumers utilizing this system L_s is indicated by the sum L_{s_0} - The expected count of units or customers utilizing this system when the rate of arrivals is faster and L_{s_1} - The expected count of units or customers utilizing this system when the rate of arrivals is slower.

$L_s = L_{s_0} + L_{s_1}$

Where,

$L_{s_0} = \sum_{n=0}^{F-1} n\left\{P_{0,n}(0) + P_{1,n}(0)\right\}$ (3.3.7)

$L_{s_1} = \sum_{n=0}^{\infty} n\left\{P_{1,n}(1) + P_{0,n}(1)\right\}$ (3.3.8)

Now, from Little's formula to this model, the expected waiting time of the consumer utilizing the system can be computed by

$w_s = \frac{L_s}{\bar{\lambda}}$ (3.3..9)

Where, $\bar{\lambda} = \lambda_0 \left(P_{0,n}(0) + P_{1,n}(0)\right) + \lambda_1 \left(P_{0,n}(1) + P_{1,n}(1)\right)$

4. Numerical Illustrations

In this section, the queuing system is numerically and graphically illustrated with the values of L_s and W_s for various values of λ_0, λ_1, μ and ε. Using the above-obtained equations for each value.

Let f = 4, F = 8, a = 10 and b = 15

λ_0	8	8	8	8	8	8	8	7	6	5	8
λ_1	6	6	6	5	5	5	5	5	4	3	6
μ	10	12	14	12	12	12	12	10	10	9	9
ε	0.5	0.5	0.5	0	0.25	0.75	1	0.25	0.5	0.5	0.5
L_s	62.00602	54.85204	50.02303	56.06007	55.74404	55.07304	54.71309	57.03205	51.54805	49.31501	66.82208
W_s	0.60202	0.59033	0.50755	0.6009	0.60101	0.5983	0.59073	0.68081	0.77075	0.92014	0.63046

Table 1.

Figure 1: Ls and Ws corresponds to service rates, ϵ, parameters other than ϵ are unaltered.

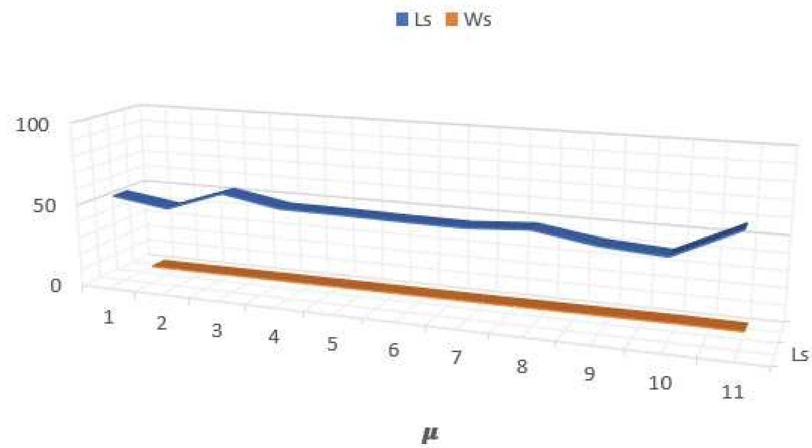

Figure 2: Ls and Ws corresponds to dependence rate, **μ**, parameters other than **μ** are unaltered.

5. Conclusion

This study offers a new approach to managing predictable arrival rates that takes into account interdependencies between the arrival and service processes as well as faster and slower rates in bulk service. Based on faster and slower arrival rates, probabilistic outcomes and related features are identified, offering important information for further research. This model demonstrates the ability to participate in mathematical modelling of real-world circumstances, which could be a direction for future research. This specific model functions as a foundational framework for the assessment and development of similar queuing models of this type. The earlier iterations are included in this model as particular cases. For example, this model reduces to the M/M/1/K when the value of b reaches 1 with finite capacity. When $\lambda_0$approaches $\lambda_1$and $v, \boldsymbol{\varepsilon}$ equals zero, the interdependent queueing model with controllable arrival rate by M. Thiagarajan and A. Srinivasan also relates to the classic M/M(a,b)/1 model as stated by J Medhi (2006) in Stochastic Models in Queueing Theory. The graph illustrates that numerically. When the service rate rises, L_s and W_s fall while all other parameters stay constant. While L_s and W_s remain unchanged, the average dependency rate rises. In essence, this research serves as a link between controlled arrival rates and bulk services

6. References

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